Visualizing the Effects of Measurements and Logic Gates On Multi-Qubit Systems Using Fractal Representation

Mate Galambos
Department of Telecommunications
Budapest University of Technology and Economics
Budapest, Hungary
e-mail: mate.galambos@mcl.hu

Sandor Imre
Department of Telecommunications
Budapest University of Technology and Economics
Budapest, Hungary
e-mail: imre@hit.bme.hu

Abstract—**Visual representation is essential to share ideas, interpret previous achievements or formulate new algorithms quickly and intuitively. Fractal representations of multi-qubit systems can visualize individual qubits even in case of entanglement. The proposed representation can be used to easily determine measurement probabilities. Connections with density matrices for pure and mixed states are also discussed. Finally, we visualize the effects of several single-qubit gates and controlled gates.**

Keywords - Quantum information; representation; visualization; fractals; binary trees

I. INTRODUCTION

Quantum computing and communications already promises applications that outperform classical solutions, e.g. Shor's prime factorization [2], the unconditional security of quantum cryptography [3], or practical realization of quantum communication [4]. It is also likely that this discipline will become even more important during the up-coming years. However, quantum mechanics is well-known for its counterintuitive nature that is hard to visualize thus making it problematic to quickly share ideas, interpret previous achievements, or formulate new algorithms quickly and intuitively.

In order to be able to solve these issues, a visual representation could be useful. The Bloch-sphere sufficiently represents one qubit [5] [6], or more qubits that are separable, but entanglement—one of the most important phenomena in quantum informatics—eludes this type of visualization.

Another possible approach is to use objects that have enough degree of freedom to represent the whole system. However, this method usually conceals the inner structure, and does not give us an idea of what happens if we measure the state of few qubits instead of the whole system—a method used in many algorithms and protocols. This approach does not handle well those cases where the addition of more qubits is decided or when dividing the system into smaller parts.

There are existing methods to generalize the Bloch-sphere e.g. through a mathematical structure called Hopf-Fibrations [7], but the arising geometrical structures are vastly complex and hard to read, thus making the method useless as a visualization technique.

An ideal visualization scheme would preserve the mathematical structure of a multi-qubit system in a way that is easy to interpret by the naked eye using compact and two dimensional images. The ideal solution should also give at least some insight to the states of single qubits, would work for any finite number of qubits, as well as it should show entanglement. Our work aims to examine the properties of such a scheme based on fractals with emphasis on the effect of measurement and logic gates [1].

This paper is organized as follows: Section II, III present the new proposed approach using fractals in single and multi-qubit states while and IV generalizes to non-binary multipartite quantum systems. Section V discusses the measurement, while Section VI explores the question of changing the order of qubits. Section VII and VIII focus on the connection with density matrices and possible representation of mixed states. Section IX and X discuss the effect of single-qubit and controlled gates in terms of the fractal representation. Finally we conclude the paper in Section XI.

II. REPRESENTATION OF A SINGLE QUBIT

For the sake of clarity, we begin with the single-qubit representation and the case of multiple qubits will be derived from these results.

The general form of a single qubit can be formulated by means of complex-valued probability amplitudes in exponential form and orthogonal basis vectors as:

$$|\varphi\rangle = A \cdot \exp(i \cdot \alpha) \cdot |0\rangle + B \cdot \exp(i \cdot \beta) \cdot |1\rangle, \quad (1)$$

$$A^2 + B^2 = 1. \quad (2)$$

Where A B α and β are real numbers. Let us draw a horizontal bar shown in Fig 1. Using a vertical gray line let us divide it into a black and a white side with respective lengths of A^2 and B^2 where the total length of the stripe is considered 1. This should give the probabilities of a measurement on the qubit producing the value 0 or 1. To avoid ambiguity, the black part of the bar corresponding to the measurement yielding 0 should always be placed first, and the white part corresponding to the measurement value 1 should placed second, thus representing them in ascending order.

A gray frame is added to the bar so that the white part can be easily seen in front of a white background. If the phase angle is zero or equivalent to zero due to 2π periodicity the horizontal line representing the phase information is considered to be behind the grey frame and is not visible.

A is proportional to the z coordinate of the Bloch vector and the difference $\beta-\alpha$ is proportional to the azimuth angle of the Bloch vector. As these values are close to each other for close quantum states in the Bloch representation, the closeness of widths of the bars and heights of the lines representing the phase indicate fidelity.

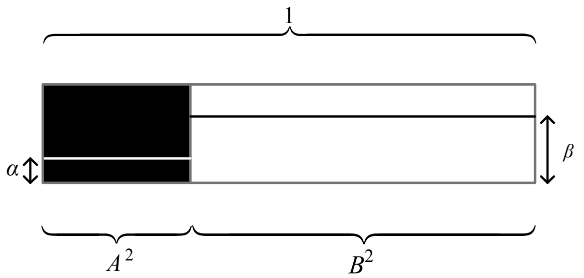

Figure 1. Representation of a single qubit. The respective lengths A^2 and B^2 of the black and white sides of the bar correspond to the probability of a measurement on the qubit yielding the bit value 0 or 1.

III. Representation of Tensor Products and Multi-Qubit Systems

Distributivity allows more than one way to mathematically formulate certain multi-qubit states, as it is illustrated in Equation 3.

$$\left(a|0\rangle + b|1\rangle\right) \otimes c|0\rangle \equiv ac|00\rangle + bc|10\rangle \tag{3}$$

The left hand side of the equation will be referred as separated, the right hand side as expanded form of the tensor product. Each formulation can be visually represented in a different way. In the following section, an introduction is given to both representation, and the connection between them will be clarified.

A. Representing Expanded Tensor Products

In case the state of the multi-qubit system is given in the form of an expanded tensor product, the probability amplitudes can be written in exponential form. The system can be represented as series of columns, each column consisting of black or white bars stuck upon each other as shown in Fig. 2. The colors of the bars represent the qubit values from top to bottom, the width of the column the probability of the state corresponding to those values, and a horizontal line dividing the lowermost bar of the column the phase. This means we associate only one phase to every bit value combination. As in the single qubit case if the phase angle is 0°, the horizontal line is not visible.

