Influence of Couple Patterns on Entropy in Multilayer Networks

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***Abstract*—Many real-world systems are networks coupled with other networks, and research on these multilayer networks about their structure properties and functions recently produced significant and remarkable findings. There is one type of multilayer network in which an individual has more than one counterpart on other layer network. For instance, important station on an infrastructure network usually has more than one supporter on its counterpart network for optional access or risk diversification. In this paper, we investigate the influence of couple patterns on information entropy and energy of two layer coupled networks with community structures. Couple patterns refer to the allocation of counterpart numbers according to nodes' degrees and the establishment of interconnections between nodes on different layer networks according to their degree of assortativity. Nodes' degrees and counterpart numbers are assorted based on the tendency of large degree nodes with more counterparts or the reverse. Additional, a pair of nodes on different layer networks can be interconnected according to their degree of assortativity. Under the scenario of a heterogeneous distribution of counterpart numbers, we have found that the influences of couple patterns on entropy and energy are negative. That is, entropy and energy of the two layer coupled network decrease when counterpart number assortativity and/or degree assortativity positively increase, while increase when the two assortativity are negatively enhanced. Moreover, networks with weak community structures extend these influences compared to networks with obvious community structures.**

Keywords-multilayer network; couple pattern; influence; entropy; energy.

I. INTRODUCTION

A large quantity of real-world complex systems are networks coupled with other networks, and recent studies on these complex systems are fruitful [1]-[13]. For instance, an infrastructure network[14][15], such as a power grid, is often coupled with water, gas, or other resource supply networks to turn various resources into power. Diseases spread in face-to-face networks, while the information of contagion spreads in the coupled online social networks[16]-[18]. The multilayer point of view can help find important layer network to improve the robustness of the whole bank networks. In[19], bank networks are composed of several layer networks which represent different types of exposure, such as unsecured overnight, unsecured short-term, unsecured long-term, secured short-term, and secured long-term. The authors found there is a high heterogeneity between these layer networks, and the unsecured overnight layer is the especially important layer network related to the stability of the overall network in a financial crisis. Researchers define the multidegrees, multilinks, cluster coefficients, and degree correlations to describe the properties of these multi-layer networks, and give deep insights into the dynamic processes of these coupled networks, such as percolation, spreading, and growth. In [14], authors investigated cascade failures on two layer coupled networks and modelled a true coupled system composed of a grid and internet. The effects of assortativity and cluster coefficient on the robustness of two layer coupled networks were clarified in [7]. These researches demonstrate that the dynamics of multi-layer networks relates directly to the topologies of these coupled networks. Among these cases, there is one kind of multilayer network in which nodes on one layer network have more than one counterpart on the other layer network. A node with multiple counterparts naturally brings advantage of optional access and adequate supply, and could disperse risk avoiding cascade of failures. The deep exploration of structures and functions of this kind of multilayer network appears in[20]. The author mathematically analysed percolation based on generating functions and concluded that multi-interconnections can significantly lower the percolation threshold. Besides several related studies, research work on this kind of multilayer network is still a little. More explorations should be carried out to look into the properties and dynamics of these complex systems. Spreading information across multilayer networks, however, always sparks interests of researchers in how to maximize information entropy [21]-[23]. Entropy in information theory characterizes uncertain sources of information. The larger the entropy is, the more random information sources are. In [24], we investigated how the overall entropy of a two layer coupled network varies based on the assumption that a group of nodes on one layer network have the same number of counterparts on the other layer network. In this paper, we explore the impacts of couple patterns on information entropy and energy in two layer coupled networks with community structures, especially in heterogeneous interconnection scenarios. Couple patterns refer to the allocation of counterpart numbers according to nodes' degrees and the establishment of interconnections between nodes on different layer networks according to their degree of assortativity. Counter number couple pattern couples nodes that have various numbers of counterparts. The counterpart number of a node is allocated according to node's degree. Nodes' degrees and counterpart numbers are assorted based on the tendency of large degree nodes with more counterparts or the reverse. Degree couple pattern interconnects a pair of nodes on different layer networks according to their degree of assortativity. In this work, the distribution of node's counterpart on the other network follows power law, and nodes interconnect with their counterparts according to the assortativity or disassortativity between them. We will find out how couple patterns affect entropy and energy when information flows on two layer coupled networks.

The outline of the paper is as follows. In Section II, we describe couple patterns in detail and establish the two layer interconnected network model. In Section III, we present the influence of couple patterns on entropy and energy. In Section IV, we give our conclusions.

II. THE MODEL

First, we use [25]-[27] to construct two standalone networks and set both of them to have the same number of nodes, number of communities, community size, and modularity strength. Then, we choose one random community in each network and establish interconnected links between nodes in the community. Figure 1 shows the illustration of this two layer couple network. As we can see, nodes in a community of the top network can interconnect several counterparts of the lower network. Their counterparts may be totally different or may partly overlap.

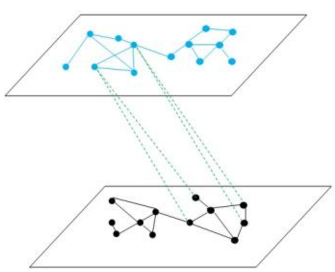

Figure 1. Illustration of a two layer coupled network model. Nodes in a community on one layer network interconnect with their counterparts on the other layer network.

