# Multiagent Genetic Optimisation to Solve the Project Scheduling Problem under Uncertainty

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Abstract—This paper considers a project scheduling problem under uncertainty, which belongs to a class of multiobjective problems of complex systems control whose decision search time grows exponentially depending on the problem dimension. In this paper, we propose a multiagent genetic optimisation method based on evolutionary and multiagent modelling by implementing different decision searching strategies, including a simulation module and numerical methods application. The comparative analysis of the scheduling methods has shown that the proposed method supports all features that might be useful in effective decision searching of the stochastic scheduling problem. The proposed multiagent genetic optimisation method, the MS Project resource reallocation method, and a heuristic simulation method were compared whilst addressing a real-world deterministic scheduling problem. The comparison has shown: firstly, the unsuitability of the MS Project planning method for solving the formulated problem; and secondly, both the advantage of the multiagent genetic optimisation method in terms of economic effect and disadvantage in terms of performance. Experimental results in conditions of uncertainty demonstrate the effectiveness of the proposed method. Some techniques to reduce the impact of the method's disadvantage are proposed in the conclusion, as well as the aims of future work.

Keywords-project scheduling; genetic algorithms; simulation; subcontract work optimisation; problem under uncertainty.

# I. INTRODUCTION

This paper is an improved and expanded version of the ICCGI 2013 conference paper "Multiagent Genetic Optimisation to Solve the Project Scheduling Problem" [1]. The paper extends the scheduling method proposed in the original paper by taking into account environment uncertainty removal with the help of the integration of numerical methods, simulation, multiagent, and evolutionary modelling. A comparison of the new method and existing scheduling methods is conducted in this paper. An application of the new method to a real scheduling problem is described.

The scheduling problem is one of the key problems in the management of organisational and technical systems. Inefficient scheduling can lead to financial losses, quality of service losses, and loss of competitiveness for the company. Companies with various different scopes are faced with the scheduling problem, for example, industrial and project companies, shopping centres, hospitals, and call centres.

There are several types of scheduling problem depending on the application sphere: operations calendar planning [2]– [6], assignment of limited resources to a set of tasks [7]–[9], and the travelling salesman problem [10].

Classical scheduling problem-solving methods have a number of disadvantages. Thus, the use of combinatorial methods and mathematical programming is associated with internal difficulties because the model of system processes is nonlinear, non-convex, and non-differentiable [11]. In addition, these methods are applied poorly to problems with dynamically changing constraints. Simulation takes into account the dynamic nature of the problem, but leads to a random search process, which does not guarantee optimal decision finding. The use of genetic optimisation allows the shortcomings of the previous methods to be overcome [10]. The application of genetic optimisation to the scheduling problem with defined constraints is widely considered in the literature [2]–[10].

In the real world, the scheduling problem is connected to the uncertainty of environment behaviour and is a stochastic version of the classical scheduling problem. It can involve many sources of uncertainty: activity duration, renewable resource availability, resource consumption, and cost of activity [11]–[16]. Mainly non-structural (parametric) uncertainty is introduced into the basic deterministic scheduling problem by researchers [17]. The design of the optimal (efficient) calendar work plan taking into account the structural uncertainty associated with the insertion of new projects is a topical task.

This paper focuses on the project scheduling problem under conditions of structural uncertainty by using evolutionary computation [18], simulation, and numerical methods of uncertainty removal [19]. The remainder of the paper is organised as follows. Section II provides an overview of the related works in the field of deterministic and stochastic scheduling. Section III formulates the deterministic project scheduling problem with time constraint. Section IV introduces the genetic algorithm based on an annealing simulation and novelty search. Section V describes a dynamic model of multiagent resource conversion processes that has been selected as a system formalisation model. Section VI presents the algorithm for the multiagent genetic optimisation program based on the integration of evolutionary computation and multiagent simulation. Section VII introduces the multiagent genetic optimisation method under uncertainty. Section VIII presents the algorithm of the multiagent genetic optimisation program under uncertainty based on the integration of the evolutionary computation, multiagent simulation, and numerical optimisation methods. Section IX presents a comparative analysis of the existing methods and the proposed method of solving the deterministic and stochastic scheduling problem. Section X evaluates the practical implementation of the multiagent genetic optimisation program to solve a real-world scheduling problem, both deterministic and stochastic. Section XI concludes this paper and explores future work.

## II. RELATED WORK

In general, the deterministic scheduling problem is connected to the problem of seeking an operations sequence that satisfies the constraints and optimises the objective functions. Renewable resources (such as staff or equipment) are usually considered when studying the scheduling problem. For certain tasks (for example, production planning) nonrenewable resources should be determined [2].

In the various scheduling problem studies different constraint sets are considered, depending on the specific task. Four constraint types were identified in [4]: resource, precedence, physical layout, and information constraints. The time constraint type should be added to the list of constraint types when analysing workflow inside a project development company. Time limitation is associated with having a time frame for the operations start date.

All constraints, except precedence ones, have been studied by Brezuliani et al. [7]. Precedence and resource constraints were considered by Okada et al. [2], Klimek [3], Abdel-Khalek et al. [5], and Dhingra and Chandna [8]. Resource and information constraints were studied by Yang and Wu [9]. Resource, precedence, and time constraints were considered by Karova et al. [6]. A study of scheduling with a resource constraint to determine a public transport route was presented by Osaba et al. [10].

The optimisation objects are different in the studies reviewed. The classical objective function of working time (makespan) minimization was considered by Sriprasert and Dawood [4], Osaba et al. [10], He and Wan [12], Zhang and Chen [13], Csebfalvi [14], and Artigues et al. [15]. The objective function of constraints violation penalty minimization has been considered by Karova et al. [6] and Yang and Wu [9]. Both mentioned objective functions were considered by Okada et al. [2], Brezuliani et al. [7], and Dhingra and Chandna [8]. The objective function of net present value of discounted cash flow maximization was considered by Chen and Zhang [16].

There are different ways of conducting an objective function evaluation: analytical methods, simulation, artificial neural networks, fuzzy systems, and component modelling. Analytical methods are the most widely used; the drawback of this approach is the lack of analysis of the dynamic behaviour of complex systems. This drawback is overcome by using the simulation model of Osaba et al. [10] to evaluate the objective function. The integration of evolutionary modelling and simulation can limit the random search space and enhance heuristic optimisation by taking into account the dynamically changing constraints of the scheduling problem.

The reviewed studies do not consider subcontracted workforce optimisation, while this problem is very real to developers and even to mass production enterprises. The optimisation problem of the subcontracted workforce is connected to scheduling subcontractors in order to maximise the utilisation of the company's own resources. In the literature, a problem regarding the appropriate selection of subcontractors using artificial intelligence methods was studied by Chen et al. [20]. A subcontract optimisation technique based on a simulation and heuristics has been suggested by Aksyonov and Antonova [21]. The current article considers new subcontract optimisation techniques for a deterministic scheduling problem with the use of a genetic algorithm.

An unexpected external influence may result in deterministic schedules becoming more expensive and longer than expected, or even becoming unfeasible. Many researches in previous years have been dedicated to solving the stochastic scheduling problem. They have analysed different non-structural (parametric) sources of uncertainty, such as the examination of renewable resource availability and resource consumption by He and Wan [12] and the cost of activity by Chen and Zhang [16] and Xu and Feng [11]. A stochastic activity duration analysis was applied by all the authors [11]–[16].

There are three groups of methods for solving the stochastic scheduling problem: predictive, proactive, and reactive methods [17]. Predictive methods ignore uncertainty, so the predictive schedules can be late, over the budget, or even become infeasible. Proactive methods are intended to construct a predictive schedule that will perform well under a wide variety of external situations. Reactive methods are intended for online scheduling at the time of job execution, incorporating up-to-date information, and changing the schedule when disruptions take place [22].

Proactive methods are the most popular in researches on the stochastic scheduling problem. The main idea of proactive methods is to distinguish the two decision searching stages: the stage of the uncertainty removal and the stage of the deterministic problem solving. The direct order of the stages is used in most of the researches. The following techniques of uncertainty removal are considered by different authors: two-stage algorithm based on chanceconstrained programming by He and Wan [12], 99-methods by Zhang and Chen [13], heuristic algorithm with forbidden sets and forward-backward list scheduling by Csebfalvi [14], and Monte-Carlo simulation by Chen and Zhang [16]. The reverse order of the stages was used by Artigues et al. [15]. In this research, first, the initial schedule is found by solving the deterministic equivalent of the stochastic problem obtained by replacing the uncertain parameters with their average values. Second, the schedule is modified by inserting buffer times into the schedule to discourage the propagation of schedule disruptions.

The reviewed studies do not consider the structural uncertainty associated with the insertion of new projects into the schedule, while this problem is real in small and medium-sized enterprises. The current article considers a new proactive scheduling method under structural uncertainty with the use of a genetic algorithm, simulation, and numerical methods.