The quantum system as a whole can be represented by placing these columns next to each other in ascending bit value order and merge those neighboring bars that has the same bit value and phase. This merging step produces one bar with black and white parts for the first qubit, two bars for the second etc. because of the ascending order of qubits. Since the lowermost bars are the most likely to have differently colored neighbors they are the most logical place for the lines indicating the phase.

$$|\varphi\rangle = C_1\exp(i\gamma_1)|00\ldots0\rangle + C_2\exp(i\gamma_2)|00\ldots1\rangle + \ldots + C_n\exp(i\gamma_n)|11\ldots1\rangle$$

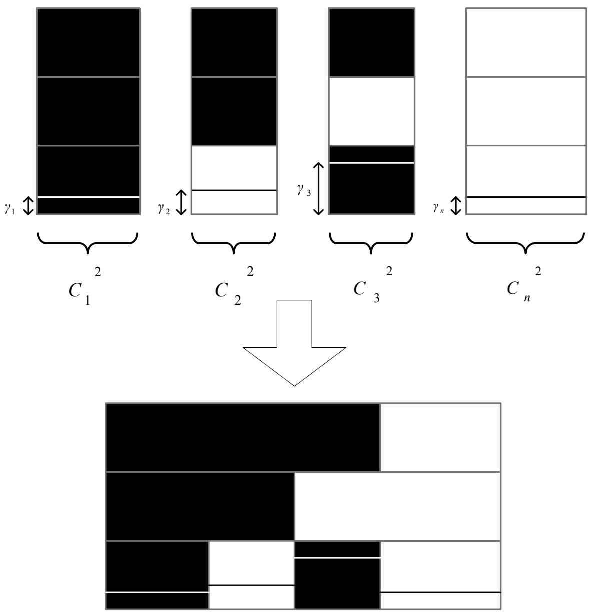

Figure 2. Representation of a multi-qubit system. Columns of black and white bars corresponding to the expanded tensor product describing the system. Probability amplitudes are written in Eulerian form, the width of each column is given by the square of the Eulerian amplitude and horizontal lines added to the lowermost bars to indicate the phase. The color of the bars in the columns will be determined by the bit values, from top to bottom black corresponding to 0 and white corresponding to 1.

B. Representing Separated Tensor Products

In order to represent separated tensor product of single qubits, the scaled down version of the bar representing the qubits should be copied under each black and white halves of the previous qubits as shown in Figure 3 [1]. This can be useful when the tensor product of known single qubit states have to be calculated.

If the system can be described as a separated tensor product of groups of inseparable qubits, then instead of single bars the representation of expanded tensor products should be copied under each other.

C. Connection Between the Representation of Expanded and Separated Tendor Products

The representations of separated tensor products are very similar to the expanded tensor products the only difference being the position of the lines indicating the phase. This follows from the definition of the two representations and the properties of the tensor product. Thus the representation of the separated tensor product can be transformed to the representation of expanded tensor product by copying the horizontal lines to the lowermost bars and adding their heights taking 2π periodicity into consideration practically adding the phase angles of the qubits.

If the lowermost bars inherit the phase information, properties of the system as a whole can be read from the representation, while the non inheriting form makes it easy to make conclusions regarding the phase of the subsystems.

The representation of expanded tensor products can be transformed to the non inheriting representation of separated tensor products by reversing the process. This should be done by ensuring that the bars or groups of bars having the same phase are exact copies of each other as described in Section III B. If this step cannot be done that is an indication of the qubits being inseparable. Finally, in both cases the resulting structures are statistically self similar with the bars serving as unit fractal objects.

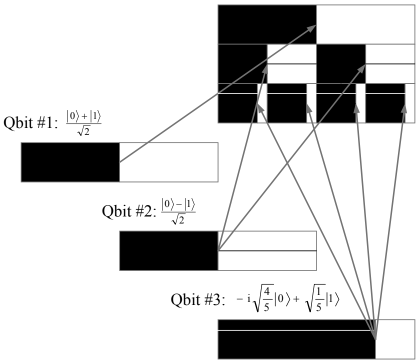

Figure 3. Fractal representation of a multi-qubit system and the separable qubits that serve as its building blocks. Note that after copying the phase information to the lowermost bars and adding their heights the same picture arises as from the representation of the expanded tensor product described in Section III A.

IV. Generalization for Multipartite System

In case of a multipartite quantum system whose parts are not qubits, but quantum systems with a small number of states, the representation can be generalized to describe this non-binary system. For example: three particles each with four excitation state and a ground state.

To represent the extra states, more bars with different colors should be added to the representation. These bars are packed above each other to create columns whose width represents the probability of a measurement finding the system in a certain state, while horizontal lines in the lowermost bars with complementary color [8] to the color of the bar represent the phase of the state.

In the example of the three particles with the five states each, the color black should be assigned to the ground state, white to the first excitation state, red green and blue to the second third and fourth excitation state. In this case a column whose colors from top to bottom are green, black and blue, with one half width and an orange horizontal line in the middle of the blue bar, means 50% probability of a measurement on the whole system finding the first particle in second excitation state, the second particle in ground state and the third particle in fourth excitation state while the phase of the total system is -1.

Since the color grey is it is own complementary color it should not be used for bars, only for the frame around the bars.

V. Conditional Probablilities

Probabilities of a measurement performed on the system as a whole yielding certain bit values can be read from the width of bars. However if measurements are performed on individual qubits, conditional probabilities can be read from the representation and changes introduced by the measurement can be anticipated. For this the qubits should be ordered from top to bottom in the order of the measurement.