Due to the fact that distributions of the wide variety of phenomenon in nature and man-made world follow power laws[28], a series of counterpart numbers that are distributed heterogeneously are generated. The tendency of assorting counterpart number with node's degree indicates couple strength between nodes of different layer networks. Positive assortativity means that large degree nodes have more counterparts than small degree nodes, while negative assortativity means that small degree nodes prefer to link more nodes on the other layer network. After the above two steps, we get a sequence of nodes' degrees and a sequence of nodes' counterpart numbers. Each node in the community will be allocated a certain number of counterparts from the counterpart number sequence according to the method proposed in [29]. In the simulation, we range assortativity from -0.05 to 0.1, which can significantly affect entropy.

On the other hand, the tendencies of interconnecting nodes in different layer networks are various. Large degree nodes may tend to couple with nodes similar to themselves, while the opposite is also possible. Two layer coupled networks are established after nodes of different layer networks interconnect with each other according to their degree assortativity. Couple patterns are made of counterpart number assortativity and/or degree assortativity. Then we use the method introduced in [24] to compute the entropy and energy expended in these two layer coupled networks in order to find out how they are affected by couple patterns. To describe our model completely, we retell this method. Nodes collect information most from their first neighbour nodes, second neighbour nodes, and their couple nodes on the other layer network. q_i is the frequency of node i spreading information, then we use

$$H(x) = -\sum_{i=1}^{n} q_i \log q_i \quad (1)$$

to compute entropy [30]. In order to determine about how much energy is expended, we first interpret asymmetry and similarity between two nodes. Asymmetry is defined according to the covariance of degree between two nodes, given by

$$A(x_{i,} x_j) = \left|(K_{x_i} - \mu)(K_{x_j} - \mu)\right| \quad (2)$$

where μ is the mean value of nodes' degrees of community x_i belongs to. Similarity describes common properties between a pair of nodes. In this paper, node similarity is quantified by the portion of common friends to total friends when two nodes are on the same network. If two nodes belong to different layer networks, then we regard their common friends as the portion of their mutual coupled neighbours to their total friends. For a pair of nodes that are asymmetrical and dissimilar to each other, it could be deduced that they need more energy in order to share information. Hence, we calculate energy by

$$E = \sum_{j \in N(i), Cou(i)} A_{i,j} (1 - S_{i,j}) \quad (3)$$

$N(i)$ and $Cou(i)$ are the neighbour set and counterpart set of node i. $S_{i,j}$ is the similarity between node i and j, defined by

$$S_{i,j} = \frac{N(i) \cap N(j)}{N(i) \cup N(j)} \quad (4)$$

We expect to find out how the allocation of counterparts and the establishment of interconnections among nodes affect information entropy and energy.

III. RESULTS

We construct a two layer coupled network with the first couple pattern in which there is only counterpart number assortativity (the assortativity between node's degree and its counterpart number) and observe how entropy varies with this couple pattern. Because correlations of entropy and couple pattern have no significant difference under values of counterpart number assortativity larger than 0.5, in the simulation, the counterpart number assortativity ranged from 0.05 to 0.5. Each node interconnects its counterparts on the other layer network regardless of degree assortative mixing between the mutually connecting nodes. Pearson correlation coefficients of entropy and the counterpart number assortativity are calculated as influences of couple pattern. As shown in the top of Figure 2, when counterpart number assortativity is positive, the influence is negative, and the absolute value increases with the absolute value of counterpart number assortativity. When counterpart number assortativity is negative, there is the same tendency. Hence, the overall influence of this couple pattern on entropy is negative. That is to say, if more counterparts are allocated to nodes with larger degrees, then the entropy of whole system tends to decrease. Conversely, if nodes with small degrees have more counterparts, then the entropy of whole system tends to increase. We also notice that weak community structure on each layer network stretches this effect when compared to networks with obvious community structures. The range of correlation is from 0.21 to -0.14 when modularity is 0.2, which is wider than the range from 0.16 to -0.06 if modularity is 0.8.

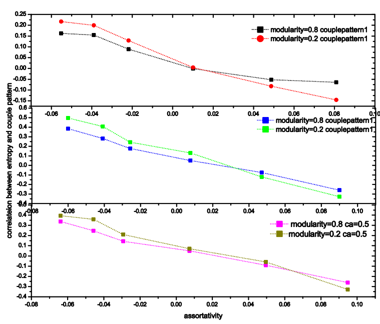

Figure 2. Pearson correlation coefficient of entropy and couple patterns. Degree assortativity between node pairs ranged from -0.08 to 0.1. Ca refers to counterpart number assortativity for short.