## III. DETERMINISTIC PROBLEM STATEMENT

Let us consider the deterministic problem of project scheduling aimed at the calendar planning of operations. All project operations have to be carried out in combination with a set of time constraints. The set of time constraints is defined through negotiations with customers. In the case of the organisation's own lack of resources, subcontracted resources have to be involved to meet the time constraints.

The objective functions of the considered problem are: 1) subcontract cost minimization and 2) minimization of total downtime of own resources. The second objective function is associated with the fixed labour costs in the project companies. If the salaries are fixed then downtime is also paid, which is not profitable for the company.

For the project scheduling problem considered in this study, the following assumptions have been made:

1. A single project consists of a number of operations with a known processing time, early and late start dates, labour input, and labour cost.

2. The operation requires the availability of renewable resources (own or subcontracted workforces).

3. Nonrenewable resources are not considered.

4. Operations cannot be interrupted.

5. Subcontractors can be involved in performing part of the operation.

6. Subcontractors can be interrupted and the operation can continue with the use of the company's own resources in the event of the reappearance of its own available resources.

7. Subcontractors are available every day on request in unlimited quantities.

Let us describe the problem of project portfolio scheduling with the use of the following designations.

- Indices:
  - *i*: project index, i = 1, 2, ..., P.
  - *j*: operation index,  $j = 1, 2, ..., Op_i$ .
  - w: department index, w = 1, 2, ..., V.
  - *t*: time index, t = 0, 1, 2, ..., T.

#### Decision variables:

TB(i,j): set of start dates of operations.

## Initial parameters:

ES(i,j): early start date of the operation (i,j).

LS(i,j): last start date of the operation (i,j).

 $SL_w$ : number of persons in the department w.

SLO(i,j,w):amount of workforce (persons) needed in department *w* to fulfil the operation (i,j).

# SS(i,j): operation (i,j) subcontracting cost per day. Parameters obtained in the decision-making process:

Active(i,j,t): a sign of the operation (i,j) execution at time t.

 $Active(i, j, t) = \begin{cases} 1, & \text{if operation } (i, j) & \text{is executed at the moment } t \\ 0, & \text{otherwise} \end{cases}$ 

RD(t,w): resources from department *w* demanded to fulfil the active operations at the time *t*.

$$RD(t,w) = \sum_{i=1}^{P} \sum_{j=1}^{Op_i} \left[Active(i, j, t) \cdot SLO(i, j, w)\right]$$

VF(t,w): amount of free workforce of department w at the time t.

$$VF(t,w) = \begin{cases} SL_w - RD(t,w), & \text{if } RD(t,w) \le SL_w \\ 0, & \text{otherwise} \end{cases}$$

 $V_{SC}(i,j)$ : volume of subcontracted workforces on operation (i,j).

**Problem description:** 

$$OF_{1} = \sum_{i=1}^{P} \sum_{j=1}^{Opi} (SS(i, j) \cdot V_{SC}(i, j)) \to \min, \qquad (1)$$

$$OF_2 = \frac{\sum_{t=0}^{T} \sum_{w=1}^{V} VF(t,w)}{T \cdot V} \to \min, \qquad (2)$$

$$TB(i, j) \in [ES(i, j); LS(i, j)] \quad \forall i, \forall j$$
(3)

Objective function (1) minimises the total subcontracting cost. Objective function (2) minimises the total downtime of own resources. Constraint (3) maintains the time frame of the operations' start.

### IV. GENETIC ALGORITHM BASED ON ANNEALING SIMULATION AND NOVELTY SEARCH

The genetic algorithm (GA) is one of the evolutionary approaches that can be used to solve complex system management problems in a short time [18]. The technique of the GA application includes the following steps: 1) selecting the method of encoding the problem decision (phenotype) into a chromosome (genotype); 2) definition of the evaluation method of the chromosome fitness function (FF); 3) the genetic operator's description; and 4) the initial population generation and GA work. The modification of the GA on the basis of an annealing simulation and novelty search is considered in the article in order to enhance the quality of the decisions on the scheduling problem.

#### A. Chromosome Encoding

There are various techniques for decision encoding presented in the literature: operations sequence encoding [3][6][10], operations precedence encoding [2][4], operations start dates encoding [5][7], and encoding of resource assignment for the operation [7]–[9]. We used the encoding of the shifting of the operation start dates because this technique supports time constraints, is not redundant, and is simple to implement.

The GA chromosome encodes the operations' start dates, shifting from the initial work plan to the right or left on the time axis via binary code (0/1). The shift range is two weeks on either side of the initial operation start date. The chromosome size is 5·*r* genes, where *r* is the number of analysed operations, 5 is the number of genes needed to encode a single operation shifting (4 genes to encode  $2^4 = 16$ shifting days and 1 gene to encode the shifting direction).

#### B. Genetic Algorithm Modification

The concept of a novelty is a major GA concept. This concept is connected with the emergence of new elements and interactions in the environment during evolution. Two novelty types are distinguished in [23]: 1) *combinatorial novelty*, when the new species emerge by combining the existing species; and 2) *creative novelty*, when the new species are not reproducible by a combination of the species. The validity of the fundamental feasibility of the second novelty type is still open.

Let us consider the case of a combinatorial novelty search as an adaptation mean in an open system. To implement this approach we modify a simple GA by introducing the concept of "decision originality" as a measure of the decision fitness to the environmental conditions [23]. The decision-chromosome's originality in the population is determined via the numerical transformation of the Hamming distance matrix.

Let us define the Hamming distance matrix as follows:

$$H = (h_{ij})_{i=1, j=1}^{N},$$
(4)

where  $h_{ij}$  is the Hamming distance between the *i*-th and *j*-th chromosomes ( $Ch_i$  and  $Ch_i$ ), equal to the number of positions, at which the corresponding gene values are different in chromosomes  $Ch_i$  and  $Ch_j$ ; N is the number of chromosomes.

We associate the matrix H with the matrix of originality weights W defined as follows:

$$W = \left( w_{ij} \right)_{i=1, \, j=1}^{N}, \tag{5}$$

where  $w_{ii}$  is the weight of the corresponding value of the Hamming distance determined as a quadratic function, increasing in the range from 1 to *R* as element  $h_{ij}$  is changed in the range of 0 to *L*:

$$w_{ij} = \frac{R-1}{\sqrt{L}} \cdot \sqrt{h_{ij}} + 1, \qquad (6)$$

where *L* is the chromosome size, and *R* is the maximum weight of the chromosome in the pair, R > 0.

The two strategies of chromosome crossing have been described using the concept of originality. The first strategy – *the originality search strategy (OSS)* [24] – focuses on the combinatorial search for the new decisions in the population by crossing chromosomes that have different encodings. The second strategy – *the maximum search strategy (MSS)* [18] – focuses on the targeted search for the best chromosomes by crossing chromosomes that are the most adapted to the

environment. The fitness of the *i*-th chromosome to the environment is evaluated by the fitness function  $FF_i$ , i=1...N.

Let us define the chromosome crossing probability matrices on the basis of the proposed strategies as follows:

$$P_{oss} = \left(p_{ij}^{oss}\right)_{i=1,j=1}^{N}, \ p_{ij}^{oss} = \frac{w_{ij}}{\sum_{j=1}^{N} w_{ij}},$$
(7)

$$P_{MSS} = (p_i^{MSS})_{i=1}^{N}, \ p_i^{MSS} = \frac{FF_i}{\sum_{i=1}^{N} FF_i}.$$
 (8)

In formulas (7) and (8) the matrices' cells are filled by probability values in accordance with the roulette law [18]. In the case of the OSS strategy, the weight of the chromosome originality serves as a measure of chromosome importance. In the case of the MSS strategy, the chromosome FF serves as a measure of chromosome importance.

An annealing simulation algorithm (ASA) [25] is intended to implement the proposed chromosome crossing strategies during the GA work. This algorithm is based on the analogy of the metal annealing process, which results in the appearance of new metal properties. The technique for ASA and GA integration is proposed below.

Step 1. Set the annealing simulation algorithm parameters: the initial value of the parameter  $t_Z$ ; the value of the parameter  $\alpha$ , which controls the rate of annealing temperature decrease,  $0 \le \alpha \le 1$ .

Step 2. Set the GA parameters: the number of generations *K*; the chromosome size *L*; the likelihood of the genetic operators being applied. Set the number of the current population *Z*: Z = 1. Generate the initial population.

Step 3. Apply the genetic operators to the current population *Z* with a probability that depends on the value of parameter  $t_Z$ . Increase the number of the current population Z = Z + 1. Change the value of parameter  $t_Z$  [25]:

$$t_{Z+1} = t_Z + \alpha \cdot t_Z \,. \tag{9}$$

Step 4. Check the condition: Z > K. If the condition is satisfied then go to Step 5, otherwise return to Step 3.