Using the column vector formalism an n-qubit state has writes as:

$$|\varphi\rangle = \begin{bmatrix} C_1 \exp(i\gamma_1) \\ C_2 \exp(i\gamma_2) \\ \vdots \\ C_n \exp(i\gamma_n) \end{bmatrix} \tag{4}$$

and the state of the system after the measurement is shown in (5).

$$|\varphi'\rangle = \frac{M|\varphi\rangle}{\sqrt{\langle\varphi|M|\varphi\rangle}} \tag{5}$$

If the first qubit is measured and the measurement corresponds to one of the states used as the basis than the matrix of the measurement on the whole system can be written in the form of Equation 6 and 7.

$$M = \begin{bmatrix} m_0 & 0 \\ 0 & m_1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

$$M = \begin{bmatrix} m_0 & & & & & 0 \\ & \ddots & & & & \\ & & m_0 & & & \\ & & & m_1 & & \\ & 0 & & & \ddots & \\ & & & & & m_1 \end{bmatrix} \tag{7}$$

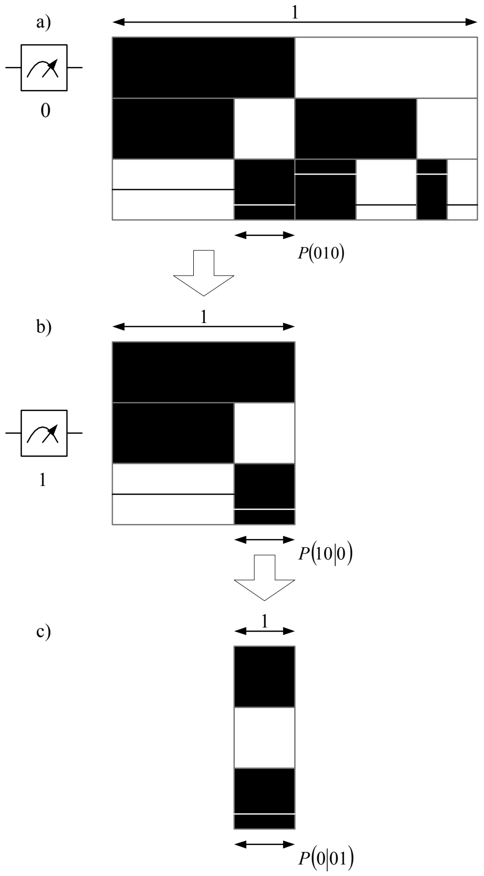

Figure 4. The effect of the measurements on single qubits. If we measure the first qubit in the system described by the fractal representation in part (a) of the figure and the measurement yields a zero then after the measurement the half under the black part of the first row shown in part (b) will describe the system. If the second qubit is measured and the measurement yields a one, then after the second measurement the system will be described by the half under the black part of the second row shown in part (c) of the figure. Thus the same width compared to the width of the different fractals in (a) (b) and (c) that are all considered unit length will give us the conditional probabilities of measurements on the rest of the system after the first few qubit was measured and found in certain states. The white arrows connect individual steps.

where either m_0 is 1 and m_1 is 0 or m_0 is 0 and m_1 is 1 depending on which basis vector was detected. Since the vector elements from top to bottom correspond to the columns of the representation from left to right, and

$$M\left|\varphi\right\rangle = \begin{bmatrix} m_0 C_1 \exp(i\gamma_1) \\ m_0 C_2 \exp(i\gamma_2) \\ \vdots \\ m_1 C_n \exp(i\gamma_n) \end{bmatrix} \tag{8}$$

this means if the measurement on the first qubit yields a zero, the half under the black part of the uppermost bar have to be examined. If the measurement yields a one, then the half under the white part will be significant as shown in Fig. 4. Taking the self similarity of the representation into consideration these halves also describe single or multi-qubit systems that will correspond to the rest of the system after the measurement is performed on the first qubit. To give the correct probabilities for these subsystems, their widths should be considered unit length according to the denominator in Equation 5. The width of individual columns will represent the conditional probability of the measurement on the rest of the system yielding the values represented by the colors of the bars.

After the measurement, the first qubit, which is now in a classical state, can be separated from the system and this logic can be recursively applied to the following qubits to get the conditional probabilities for the rest of the system after the first *n* qubit was measured.

VI. Changing the Order of Qubits and Recognizing Interchangeable Qubits

In some cases the question 'whether two qubits are in the same state or not' can be interesting. If the system is represented in a way corresponding to the separated tensor product, then two qubits can be determined to be in the same state if the bars representing them are the scaled versions of the same single qubit as shown in Fig. 5.

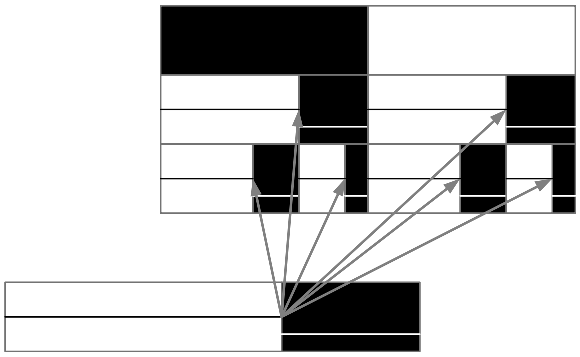

Figure 5. Since the second and third rows corresponding to the second and third qubits are consisting of scaled copies of the same single qubit representation, they are interchangeable.

If the representation is as described in Section III A, two qubits can be determined to be interchangeable if after changing the bit order the same fractal representation arises as shown in Figure 6.

The bit order can be changed by the following steps. First, changing the two lines of bars representing the two qubits, then determining the columns that make up the fractal representation by cutting the representation up at every point where two bars meet, copying the phase information to the lowermost bars, and finally reordering the columns so that the bit values represented by them are in ascending order and merging them in a way described in Section III A to form a new fractal representation.