Next, we coupled two layer networks with a more complex couple pattern in which not only nodes' degrees and nodes' counterpart numbers are assorted but also nodes interconnect their counterparts by degree assortativity. The influence of this couple pattern on entropy when counterpart number assortativity is 0.05 is shown in the middle of Figure 2. Analogously, the influence is negative and entropy decreases when the degree assortativity between nodes is positive. Therefore, if nodes prefer to interconnect to nodes similar to themselves, then the entropy of two layer network system tends to reduce. The interconnection between similar nodes cannot produce large entropy, even if both of them have large degrees. However, the interconnection between dissimilar nodes can increase entropy. The influence also is stronger in networks with weak community structures than those with obvious community structures. This is similar to the couple pattern discussed previously. The correlations of entropy and the couple pattern cover broader ranges than those with the first couple pattern.

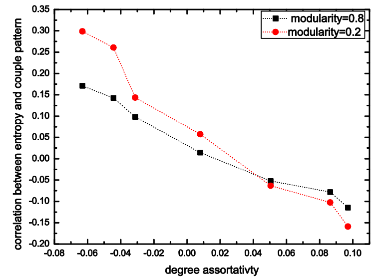

Figure 3. Pearson correlation coefficient of entropy and couple pattern as a function of degree assortativity. Network with weak community structure stretch the influence especially in disassortative degree mixing.

The influence of degree assortativity couple pattern on entropy when counterpart number assortativity is 0.5 is presented at the bottom of Figure 2. It matches our first result that counterpart assortativity has a negative effect on entropy. There is a positive correlation between a node's degree and its counterpart number, which lowers entropy; however, information entropy of a two layer coupled network is remarkably augmented as the result of adding degree assortative mixing between nodes into the couple pattern. Consequently, we can conclude that degree assortative mixing among interconnecting nodes is the dominant couple pattern affecting the entropy of the system. Therefore, we wonder what will happen if we extract the degree assortative mixing effect and allocate counterparts equally among nodes. It is straightforward that each node has only one counterpart on the other layer network. However, a node chooses its counterpart according to degree assortativity. In Figure 3, the influence holds the same tendency. Degree assortative mixing has a significant influence on entropy, and networks with weak community structures gain more entropy when nodes are disassortatively mixed.

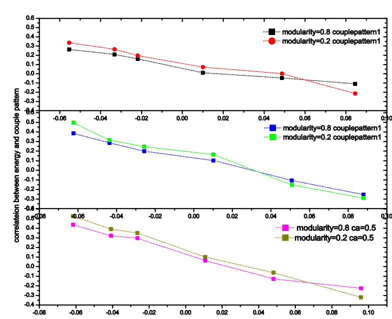

Figure 4. Pearson correlation coefficient of energy and couple pattern. The couple patterns are the same as Figure 2. The tendency is not the same but similar to that of entropy.

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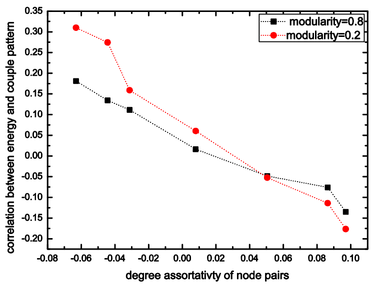

Figure 5. Pearson correlation coefficient of energy and couple pattern as a function of degree assortativity. The couple pattern is the same as Figure 3. The tendency is similar to that of entropy.

Consuming the least energy to gain information is often expected. Figure 4 shows energy expended under the above two couple patterns. It follows a similar tendency as entropy. In Figure 5, influences of couple pattern on energy are presented when each node has only one counterpart but there are degree assortative mixing between a pair of nodes. Compared to entropy, the Pearson correlation coefficients of energy and couple patterns are larger. Therefore, if we want to obtain much more diverse information, then the energy we consume is greatly affected by these couple patterns.

IV. CONCLUSION AND FUTURE WORK

In this work, we established two layer coupled network models and investigated the influences of couple patterns on entropy and energy from the aspect of counterpart number assortative mixing and degree assortative mixing. Under counterpart numbers of nodes are heterogeneously distributed scenario, a node's degree is assorted with its counterpart number in order to allocate counterparts according to node's degree. And nodes on one layer network interconnect their counterparts on the other layer networks according to degree assortative mixing among them. Couple patterns are made of one or both of the two assortative mixing couplings. We found that the influences of these couple patterns on entropy and energy are negative. Entropy and energy of the two layer coupled network decrease when counterpart number assortativity or/and degree assortativity positively increase, while increase when they are negatively enhanced. Furthermore, weak community structures stretch the influences that nodes have on networks that obtain more diverse information. Compared to the counterpart assortativity couple pattern, the degree assortativity couple pattern exerts more significant influences. We verified this phenomenon through extracting the effect of degree assortative mixing and allocating one counterpart to each node. Therefore, the degree assortativity couple pattern is the dominant pattern that affects the entropy and energy of the system greatly. Specifically, under degree assortativity couple pattern, networks with weak community structures gain much more entropy than those with obvious community structures if nodes are disassortatively mixed. Future deeper endeavours on this research should analysis available large data sets to explore more interesting new findings and propose a mathematical framework with which to theoretically support and predict the tendency of influence of corresponding couple patterns. Assortative or disassortative mixing in other nodes' properties needs to pay more attention. And particular assortative or disassortative mixing hiding only in special local structures is waiting to be discovered.

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