Step 5. Stop.

The probability of the genetic operator's application is defined on the basis of the annealing simulation in order to reflect the operator's dynamic nature.

#### C. Crossover Operator

The probability of selecting the first and second parents from the current population Z for the crossover operator (CO) is described below. The probability of selecting the first parent has to take into account both random selection and targeted selection based on the MSS strategy (8). The probability of random selection should be reduced in the population's evolution, and the probability of the MSS strategy should be increased. This fact is reflected in the probability of selecting the first parent i in the population Z:

$$P_i^{Z}(CO) = \frac{1}{N} \cdot \left( 1 - \exp\left(-\frac{1}{t_z}\right) \right) + p_i^{MSS} \cdot \exp\left(-\frac{1}{t_z}\right).$$
(10)

The probability of selecting the second parent has to take into account the OSS and MSS strategies. The probability of the OSS applying (7) should be reduced during the population's evolution, and the probability of the MSS applying (8) should be increased. This circumstance is reflected in the probability formula for selecting the second parent *j* for the first parent *i* in population *Z*:

$$P_j^Z(CO) = p_{ij}^{OSS} \cdot \left(1 - \exp\left(-\frac{1}{t_z}\right)\right) + p_j^{MSS} \cdot \exp\left(-\frac{1}{t_z}\right). \quad (11)$$

#### D. Mutation and Inversion Operators

The applied probability of the mutation operator (MO) in population Z is described below. This formula has to take into account the probability reducing during evolution in order to save genetic material [24]:

$$P_{Z}(MO) = P_{0}(MO) \cdot \left(1 - \exp\left(-\frac{1}{t_{Z}}\right)\right), \qquad (12)$$

where  $P_0(MO)$  is the initial value of the mutation operator applied probability.

The applied probability of the inversion operator in population Z is described by analogy with the mutation operator applied probability.

## E. Fitness Function

The following fitness function considers both objective functions (1) and (2) described in Section III:

$$FF = \omega_1 \cdot (\frac{OF_1^{\text{lnit}}}{OF_1}) + \omega_2 \cdot (\frac{OF_2^{\text{lnit}}}{OF_2}) \to \max, \quad (13)$$

where  $\omega_1$ ,  $\omega_2$  are weight coefficients,  $\omega_1+\omega_2=1$ ;  $OF_1^{Init}$ ,  $OF_2^{Init}$  are objective function initial values obtained by expert evaluation of the operation start date.

Used FF is described with the use of the linear convolution of normalised heterogeneous criteria (1) and (2).

## V. THE DYNAMIC MODEL OF MULTIAGENT RESOURCE CONVERSION PROCESSES (MRCP)

The processes of the project's work execution have to be formalised via a simulation model in order to evaluate the objective function values. The use of multiagent approach at the stage of business process model formalisation is caused by the presence of decision makers in the system; their behaviour is motivated, they cooperate with each other, and accumulate knowledge of the problem domain. The problem of the model selection for business process formalisation was addressed by the authors in [26]. The following models for supporting agent representation of business processes were considered: Gaia model, Bugaichenko's model, Masloboev's model, simulation model of intelligent agents interaction (SMIAI), resource-activity-operation model (RAO), and multiagent resource conversion processes model (MRCP). The comparison showed that the MRCP model has the fullest functionality in area of business process formalisation: it includes a hybrid agent model (intelligent and reactive), a model of a resource converter and a queue system, allowing the analyst to examine the dynamic features of processes [26].

Let us consider the basic principles of the MRCP model.

The MRCP model [27] was developed on the basis of resource conversion process (RCP) model [28] and it targets the modelling of business processes and decision support for management and control processes. A key concept of the RCP model is a resource converter consisting of input, launch condition, conversion module, control block, and output.

The launch condition, once it becomes true, enables the conversion process to take place based on the state of input resources, control commands, available conversion tools, and other external environment events. Conversion time becomes known right before the start of the conversion process as a function of the control commands and active resources limitation.

The MRCP model may be considered as an extension of the basic RCP model, adding the functionality of agents. The main objects of the discrete multiagent RCP are: operations (Op), resources (Res), control commands (U), conversion devices (Mech), processes (PR), sources (Sender) and junctions resource receivers (Receiver), (Junction), parameters (P), agents (Agent), and coalitions (C). Process parameters are set by the object characteristics function. Relations between the resources and conversion device are set by the link object (*Relation*). The agents and coalitions' existence assumes availability of the situations (Situation) and decisions (action plan) (Decision).

The MRCP model has a hierarchical structure, defined by high-level integration system graphs. Agents control the RCP objects. Every agent includes a unique model of a decision maker. The agent (software or hardware entity) is defined as an autonomous artificial being with active and motivated behaviour, capable of interacting with other objects within a given virtual environment. With every system tick the agent performs the following operations [27]: environment (current system state) analysis, state diagnosis, knowledge base access (knowledge base [KB] and database [DB] interaction), and decision-making. Thus, the functions of analysis, situation structuring and abstraction, as well as the control commands generation of the resource conversion process are performed by agents.

Consequently, coalition is generated after the union of several agents. Figure 1 shows an example of  $C_1$  coalition formation after the union of  $A_2$  and  $A_3$  agents.

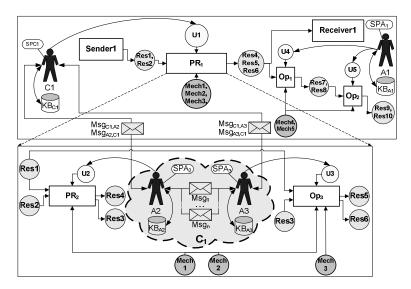


Figure 1. Interaction and coalition formation.

Agent coalition has the following structure:  

$$C = \langle Name, \{A_1, \dots, A_m\}, G_C, KB_C, M\_In, M\_Out, SPC, Control O >.$$

where *Name* is coalition name;  $\{A_1, ..., A_m\}$  is a collection of agents forming a coalition;  $G_C$  is coalition goal;  $KB_C$  is coalition knowledge base;  $M\_In$  is a collection of incoming messages;  $M\_Out$  is a collection of outgoing messages; SPC is a collection of behaviour scenarios acceptable within coalition; *Control\_O* is a collection of controlled objects of the resource conversion process.

The simulation algorithm of the agent-containing model comprises the following main stages: system time determination, agent and coalition actions processing (state diagnosis, control commands generation), conversion rules queue generation, conversion rules execution, and operation memory state (i.e., resources and mechanism values) modification. The simulator makes use of the expert system unit for diagnosis of situations and generation of control commands.

Each agent possesses its knowledge base, set of goals needed for the behaviour configuration setting, and priority that defines agent order in control gaining queue.

The following agent behaviour rules structure was used in the resource conversion processes subject area:

## Name <Rule Name>

**If** *<Message Conditions, RCP Conditions, G\_Ag Conditions>* 

**Then** <*G*\_*Ag* Changes, Message Actions, Private Actions>,

where *Message Conditions* are message-related conditions; *RCP Conditions* are resource conversion process-related conditions;  $G_Ag$  Conditions are goal-related conditions;  $G_Ag$  Changes are agent current goals modifying actions; *Message Actions* are message generation actions; *Private Actions* are converters and resource-related actions (activity plan), targeting the achievement of set goals.

The MRCP agent behaviour rules have been developed on the basis of the special-purpose object-oriented Reticular Agent Definition Language (RADL) in the form of When-If-Then [29].

Generally, in the case of any situation corresponding to the agent's activity, the agent tries to find a decision (action scenario) in the knowledge base or work it out itself according to the existing behaviour rules, makes a decision, controls goals' achievement, delegates the goals to its own or another agent's RCP objects, and exchanges messages with others.

The MRCP model was chosen to evaluate the chromosome FF value (13). The decision variables and input parameters described in Section III are fed in the model input. The parameters obtained in the decision-making process are the model output. In the MRCP model, we use agents to implement the resource allocation algorithm and use simulation to perform the operation's execution. The resource allocation algorithm is described in [23] and allows executors of operations to be appointed in accordance with the assumptions made in Section III.

#### VI. MULTIAGENT GENETIC OPTIMISATION PROGRAM

One of the software development problems when addressing the implementation of the proposed scheduling method is the choice of modelling tool. The modelling tool should support the RCP, multiagent, and expert models' description and have built-in, object-oriented development tools in order the additional tool functions to be developed by the systems analyst (programmer).

#### A. Comparative Analysis of Modelling Systems

Let us consider the following modelling systems: simulation modelling tools PlantSimulation (P) [30] and Simio (S) [31], particularly real-time expert system G2 (G) [32], multiagent simulation systems AnyLogic (A) [33], RepastJ (R) [34], MagentA (M) [35], and BPsim (B) [26]. The results of the comparative analysis of these tools are presented in Table I.