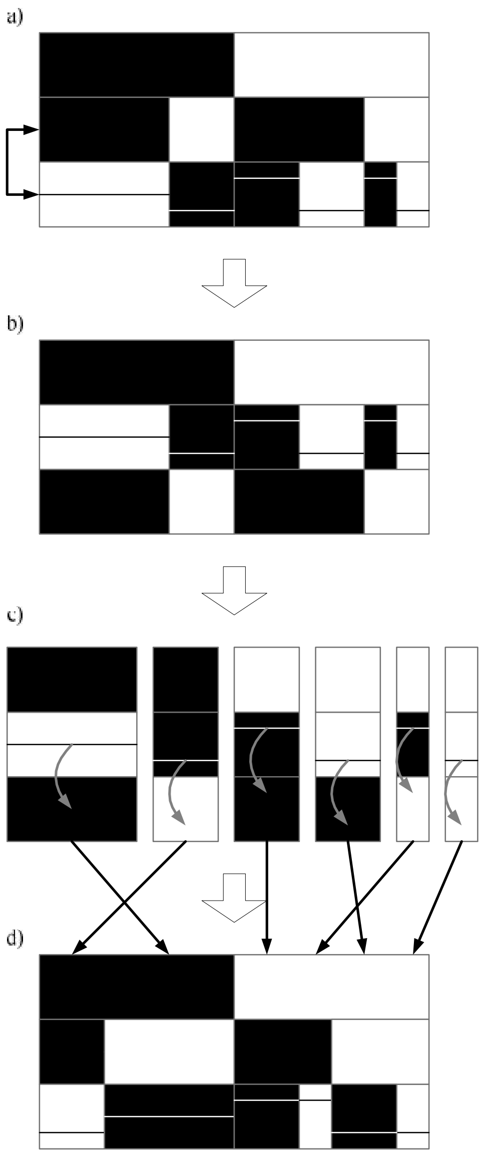

Figure 6. To change the order of two qubits, first the order of the corresponding rows have to be changed as shown in part (a) and (b) of the picture. After that the columns making up the representation have to be identified, and the phase information copied in the lowermost bars as shown by the gray arrwes in part (c). Finally, the columns have to be rearranged to ascending order as shown by the black arrows and the neighboring bars with the same color and phase remerged. Since the vector representations in this example before and after the reordering look differently, the two qubit was not interchangeable.

VII. Connection With the Vector Representation of Complex Numbers and the Density Matrix

In this section, the connections with other representations will be explained. Although using the absolute value square of the probability amplitude for the widths has its advantages, often the complex values of the probability amplitudes have to be represented in a vector form. The phase angle and thus the angle in polar coordinates can be read from the representation however for the length of the vector the square root of the columns have to be calculated. In a purely geometrical approach, this can be constructed using a parabola shown in Fig 7., whose equation is

$$y = x^2 . \tag{9}$$

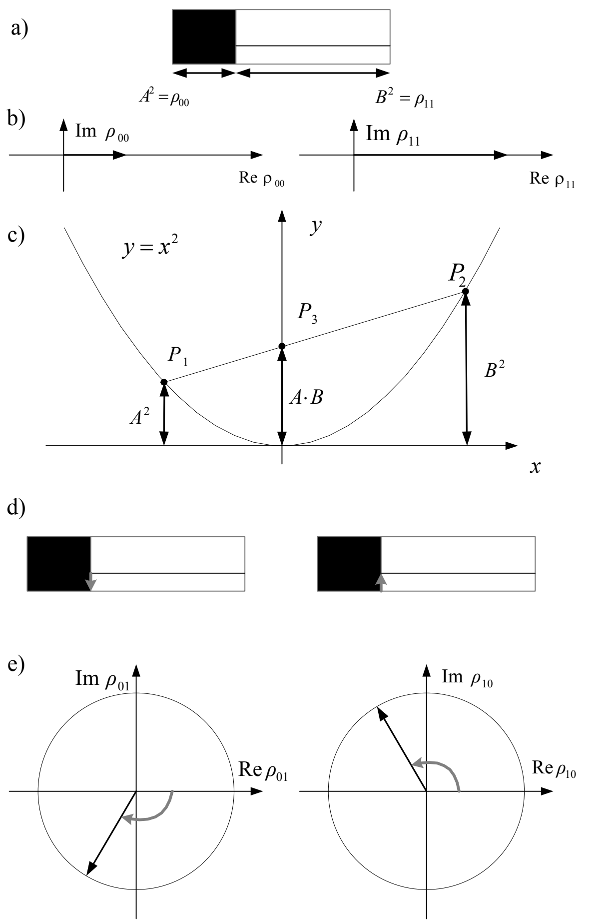

Figure 7. Geometric steps to determine the vector representation of the elements in the density matrix. The widths of the bars in part (a) are used represent the diagonal elements shown in part (b). A parabola shown in part (c) can be used to draft the squares square roots and products of certain lengths. The phase difference shown in part (d) will provide the angle of the vectors in part (e) corresponding to offdiagonal elements of the density matrix. The length of the vectors are given by the distance between the origin and P_3 in part (c).

This parabola can also be helpful if product of lengths or the square root of their product has to be calculated. It is easy to show that the points P_1 P_2 and P_3 are collinear, whose Cartesian coordinates are:

$$P_1 = \left(-A; A^2\right) \tag{10}$$

$$P_2 = \left(B; B^2\right) \tag{11}$$

$$P_3 = (0;AB) \qquad (12)$$

This method can be useful when information regarding the density matrix is needed. The width of the columns will correspond to the elements in the main diagonal of the density matrix, and off-diagonal elements can be calculated from the difference in the heights of the horizontal lines representing the phase angles, and the square root of the product of the columns widths. As mentioned previously, this can be geometrically achieved by the following steps: 1, measuring them on the *y* axis, 2, projecting these heights onto the parabola, 3, connecting the resulting points with a line as shown in Figure 7.