Comparison criteria	Р	S	G	Α	R	Μ	В				
RCP modelling availability											
Subject area conceptual model	0	0	0	0	0	•	•				
design	Ŭ	Ŭ	Ŭ	Ŭ	Ŭ	•	•				
RCP description language	٠	٠	٠	•	٠	•	٠				
Hierarchical process model	•	•	•	•	0	0	•				
Use of natural language for	0	0	•	0	0	0	0				
model definition			•	_	0	0	0				
Multiagent r	nodelli	ing ava	ilabili	ty	-						
"Agent" element	0	•	0	•	•	•	•				
Agents behaviour models	0	•	0	•	•	•	•				
Agent's knowledge base	0	0	0	0	0	•	•				
support	0	0	Ŭ	0	0	•	•				
Message exchange language	0	0	0	•	•	•	•				
Other modelling	g techr	iiques	availal	bility	-						
Simulation modelling	0	0	٠	•	0	0	•				
Expert modelling	0	0	•	0	0	0	•				
Situational modelling	0	0	•	0	0	0	•				
Evolution modelling	0	0	0	0	0	0	0				
Object-o	rienteo	l appro	oach								
Use of UML language	0	0	0	0	•	0	•				
Object-oriented programming	•	•	•	•	•	0	•				
Wizard technology for agent	0	0	0	0	0	0	•				
design	0	0	0	0	0	0	•				
Object-oriented simulation	0	0	•	•	0	0	•				
Subject area conceptual model											
and object-oriented simulation	0	0	0	0	0	0	•				
integration											

TABLE I. ANALYSIS OF THE MODELLING TOOLS

As we can see, all the current systems lack the support of some features that might be useful for effective simulation. For example, the problem domain conceptual model design and agent-based implementation approach is limited. Also, some systems, e.g., AnyLogic and G2, require users to have several programming skills. So, for a non-programming user there is no system that can offer a convenient description of a multiagent resource conversion process. Again, AnyLogic and G2 make use of high-level programming language, which results in these products being highly functional.

Not all of the systems support the evolutionary modelling methods. Only the BPsim system includes wizard technology for agent design and tools for the integration of the subject area conceptual model and object-oriented simulation. As a result of the comparison of modelling systems, the BPsim system was selected as the basis for the proposed scheduling method implementation because the system supports the development and integration of the intelligent agents (wizards) with the object-oriented simulation using a common database.

#### B. BPsim Agent Architecture

The BPsim system consists of the following subsystems: BPsim.MAS dynamic situations modelling system and BPsim.MSN technical and economic development system [26]. BPsim.MAS supports the MRCP model (including reactive agents) description via graphical notation of the resource conversion processes. BPsim.MSN ensures the development of the decision search information technology (intelligent agent) based on the UML sequence diagrams [36] and Transact-SQL database management language [37]. So, the BPsim agent may have a hybrid nature, and it contains two components:

- Intelligent (agent is described via a decision search diagram defined on the UML sequence diagram).
- Reactive (agent is described via production rules and/or frame-based expert system).

Two main agent architecture classes are distinguished [38]:

1. Deliberative agent architecture based on artificial intelligence principles and methods, i.e., knowledge-based systems.

2. Reactive architecture based on a system reaction to external environment events.

The currently existing architectures cannot be defined as purely behavioural or purely knowledge-based, and any designed architecture is hybrid, offering features of both types.

Multiagent resource conversion process architecture, which is implemented in the BPsim, is based on the InteRRaP architecture [39], as the most appropriate for the problem domain.

In accordance with InteRRaP architecture's common concept, the BPsim agent model is represented in four levels (Figure 2).

1. The subsystem of cooperation with other agents corresponds to the following MRCP elements: converters, resources, tools, parameters, and goals. The subsystem of cooperation performs the following actions: generates tasks, transfers messages between agents, processes agent commands (performs resource conversion), and alters the current state of the external environment (transfers situation  $S_n$  into state  $S_{n+1}$ ).

2. The external environment interface and reactive behaviour components are implemented in the form of an agent rules base and inference machine (simulation algorithm).

3. The reactive subsystem performs the following actions: receives tasks from the external environment, places tasks in a goal stack, collates the goal stack in accordance with the adopted goal ranging strategy, selects a top goal from the stack, and searches in the knowledge base. If the appropriate rule is located, the subsystem transfers control to the corresponding resource converter from the external environment. Otherwise, the subsystem queries the local planning subsystem.

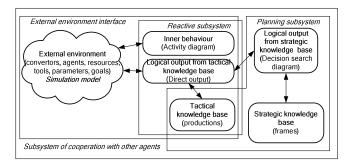


Figure 2. BPsim agent hybrid architecture.

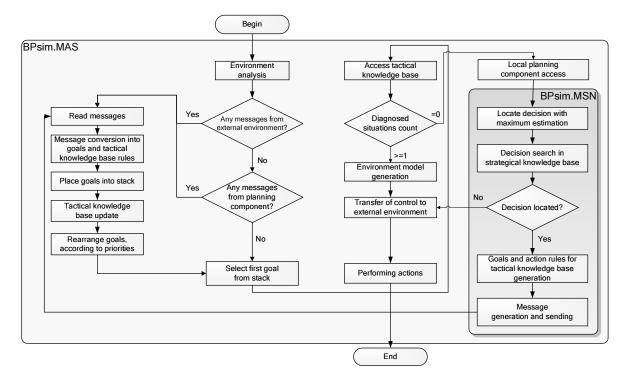


Figure 3. Hybrid BPsim agent activity algorithm.

4. The local planning subsystem's purpose is to search effectively for decisions in complex situations (e.g., when goal achievement requires several steps or several ways of goal achievement are available). The local planning component is built on a frame-based expert system. The frame concept and conceptual graph-based approach are utilised for knowledge formalisation.

5. The scheme presented on Figure 3 shows the interaction of separate units during agent activity within BPsim.MAS and BPsim.MSN.

The problem domain conceptual model and agent knowledge base design are based on the UML class diagram extension. Semantically, this notion may be interpreted as the definition of the full decision search graph, containing all the available ways of goal achievement (pre-defined by experts). The current knowledge base inference machine is implemented in the decision search diagram, based on the UML sequence diagram. Each decision represents an agent activity plan. Each plan consists of a set of rules from a reactive knowledge base. Based on the located decision, the current agent plan is updated. Examination of all available options contained in the knowledge base generates an agent plans library.

If an agent, when processing a task or message received from the external environment, is unable to locate the appropriate rule in its knowledge base (e.g., select an option from several ones), the reactive behaviour component queries the plans library, indicating goal (i.e., task to execute, or external environment state to bring into). The planning subsystem searches the plans library, selects an appropriate plan, and places the first rule of the selected plan into the reactive goals stack.

The problem of the implementation of the BPsim.MAS and BPsim.MSN systems integration is solved by implementing the communication between BPsim agents using a single database.

The communication between BPsim agents is implemented in different ways on the different levels:

1. The message exchange between reactive agents within the dynamic model MRCP is implemented by the transacts' (messages) introduction into the process model and by the description of the message processing rules in the agent's model.

2. The message exchange between reactive and intelligent agents within the dynamic model (MRCP) is implemented via applying the clipboard messages containing common variables used in BPsim.MAS and BPsim.MSN systems.

3. The message exchange between BPsim agents and external systems (in cases when the interaction is necessary to transmit not only data but also knowledge) is implemented by applying the communication protocols. As an interaction standard, the Foundation for Intelligent Physical Agents (FIPA) standard has been selected because it has the following advantages: highest reliability, ontology description availability, problem area compliance, and easy implementation. The Agent Communication Language (ACL) message type, supported in the FIPA standard [40], is used in the message exchange between BPsim agents and the environment.

## C. Multiagent Genetic Optimisation Program Development

The multiagent genetic optimisation (MGO) program has been developed on the basis of the BPsim.MAS and BPsim.MSN systems. The MGO program is intended to solve the problem of simulation and evolution modelling integration. The genetic optimisation information technology (IT) has been designed on the basis of BPsim.MSN, and is intended to aid GA setting and GA execution.

The algorithm for the interaction between the decision maker and MGO program during the decision-making process is shown in Figure 4. The MRCP model is intended to conduct the chromosome's FF evaluation by carrying out an experiment with the model. The decoded chromosome phenotype (operations calendar planning) is fed into the model input. The FF evaluation in accordance with (13) is obtained in the model output. Agents in the MRCP model are used to allocate the renewable resources (both own and subcontracted). The decision maker carries out the problem statement and solution choice among the solutions obtained by the use of the MGO program.

The MGO program has a number of advantages (key strengths) compared to existing evolutionary scheduling optimisation software [3][5][9][10]:

1. The integration of simulation, expert, multiagent, conceptual, and evolutionary approaches in order to decide the scheduling optimisation problem.