From the density matrix of pure states, the fractal representation can be created using the elements in the main diagonal as widths of the columns and the negative phase angles of the first row in the density matrix as heights of the horizontal lines.

Since the vector representation of the quantum system cannot always be constructed from the density matrix, the fractal representation has a one-to-one correspondence only to the vector representation but not to the density matrix.

VIII. Representation of Mixed States

The fractal representation of these pure states can be used to represent the mixed state, if the mixed state is described by an ensemble of a small number of differing pure states.

$$|\varphi_{21}\rangle = |00\rangle \qquad |\varphi_{22}\rangle = |11\rangle$$

$$\rho_2 = P_1|\varphi_{21}\rangle\langle\varphi_{21}| + P_2|\varphi_{22}\rangle\langle\varphi_{22}|$$

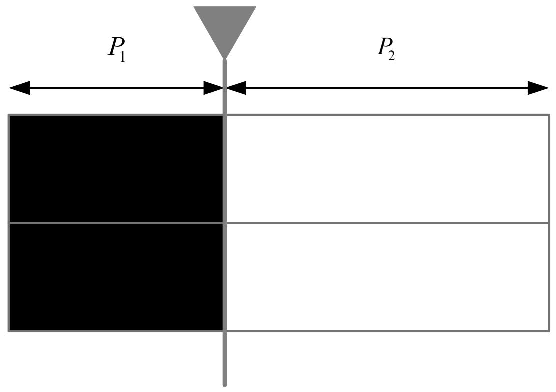

Figure 8. Representation of a mixed state that can be described as an ensemble of quantum systems in two possible pure states. The widths of the representations of pure states are scaled down by the factor of their weight in the ensemble.

The representation can be created by scaling the width of the fractal representation of each pure state by the factor of their probability in the ensemble and drawing them next to each other as shown in Fig. 8. The pure states are separated by grey lines extending above and below the fractal representation. For the sake of visibility, a gray triangle is added above the points where the representations of pure states meet.

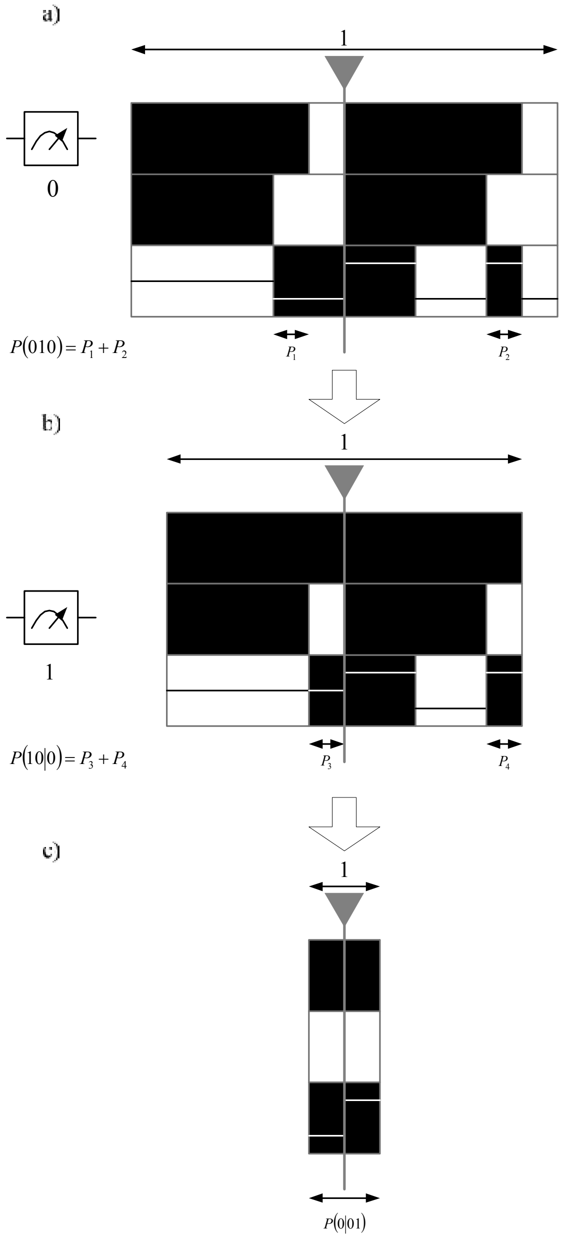

Figure 9. If a measurement is performed on the mixed state as a whole, the probability of the measurement collapses it into a certain state equals the sum of all the widths of the columns whose colors correspond to the state. If measurements are performed on single qubits, the logic described in Section V has to be applied to all of the representations of pure states separately, and the conditional probabilities will be given by the combined widths of all the columns with the corresponding colors. Note the similarities and differences between Figure 4 and Figure 9.

If a measurement is performed on all the qubits in the ensemble, the probability of a measurement yielding a string of zeroes or ones equals the combined widths of all the columns whose color corresponds to the bit values in the string.

If the qubits of the ensemble are measured one by one, then the logic described in Section V has to be applied to all the fractal representations of pure states, and the conditional probability on the ensemble will be given by the combined widths of all the columns corresponding to the strings of bit values (see in Fig. 9.).