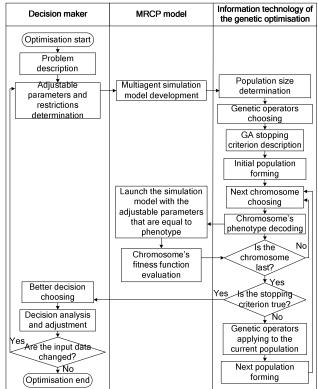


Figure 4. Interaction between the decision maker and MGO program.

2. The availability of the hybrid multiagent system architecture, which allows complex scheduling models to be built consisting of two interacting elements: 1) the dynamic model MRCP and 2) the genetic optimisation model intended for control of model MRCP parameters.

3. Description of the system models using MRCP and UML graphical notation.

4. The evolutionary and simulation models' integration via wizard technology for users without programming skills.

5. The modified genetic algorithm implementation in the genetic optimisation model.

6. Support for the development of the user's own ways of solving the scheduling problem by the use of a modified GA through the decision's phenotype encoding description using Transact SQL query language and decision search diagrams based on the UML (only for user-programmers).

The MGO program is intended to decide the scheduling problem under certainty. Let us consider the modification of the MGO method in order to solve the scheduling problem under uncertainty.

## VII. THE MGOU METHOD OF PROJECT SCHEDULING UNDER UNCERTAINTY

The project scheduling process is a time-consuming task that is complicated by the incompleteness of the initial information, the reduction of decision-making time, and increased requirements for the experience and expertise of decision makers (DM). The incompleteness of initial information is related to the uncertainty of the situation in which the decision should operate.

Two different types of uncertainty are allocated in [19]: the uncertainty of the environment state and the "active partner" uncertainty, reflecting the behaviour of the other decision makers. Accounting for the "active partner" uncertainty leads to the problem statement in conflict situations; methods for solving such problems are considered by the theory of games [19].

#### A. Problem Statement Under Uncertainty

We considered the scheduling problem under the environment's behaviour uncertainty. We associated the environment's behaviour uncertainty in the project scheduling with the lack of information on the number and size of additional projects that may arise during the planning period. The statement of the problem given in Section III takes the following form:

$$y = \{OF_1(\varphi(x, z)), OF_2(\varphi(x, z))\} \rightarrow \max, x \in X, z \in Z, y \in Y$$
(14)

where  $X = \{x_1, ..., x_n\}$  is a set of the alternative schedules;  $Z=\{z_1, ..., z_S\}$  is a set of the environment states;  $\varphi(x, z)$  is a transformation function of the alternative schedule *x* at the environment state *z* in some results;  $OF_1, OF_2$  are objective functions of the problem considered;  $Y = \{y_{ij}\}_{i=1..n}^{j=1..s}$  is a set of the decisions evaluation (matrix of decisions). Each element of the matrix (14)  $y_{ij}$  is a collection of 2 estimates of the alternative  $x_i$  outcome for environmental condition  $z_j$ .

We denoted the set of the probabilities that the environment will be in the states *Z* as follows:  $P = \{p_1, ..., p_S\}$ ,  $\Sigma p_j = 1$ . The vector *P* can be unknown for the selected environmental states *Z*.

The main difficulty in solving the problem (14) is the function  $\varphi$  definition, since this function should consider the cumulative system behaviour statistics, dynamic system processes, and DM behaviour scenario that it is not always possible to represent analytically for real management problems. The use of multiagent simulation allows the imposed requirements of the transformation function  $\varphi$  to be taken into account. The MRCP model is used for the system processes' formalisation and representation of the function  $\varphi(x,z)$ .

There are various methods of multi-criteria decision making that reduce the multi-criteria problem to the single-criterion problem [19].

In the main criterion method, one of the functions is selected as the objective function. This function best reflects the purpose of the decision making from the user's point of view. Such a transition is not always equivalent to the original problem.

The linear convolution method allows the criterion vector to be replaced by a scalar via a linear combination of all the weighted criteria functions. It is a requirement that all the functions' values should be presented on a numerical scale.

The maximin convolution method is focused on the worst case and chooses the optimality criterion, which corresponds to the smallest value of all the criteria. The disadvantage of this method is its focus on the worst criteria.

In the presence of knowledge about the decision maker preferences, the criteria coefficients can be determined using the Saaty method [41]. The main object of the Saaty method is a triangular matrix S of pairwise comparisons, each element of which is interpreted as the superiority ratio of the one criterion over the other. Coefficients of superiority can be selected by the user from a fixed Saaty scale. Further, the matrix S is transformed to the new matrix S', for which the maximum eigenvalue is determined through a linear equations system. The weighted coefficients vector of the individual criteria significance is an eigenvector of the matrix S' corresponding to the maximum eigenvalue of the matrix S'. The Saaty method's disadvantages are the need to solve linear systems of equations and the quadratic dependence of the pairwise comparison's number on the criteria and number of alternatives [41].

The linear convolution method was selected to reduce the considered multi-criteria scheduling problem to the singlecriterion problem using normalisation of the criteria with respect to the reference values (13).

Reduction of the problem under uncertainty (14) to the deterministic problem implies the application of numerical criteria. These criteria reflect the optimistic/pessimistic perspective of the decision maker on the processes [19]. The optimistic criterion is the Hurwitz criterion with the coefficient equal to 1. The pessimistic criteria are: Wald

criterion, Savage criterion, and Hurwitz criterion with coefficients equal to 0. The neutral criteria are: Bayes-Laplace criterion, Bernoulli criterion, and Hurwitz criterion with coefficients equal to 0.5.

All the mentioned criteria are used for uncertainty removal in the scheduling method described below.

## B. Multiagent Genetic Optimisation Method Under Uncertainty

The multiagent genetic optimisation method under uncertainty (MGOU method) integrates simulation, genetic algorithms, and numerical methods. The MGOU method includes the following steps.

**Step 1.** Definition of the input information: a) sets Z of the environment state and sets P of the probabilities that the environment is in the certain state; b) the function  $\varphi$  with the use of MRCP model. The alternative work schedule x and environment state z are fed into the model input. The  $OF_1$  and  $OF_2$  evaluations are obtained in the model output.

Step 2. Formation of a set of alternative schedules X, that include the efficient (optimal) solution of the problem (1–3) under conditions of certainty. At this step the MGO method is used to find the chromosome population (set of alternative work schedules) including the efficient (optimal) solution of the problem (1–3).

Step 3. Formation of the matrix of decisions Y for the problem (14) via conducting  $n \cdot s$  experiments with the MRCP model with *n* alternatives from the set X and s alternatives from the set Z. The matrix of decisions Y for the problem (14) is presented in Table II.

**Step 4.** Replacement of the optimality criterion vector  $\{OF_1, OF_2\}$  of the problem (14) on the scalar value. The replacement can be performed using known numerical methods, such as the linear convolution as in formula (13). The statement of the problem (14) after the step's fulfilment takes the form:

$$y = F(x, z) \to \max, y \in Y, x \in X, z \in Z, \qquad (15)$$

where F(x,z) is the function of implementing an alternative *x* in environment state *z* to decision evaluation *y*.

Step 5. Reduction of the problem under uncertainty (15) to the deterministic problem using numerical functions J. There are several numerical functions, depending on the knowledge of probabilities vector P and the strategy used for the uncertainty removal: the Bayes-Laplace criterion (mathematical expectation criterion), Wald criterion (criterion of the guaranteed result, maximin criterion), Savage criterion (criterion of minimum regret), Bernoulli criterion (principle of insufficient grounds), and Hurwitz criterion (criterion of pessimism-optimism) [19].

 TABLE II.
 MATRIX OF DECISIONS FOR THE MULTI-CRITERIA

 PROBLEM UNDER UNCERTAINTY
 PROBLEM UNDER UNCERTAINTY

	<b>Z</b> 1	•••	ZS
<b>X</b> 1	$OF_1(\phi(x_1,z_1)), OF_2(\phi(x_1,z_1))$		$OF_1(\phi(x_1,z_s)), OF_2(\phi(x_1,z_s))$
•••			
Xn	$OF_1(\phi(x_n,z_1)), OF_2(\phi(x_n,z_1))$		$OF_1(\varphi(x_n,z_s)), OF_2(\varphi(x_n,z_s))$

Let us consider the description of selected numerical functions of uncertainty removal.

The Bayes-Laplace criterion is applied with knowledge of probabilities P and characterises the "average income" when making an alternative work schedule x [19]:

$$J_{BL}(x) = \overline{F(x,z)} = \sum_{i=1}^{S} p_i \cdot F(x,z_i) \to \max_{x \in X}, \quad (16)$$

where the line on top of the symbol denotes the mathematical expectation. The decision of the problem (16) will be an alternative work schedule  $x_{i*}$ :  $i^* = \arg(\max J_{BL}(x_i))$ .