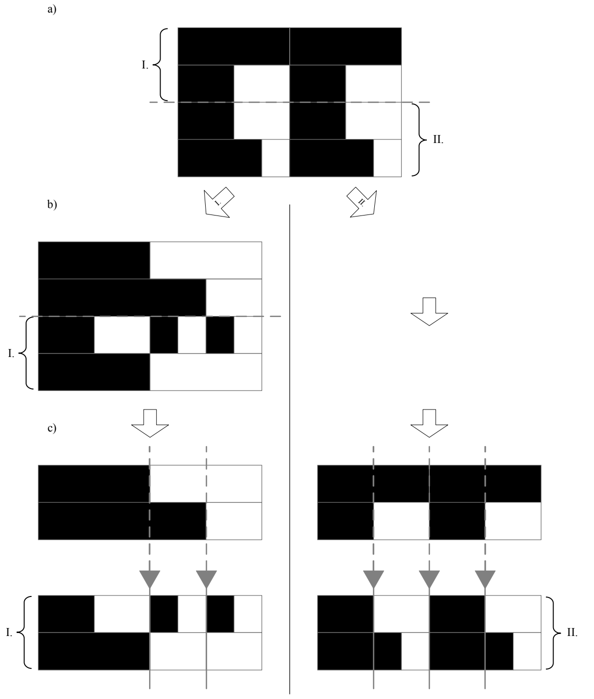

Figure 10. It seems that if a multi-qubit system is divided into two subsystems the parts that have to be handled as they are part of the same pure state are determined by color of the other qubits above them. For this the reordering of the qubits shown in part (b) is necessary for subsystem I since its qubits are not on the bottom of the representation. The same is not true to subsystem II.

The density matrix can be easily constructed from the fractal representation of mixed states. The steps are the following: 1, creating the density matrices of individual pure systems as described in Section VII., 2adding up the same matrix elements weighted by their probability in the ensemble.

Although we do not have a general proof yet, it seems that the subsystem of an entangled pure multi-qubit system can only be represented by copying the bits of the subsystem in the fractal representation below the rest of the qubits and handling them as if they would represent different pure states if the bars above them have the same colors.

It seems that if the pure multi-qubit system has to be divided into two or more subsystems that are all need to be examined, then as many copies with reordered rows of the original version of the fractal representation are needed as the number of subsystems (see in Fig. 10.).

This means that the usage of these extended grey lines could indicate more qubits not shown above the fractal representation whose bars meet where the extended lines indicate.

Because the density matrix is easily constructed from the fractal representation, the density matrix of subsystems seems to be created with this method without actually calculating the partial traces.

IX. Single Qubit Gates

In this section, the effects of the most common quantum gates are discussed in terms of the fractal representation.

A. Pauli X Gate

The Pauli X gate swaps the bit values thus effectively changing the color of the bars to the opposite. This means the bars should be rearranged with the purpose of satisfying the convention of ascending bit value order.

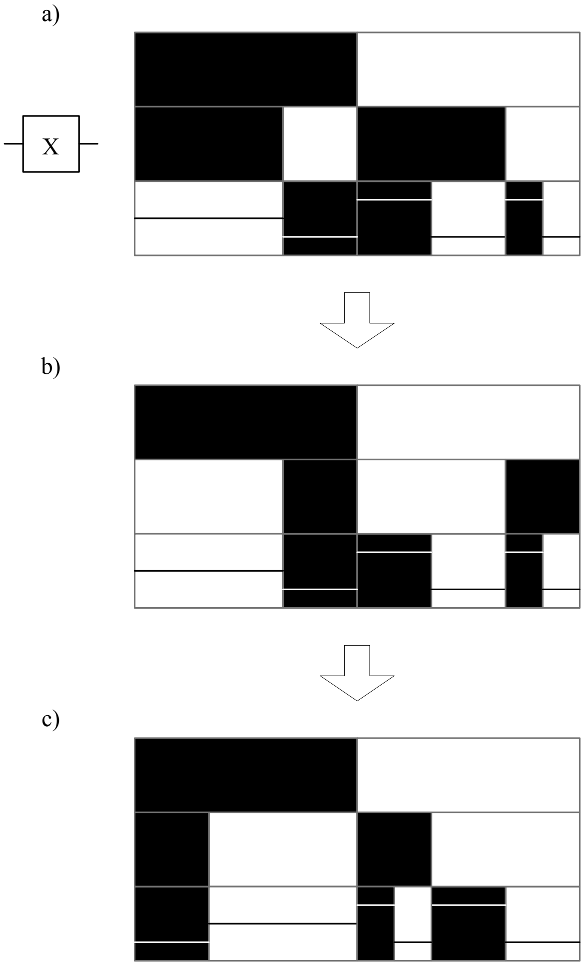

Figure 11. Effect of the Pauli X gate. All the colors of the bars in the row corresponding to the effected qubit are changed to the opposite. After the change an additional step of rearranging the columns in ascending order is required.

If the Pauli X gate affects one qubit in a multi-qubit system, the color of each bar in the row corresponding to the affected qubit has to be changed, followed by the step of reordering the columns and remerging the neighboring bars with the same color and phase (Fig. 11.).

B. Pauli Y Gate

The Pauli Y gate acting on a single qubit changes the color of the bars to the opposite, and shifts upward the horizontal line indicating the phase in the bar changing from black to white with three fourths of the bars height while the in the bar changing from white to black the shift is only one fourth. During the shifting, the 2π periodicity of the phase has to be taken into consideration. Since the color is changed, an additional step of reordering is necessary (see Fig. 12.).

In case of a multi-qubit system, this color change affects every bar in the row corresponding to the qubit, while the phase change affects the lines I the lowermost bars. A step of reordering and remerging the columns is also necessary.