The Wald criterion characterises the best solution for the most unfavourable situation [19]:

$$Jv(x) = \min_{z_i \in Z} F(x, z_i) \to \max_{x \in X}$$
 (17)

This formula is valid if the function F(x,z) characterises the "income". Otherwise, the maximin criterion is transformed into a minimax criterion.

The Savage criterion characterises the best solution when comparing the worst losses. The losses emerge when there is preference of others for one alternative x at a fixed environment state z [19]:

$$Js(x) = \max_{z_i \in Z} \left( \max_{x_j \in X} F(x_j, z_i) - x_j \right) \to \min_{x \in X}$$
 (18)

The Bernoulli criterion characterises decisions by considering equiprobable external environment events [19]:

$$Jb(x) = \frac{1}{S} \cdot \sum_{i=1}^{S} F(x, z_i) \to \max_{x \in X} \cdot$$
(19)

The disadvantage of this criterion is that the unknown distribution law of the magnitude P is replaced by the uniform distribution law.

The Hurwitz criterion characterises the solution for a given propensity DM to pessimism or optimism [19]:

$$Jh(x) = \alpha \cdot \max_{z_i \in Z} F(x, z_i) + (1 - \alpha) \cdot \min_{z_i \in Z} F(x, z_i) \to \max_{x \in X}, \quad (20)$$

where  $\alpha$  is the indicator of pessimism or optimism. If  $\alpha = 0$  the case of extreme pessimism comes about, if  $\alpha = 1$  the case of extreme optimism comes about.

#### VIII. MULTIAGENT GENETIC OPTIMISATION PROGRAM UNDER UNCERTAINTY

The program for multiagent genetic optimisation under uncertainty (MGOU program) has been developed on the basis of an MGO program, BPsim.MAS dynamic situations modelling system and BPsim.MSN technical and economic development system. The information technology for uncertainty removal has been designed on the basis of BPsim.MSN. The algorithm for the interaction between the decision maker and MGOU program during the decision-making process under uncertainty is shown in Figure 5. The decision maker carries out the problem statement, definition of the different environment condition, and alternative work scheduling set formed with the help of the genetic optimisation IT.

The uncertainty removing IT is intended to aid risk assessment under the uncertainty behaviour of the environment. The MRCP model is intended to conduct the experiments according to the plan for different values of work scheduling and project size (environment condition).

The MGOU program has a number of advantages (key strengths) compared to existing scheduling optimisation under uncertainty software [11]–[13][15]–[17], as well as the advantages of the MGO program that have already been described in the Section VI:

1. The integration of simulation and evolutionary approaches with numerical decision support methods to decide the optimisation problem of scheduling under uncertainty.

2. Consideration and removal of parametric and structural environmental uncertainty.

3. The availability of uncertainty removal via wizard technology for users without programming skills.

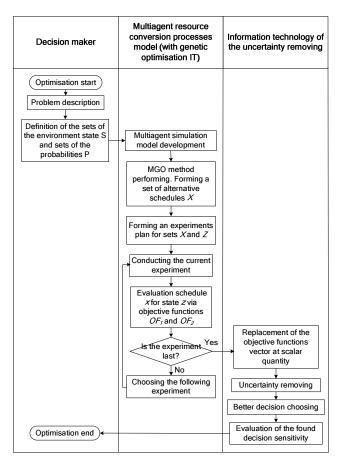


Figure 5. Interaction between the decision maker and MGOU program.

## IX. COMPARATIVE ANALYSIS OF THE PROJECT SCHEDULING METHODS

Let us consider the following project scheduling methods: critical path method (CPM) and program evaluation and review technique (PERT), branch and bound method (B&B), genetic algorithms, Xu and Feng's method, Osaba's method, and the MGOU method. A comparison of the selected methods is presented in Table III.

The CPM and PERT methods allow the reserves of time for the execution of certain works to be determined [17][42]. CPM assumes the deterministic duration of activities and PERT incorporates uncertainty into activity durations.

The branch and bound method uses the evaluation of upper and lower bounds to cut a set of solutions via removal of the subsets containing no optimal solutions. The upper bound is obtained using heuristics, while the lower bound can be found using mathematical programming [17][42][43].

Genetic algorithms are used to find the optimal schedule via the evolution of populations of schedules with the help of genetic operators [2]–[10][18]. The optimal solution can be found in GA by considering not only the one improvement decision but many improvement decisions.

Xu and Feng's method is intended to optimise project scheduling with uncertain activity durations and activity costs [11]. First, fuzzy random parameters are transformed into fuzzy variables that are subsequently defuzzied using an expected value operator with an optimistic-pessimistic index. Second, the deterministic problem is solved with the use of a hybrid particle swarm optimisation algorithm.

The Osaba's method is intended to solve the dynamic travelling salesman problem with the use of integrated simulation and genetic algorithms [10]. Simulation is used to reflect the dynamic nature of the system processes and objective function.

Comparison criteria	CPM, PERT B&B		GA	Xu/Feng meth.	Osaba meth.	MGOU meth.						
Problem statement												
Scheduling	•	٠	•	٠	٠	•						
Renewable	•	•	•	•	•	•						
resources	•	•	•	•	•	•						
Nonrenewable	0	0	0	•	0	•						
resources	0	0	0	•	0	•						
Subcontract	•	0	0	0	0	•						
optimisation	•	-	-			•						
	Methods for solving the deterministic problem											
Simulation	0	0	0	0	•	•						
Multiagent	0	0	0	0	0	•						
modelling	0	0	Ŭ	0	0	•						
Optimisation												
methods:	●/O	•/o	0/●	0/●	0/●	0/●						
exact/heuristic												
	Un	certainty	consid	eration	-							
Parametric												
uncertainty												
with:	●/o	0/0	0/0	0/●	0/0	•/0						
distribution	•, •	0/0	0/0	0/•	0/0	•, •						
law/fuzzy												
logic												
Structural	0	0	0	0	0	•						
uncertainty	ÿ	~	Ŭ	ý	ÿ	•						

TABLE III. ANALYSIS OF THE SCHEDULING METHODS

The MGOU method integrates the simulation, multiagent, and evolutionary modelling and numerical methods in order to solve the project scheduling problem under uncertainty.

The following comparison criteria were distinguished: application of renewable and nonrenewable resources; optimisation of subcontracting volume in order to decrease the project costs; application of simulation in order to adequately formalise the nonlinear, non-convex, and nondifferentiable system processes model; application of multiagent model in order to reflect the decision makers model; application of exact and heuristic optimisation methods in order to conduct the optimisation experiment for optimal solution finding; consideration of the uncertainty with different description in order to reflect unexpected external influences on the schedule.

As we can see from the table, all methods except the MGOU method lack the support of some features that might be useful in effective decision searching of the scheduling problem. For example, subcontract optimisation (except CPM and PERT), agent-based approach implementation, and structural uncertainty evaluation are limited. Another disadvantage of the four most popular scheduling methods (CPM, PERT, B&B, GA) are lack of nonrenewable resource consumption (including the resource life cycle description), lack of simulation that helps to analyse the dynamic system processes of the resources allocation, and lack of uncertainty consideration (except PERT). The Xu and Feng's method nonrenewable resource considers consumption and uncertainty with fuzzy logic, but does not use simulation and multiagent models to optimise subcontracted work. The Osaba's method includes simulation but does not consider uncertainty.

The full potential of scheduling under uncertainty is implemented in the MGOU method. The disadvantage of the method is its lack of fuzzy uncertainty description.

## X. EXPERIMENTAL RESULTS

The application of the MGO and MGOU programs to solve the project scheduling problem is presented in this section. Let us consider a company, Telesystems, which consists of project, manufacturing, and supply departments. The goal is the minimization of the company departments' total downtime and the total cost of the subcontract. In this section we consider the application of the MGO method to both the project scheduling problem without uncertainty and to the project scheduling problem under uncertainty.

#### A. Experimental Results for the Deterministic Problem

A detailed statement of the considered problem is given in [21]. The MRCP model has been developed to evaluate the chromosome FF (13). The MRCP model implements the resource allocation model, which satisfies the assumptions determined in Section III. The model adequacy has been proven in [21] through the evaluation of 5 projects. The following input information has been used in the model: 1) number of projects – 10 with 35 operations; 2) time interval – T = 430 days (1 year and 3 months); and 3) time limit – the early and late start of the operations is determined by the shift in the provisional operational start dates by 2 weeks to the right or left along the time axis.

The following GA parameters have been determined in the course of the genetic optimisation IT work: the population size – 10 chromosomes; the chromosome size – 175 genes (5 genes to encode the 35 project operations); the following genetic operators – reproduction based on roulette, five-point crossover with probabilities determined by (10) and (11), five-point mutation with an initial probability equal to 10% and dynamic probability determined by (12), inversion with initial probability equal to 5%; algorithm stopping criterion – a change of 10 populations; random initial population; and following the ASA parameters values of  $t_{Z0}$  = 1,  $\alpha$  = 0.9, K = 10.