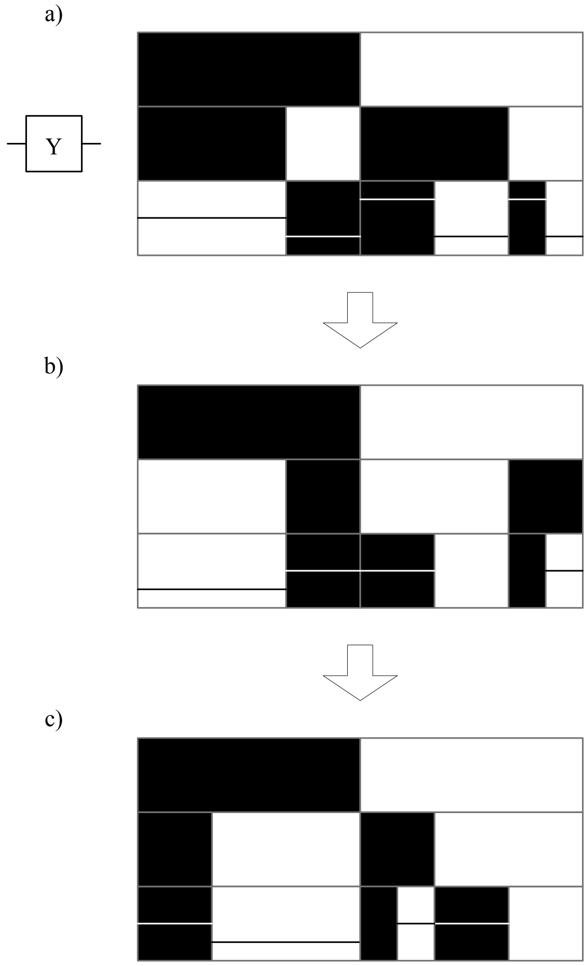

Figure 12. Effect of the Pauli Y gate. The colors in the row corresponding to the affected qubit change color and changes in the phase are introduced depending on the original color of the bars. Under black bars changing white the horizontal lines indicating the phase shift upward with three fourth of the bars height while under bars changing form while to black the shift is only one fourth. If the lines would shift above the bar the 2π periodicity has to be taken into consideration.

C. Pauli Z Gate

The Pauli Z gate does not change the color but shifts the line indicating the phase in the white colored bar upward with half the height of the bar. The 2π periodicity has to be taken into consideration, but reordering is not necessary.

In case of a multi-qubit system, the change will affect all the lines in the lowermost bars under the white bars in the row corresponding to the qubit as shown in Fig. 13.

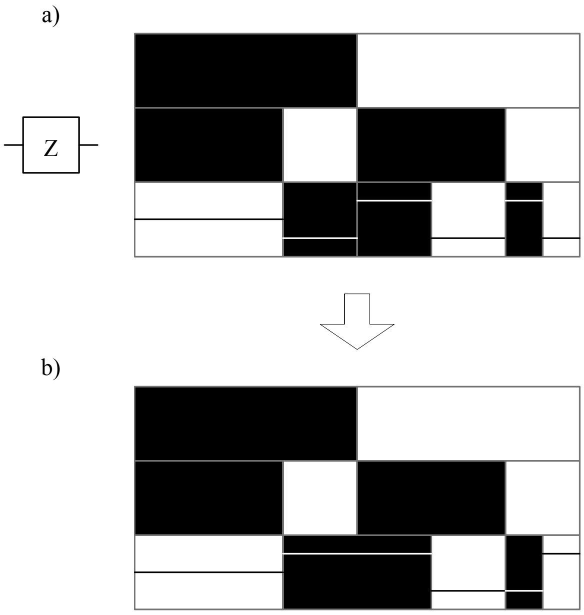

Figure 13. Effect of the Pauli Y gate. The color in the affected row remains unchanged while the lines representing the phase shif upwards with half the height of the bars below the white bars in row corresponding to the affected qubit. Since the colors are unchanged the additional step of reordering is not necessary but merging or cutting of bars can be necessary because of the phase changes introduced by the operation.

D. Hadamard Gate

The effect of the Hadamard gate on a single qubit can be easily calculated using the sum and difference of the probability amplitudes represented in vector form. This can be constructed using the method introduced in Section VII.

If a Hadamard operation is performed on one qubit of a multi-qubit system (shown in Fig. 14.), first the order of the qubits in the representation has to be changed so that the qubit affected by the operation becomes the lowermost. To perform the Hadamard operation, the lowermost bars should be grouped in way that those under the same colors are in the same group. Than the operation can be performed on each group in a way like they are all single qubits. Next the order of the qubits can be changed again, meaning the line representing the affected qubit does not have to be the lowermost.

a)

b)

c)

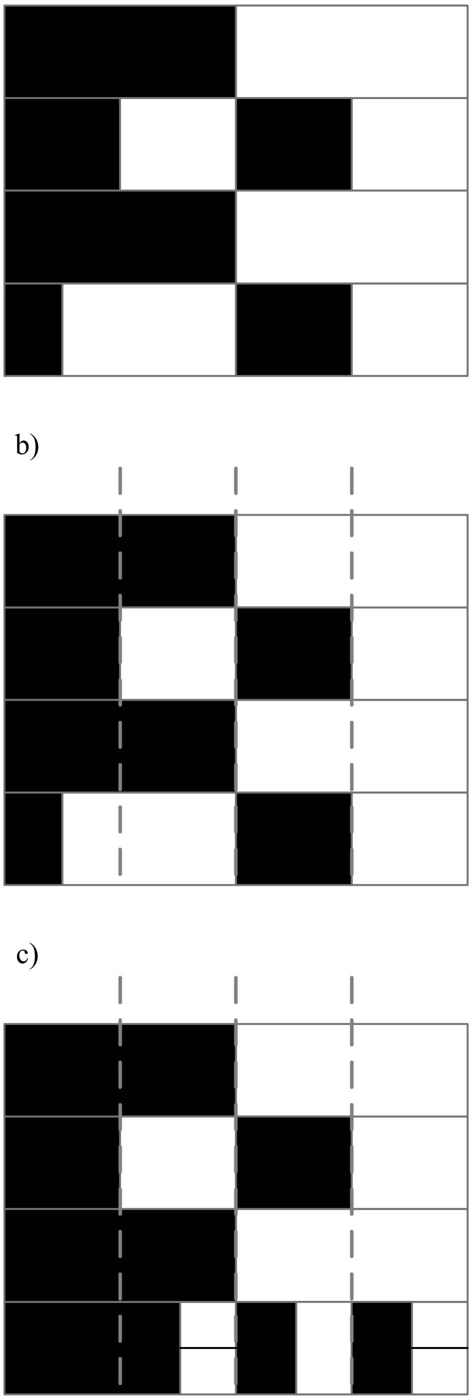

Figure 14. The effect of the Hadamard gate on the qubit corresponding to the lowermost row. The parts of bars in the lowermost row should be grouped together that are under the columns of bars with the same colors as shown in part (b). The Hadamard gate affects the bars of the lowermost row in the same group as if they are single qubits states as shown in part (c).