A comparison of the application of simple GA and modified GA was performed. For a better comparison both algorithms proceeded from one initial population. The dependencies of the chromosome FF and the scheduling problem objective function values from the population number were obtained as a result of genetic optimisation using the developed MGO program. The change in the minimum value of the objective function (1) during genetic optimisation via simple and modified GA is shown in Figure 6.a. The change in the maximum value of the fitness function (13) during genetic optimisation via simple and modified GA is shown in Figure 6.b. The problem solution involves the maximization of the GA fitness function and minimization of subcontract cost.

At the initial stage of the modified GA (change of the generations 1–5) the search of the original decisions predominates and leads to the FF value variations, which does not always ensure the achievement of the best FF values compared with the simple GA. However, the search results are the basis of the targeted search for an extremum in the later stages of GA (change of the generations 6–10) that leads to the higher quality of the solution found. For the problem considered, the decision found with the use of the modified GA leads to the subcontract cost of 35189 rubles, which is 14% below the subcontract cost obtained by using simple GA (41050 rubles). The best decision is achieved in the ninth population.

The project scheduling problem for the Telesystems company has also been solved by use of the MS Project 2007 resources reallocation method and heuristic-simulation (HS) method described in [21]. MS Project 2007 provides the opportunity for resource reallocation (with smoothing) in order to avoid exceeding the own renewable resources availability. The percentage utilisation of the manufacturing department for the initial work plan for the Telesystems company is shown in Figure 7 by means of MS Project.

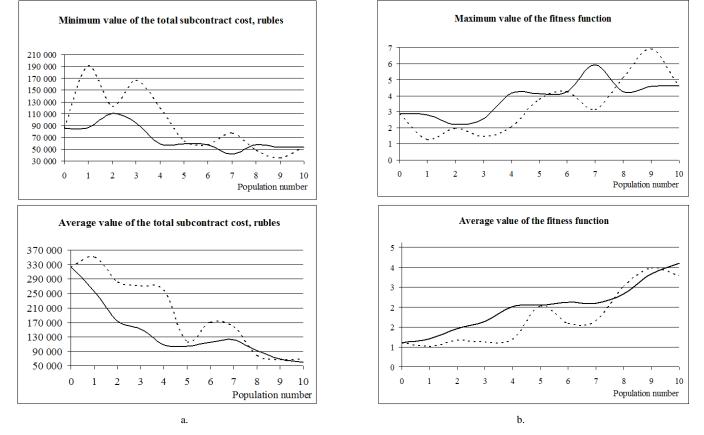


Figure 6. Dependencies of the fitness function and objective function values from the population number when applying simple GA (solid line) and modified GA (dotted line).

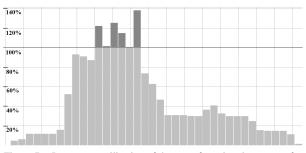


Figure 7. Percentage utilisation of the manufacturing department for initial work plan in MS Project.

The initial work plan has been formed by a decision maker. In the figure, the x-axis shows the time intervals (each of which lasts 12 days); the y-axis shows percentage utilisation. The overallocated resource availability (time intervals where the use of subcontracting is necessary) is shown in the figure by the dark shading of the stripes above the horizontal line at the 100% utilisation level. The application of the MS Project resource reallocation method has allowed the total subcontract cost to be reduced to zero; that is, the objective functions (1) and (2) have reached their optimal values. But the time constraints (3) have not been satisfied by the use of this method. In this way, the MS Project resource reallocation method is not considered suitable for the scheduling problem.

The HS method is based on the analysis of the MRCP model output parameters. In the HS method, the following steps are performed [21]: 1) modelling the results analysis of the subcontract cost and company resources utilisation; 2) search for bottlenecks associated with operations that require high costs of subcontracting; 3) shifting the start dates of operations to the period determined by HS information technology; and 4) transferring the adjusted model at the experiment stage and experiment results evaluation.

Histograms of the objective functions (1) and (2) obtained by the MGO and HS methods are shown in Figures 8.a and 8.b compared with the initial work plan. The total subcontract cost and total downtime of the manufacturing department has been consistently reduced by the use of HS and MGO methods. All time constraints have been satisfied.

Based on the analysis of the results it was concluded that the MGO method is more effective than the HS method in addressing the project scheduling problem in terms of economic effect. The total subcontract cost of the project portfolio has been reduced by 30% and the total downtime of the manufacturing department has been reduced by 1.5% for a six month period using the MGO method compared to the HS method. The total subcontract cost has been reduced by 7 times using the MGO method compared with the initial work plan. Applying the genetic optimisation based on the simulation and evolutionary modelling integration enhances the efficiency of the decision making by taking into account the dynamic resource allocation model in the simulation model and the fulfilment of the direct search in the decision space by the GA. The economic effect of applying the MGO program to solve the scheduling problem for the Telesystems company will be 430000 rubles per year, which is 9% higher than the economic effect of the use of the HS method to solve the same problem.

Let us compare the HS and MGO methods in terms of performance by measuring CPU time. The CPU time for the HS method  $T_{HSM}$  consists of the sum of the HS IT runtime  $T_{HSIT}$  and the model MRCP runtime  $T_{MRCP}$ . The sum is multiplied by the number of experiments  $X_{Iterations}$  conducted during the HS technology work. Thanks to the fact that  $T_{HSIT} \ll T_{MRCP}$  we can neglect the term  $T_{HSIT}$  and define  $T_{HSM}$  time as follows:  $T_{HSM} = X_{Iterations} \cdot T_{MRCP}$ .

The CPU time for the MGO method  $T_{MGO}$  consists of the sum of the genetic optimisation IT runtime  $T_{GOIT}$  and model MRCP runtime  $T_{MRCP}$ , which is multiplied by the chromosome number N. The sum is multiplied by the generation number K. Thanks to the fact that  $T_{GOIT} << T_{MRCP}$  we can neglect the term  $T_{GOIT}$  and define  $T_{MGO}$  time as follows:  $T_{MGO} = K \cdot N \cdot T_{MRCP}$ .

For the real-world scheduling problem the following parameter values were used:  $X_{Iterations}$ = 3, K = 10, N = 10. In this case, the HS method is more desirable in terms of performance and consumes 33 times less CPU time than the MGO method. This is connected to the use of the simulation model in the GA for fitness function evaluation, which is performed *K*·*N* times. The CPU time of the MGO method is equal to 30 minutes.

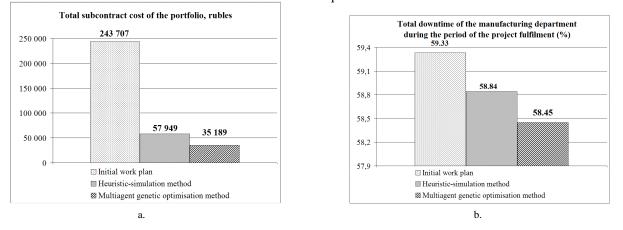


Figure 8. Dependencies of the objective function values on the decision-seeking method.

## B. Experimental Results for the Problem under Uncertainty

Let us consider the uncertainty behaviour of the environment associated with the appearance of four additional projects in the spring, summer, autumn, and winter, respectively. We defined the sets *Z* and *P* through the different events occurring in the system  $L = \{l_1, ..., l_r\}$  and a set of probabilities of the events' occurrence  $P_L = \{p_L(l_1), ..., p_L(l_r)\}$ ; and we considered the set *L* consisting of the following r = 8 events: appearance/absence of additional complex project in each of the seasons.

The graph of the determined environment states Z by using events L is shown in Figure 9.

Sixteen environment states {*SA*,...,*SP*} were allocated (*s*=16) as a result of the description of the graph of system states. Each state is characterised by the simultaneous execution of *h* events from the set *L*:  $z_i \Leftrightarrow \exists L_i^* \subseteq L: L_i^* = l_i^1 \land l_i^2 \land ... \land l_i^h$ , where  $l_i \in L$ ,  $z_i \in Z$ , i = 1..s,  $h \leq r$ .

The probability  $p_i$  of the system's being in the state  $z_i$  according to the [19] is determined by

$$p_{i} = \prod_{j=1}^{h} p_{L}(l_{i}^{j}), \ l_{i}^{j} \in L_{i}^{*}, \ p_{L}(l_{i}^{j}) \in P_{L}, \ \sum_{i=1}^{s} p_{i} = 1.$$
(20)

For the problem considered r = 8, h = 4, s = 16; that is, 8 events are considered that define the 16 states of the system, and each state is specified by the simultaneous performance of 4 events. The probabilities of the events' occurrence are specified in Table IV.