E. General Single Qubit Gate

If a generalized single qubit gate is given with the matrix of

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \quad (13)$$

then the effect of the gate on the last qubit can be described by the matrix

$$V = I \otimes \cdots \otimes I \otimes U =$$

$$= \begin{bmatrix} u_{11} & u_{12} & & & & & 0 \\ u_{21} & u_{22} & & & & & \\ & & u_{11} & u_{12} & & & \\ & & u_{21} & u_{22} & & & \\ & & & & \ddots & & \\ & 0 & & & & u_{11} & u_{12} \\ & & & & & u_{21} & u_{22} \end{bmatrix}. \quad (14)$$

Thus the effect on the state vector can be described as the original U gate affecting numerous single qubits with probability amplitudes like the neighboring ones in the state vector. Since only those neighbors are grouped together whose corresponding bit value only differs in the last digit, this means that in the fractal representation the gate affecting the last qubit acts as if it is acting on several single qubits, who constitute of the lowermost bars of columns whose color differs in only the color of the lowermost bar. In other words, the place where any two bars meet except for the lowermost bars marks the border of a group and the gate acts as if it is acting on the state described by the lowermost bars in the same group.

In case of those gates that have nonzero off-diagonal elements an additional step of reordering and remerging the columns is necessary. If the gate is acting on any other qubit than the last one then the operation can be executed by first changing the bit order and thus the order of the rows followed by the reordering and remerging of the columns, so that the qubit in question becomes the last one.

X. CONTROLLED GATES

In this section the effect of controlled quantum gates are discussed in terms of fractal representation.

A. CNOT Gate

The effect of the Controlled NOT gate is very similar to the effect on the Pauli X gate but only the color of those parts of the bars are changed in the row corresponding to the target bit that are above or below white colored bars of the row corresponding to the control bit (see in Fig.15.).

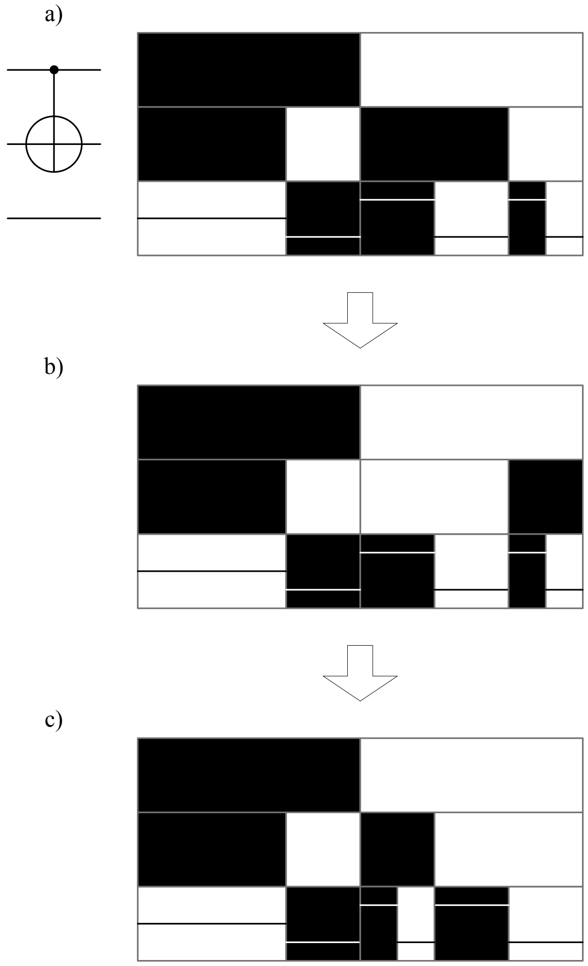

Figure 15. Representation of CNOT operation. The first qubit is the control bit and the second is the target. The effect is very similar to the effect of the Pauli X gate shown in Figure 11, but the color of the bars in the row of the target bit only changes in the columns where the color of the bars corresponding to the control bit is white. This property makes it easy to visualize how entanglement arises from a CNOT operation. Since the color of the bars are changing, an additional step of reordering shown in part (c) is required.

B. General Controlled Gate

A general controlled gate that affects the last qubit if and only if the value of the control bit is 1. In the fractal representation the effect will be similar to the uncontrolled version of the single qubit gate acting on the lowermost row, but only those parts of the bars will be affected that are under the white colored bars in the row corresponding to the control bit.

XI. CONCLUSION

Fractal representations can describe multi-qubits systems while providing insight to the state of individual qubits. In this paper, a possible generalization to non-binary multipartite quantum systems with finite number of discrete states has been presented.

By examining the effect of measurements on the whole system and on individual qubits, we concluded that conditional probabilities regarding measurements on one part of the system yielding certain qubit values after the rest of the qubits have been measured can be read by comparing the widths of the corresponding columns to the appropriate parts of the representation. The state of the system after the measurement yielding the given values is described by these parts.

We explained that the representations of the pure states can be used to represent the mixed state and measurements and operations act on the representation as if they are acting on separate pure states.

The effects of reordering the qubits and the connection with vector representation of complex numbers were discussed and used in examining the effects of certain logic gates. It has been concluded that single qubit operations on the qubit corresponding to the lowermost row act on the representation as if they are acting on single qubits described by a special grouping of the bars in the lowermost row. Controlled gates affect the qubit corresponding to the lowermost bar similarly but only those groups will change that are under white bars in the row corresponding to the control bit.

ACKNOWLEDGMENT

Special thanks to Laszlo Bacsardi and members of the BME Department of Telecommunications for their helpful comments.

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