The probabilities of the systems being in the environment states  $z_i$  calculated for the selected initial conditions are shown in Table V.

Let us calculate, for example, the probability of the systems being in the state  $z_{16} = SP$ :  $L_{16}^* = l_1 \wedge l_3 \wedge l_5 \wedge l_7$  (according to Figure 9). We use the formula (20):

$$p_{16} = p_L(l_1) \cdot p_L(l_3) \cdot p_L(l_5) \cdot p_L(l_7) = 0.75 \cdot 0.5 \cdot 0.5 \cdot 0.25 = 0.05 \cdot 0.5 \cdot 0.25 = 0.05 \cdot 0.5 \cdot 0.25 = 0.05 \cdot 0.25 \cdot 0.25 = 0.05 \cdot 0.25 \cdot 0.25 = 0.05 = 0.05 \cdot 0.25 =$$

The model MRCP of the project work performance was used to evaluate the work schedules for 16 selected states of the environment. The MRCP model, which has been described for the MGO method, was supplemented with the following input parameters: marks of the systems being in one of the analysed states.

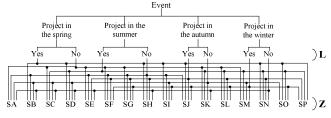


Figure 9. Graph of the system states.

TABLE IV. EVENTS L AND PROBABILITIES PL OF EVENTS' OCCURRENCE

li	Spring	Summer	Autumn	Winter		
	project l <sub>1</sub>	project l <sub>3</sub>	project l5	project l <sub>7</sub>		
$\mathbf{p}_{\mathbf{L}}(\mathbf{l}_{\mathbf{i}})$	0.75	0.5	0.5	0.25		

As a result of the application of the MGO method, 10 generations of chromosomes were obtained with information stored in the genes about the start dates of the projects' operations. Let us choose as a set of alternatives X the decoded chromosomes in the ninth population of GA, where the best solution to the problem considered was obtained according to Figure 6. Let us define the dimension of the set of alternatives n = 8.

The matrix of decisions *Y* for the project scheduling problem under uncertainty is formed by assessing, with the use of MRCP model, the selected alternatives  $\{x_1,...,x_8\}$  via the set of criteria  $\{OF_1, OF_2\}$  for each system state  $\{z_1,...,z_{16}\}$ . Table II has been filled as a result of conducting the  $n \cdot s = 128$  experiments. The criterion vector  $\{OF_1, OF_2\}$ has replaced the scalar value with the use of the formula (13) and following the values of the formula coefficients:  $\omega_1=0.5$ ;  $\omega_2=0.5$ . The removal of uncertainty has been carried out by applying the removing uncertainty IT that implements selected numerical criteria.

We applied the Bayes-Laplace criteria (16) for replacement of the matrix of decisions *Y* on the vector of decisions  $J_{BL}(x)$ . After performing the transformation, the following vector was obtained:  $J_{BL}(x) = \{104.0; 95.8; 90.4; 121.5; 120.7; 127.7; 143; 126.5\}$ . It is easy to determine that the best solution is an alternative  $x_7$  with the criterion value  $J_{BL}(x_7) = 143$ . The data obtained agree with the results of the MGO method application.

Let us investigate the stability of the solution  $x_7$  when changing the initial search conditions. By the term "stability of the solution" we mean the preservation of the solution's advantages over alternative solutions when changing the decision-maker preferences in the evaluation of the importance of objective function (13) criteria and the probabilities of events if they are known. The stability of the found solution  $x_7$  can be evaluated by the application of the Bayes-Laplace criterion to remove the uncertainty of the alternative sets. A series of experiments were carried out in order to find a vector function for evaluating the alternatives X for different initial conditions. The initial conditions were obtained by varying the degree of importance of criteria in forming the implementation function (coefficients  $\omega_1$  and  $\omega_2$ in formula (13)) and the probabilities of events pL (see Table IV).

TABLE V. ENVIRONMENT STATES Z AND PROBABILITIES P

Zi	$\mathbf{SA}$	SB	$\mathbf{SC}$	SD	SE	SF	SG	HS	IS	ſS	SK	TS	SM	NS	SO	$\mathbf{SP}$
pi	0.05	0.14	0.05	0.14	0.05	0.01	0.01	0.14	0.05	0.05	0.05	0.01	0.01	0.14	0.05	0.05

The dependence of the Bayes-Laplace function on the changes in initial conditions is shown in Figure 10. Analysis of the behaviour of the function  $J_{BL}(x)$  for different initial conditions has shown that the optimal value of the alternative  $x_7$  is preserved for all the analysed situations. Superiority of the calendar plan  $x_7$  is maximum in the case of the equiprobable occurrence of the four additional projects (with a probability of no more than 0.75) and when the objective function  $OF_1$  (which minimises the total subcontracting cost) is selected as the most important criterion of the problem considered. The superiority of the calendar plan  $x_7$  is

minimum (right side of the upper chart on Figure 10) in the case of high probabilities of four additional projects.

Let us apply the 4 remaining numerical criteria for removing uncertainty and let us evaluate the stability of the solution  $x_7$  relative to the alternatives set X by changing the coefficient values  $\omega_1$  and  $\omega_2$  in formula (13). For the Hurwitz criterion we define  $\alpha = 0.5$ . The dependence of the function J(x) behaviour (for different numerical functions) on the significance of the individual components of the criterion function (13) is shown in Figure 10.

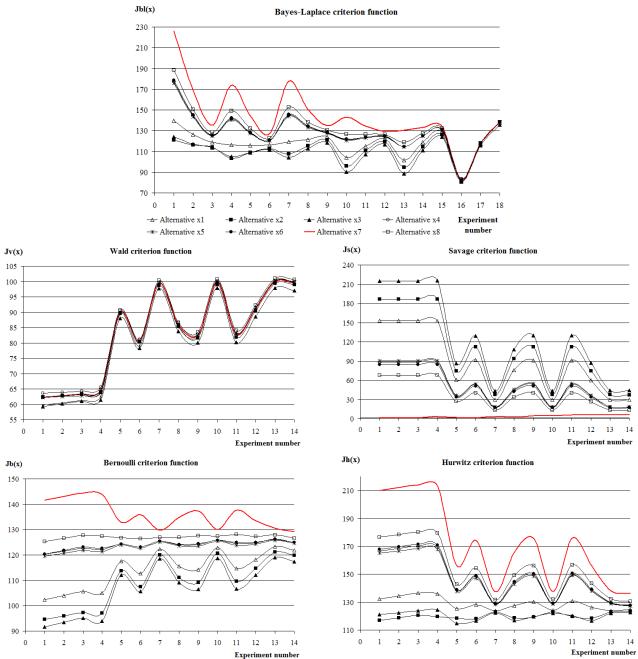


Figure 10. Values of the function J(x) for various numerical criteria when the coefficients of the objective function (13) are changed.

As follows from the charts, the use of the Savage, Bernoulli, and Hurwitz criteria reveals the optimal solution  $x_7$  that maintains stability when changing the function (13) coefficients. The scatter of the function J(x) values is observed for experiments in which function  $OF_1$  is selected as the most important criterion of the problem considered. Function J(x) values approximation is observed for the experiments in which function  $OF_1$  becomes insignificant when compared with the remaining component (function  $OF_2$ ). In applying the Wald criterion, the correlation between function J(x) values is set for each experiment. This fact is connected only to the analysis of the worst situations in which all alternatives provide comparable outcomes for each experiment.

The solution (calendar plan of the works) that provides the best outcome from the perspective of subcontract cost and the minimization of own resources downtime, and which provides resistance to external factors, was identified with the use of the MGOU method. Also, this solution is optimal for the scheduling problem under certainty. We concluded that the cost of subcontracting is the most important criterion, which greatly affects the objective function when changing environmental conditions.

#### XI. CONCLUSION AND FUTURE WORK

In this paper, a multiagent genetic optimisation method used to solve the deterministic and stochastic project scheduling problem has been described on the basis of the annealing simulation algorithm, novelty search algorithm, genetic algorithm, multiagent simulation, and numerical methods. In order to reflect the dynamic nature of the genetic operators applied, the method combines three different decision-seeking strategies: a random search strategy, originality search strategy, and maximum search strategy. The proposed integration of evolutionary modelling and simulation limits the search space and adequately evaluates the dynamic fitness functions of the chromosomes. The method described has been implemented in MGO and MGOU programs built on the basis of the BPsim.MAS multiagent modelling system and BPsim.MSN development system. The programs integrate simulation, expert, multiagent, conceptual, and evolutionary modelling with numerical methods of uncertainty removal. The comparative analysis of the scheduling methods has shown the disadvantages of the four the most popular scheduling methods (CPM, PERT, B&B, GA) with regard to the lack of nonrenewable resource consumption, simulation, and uncertainty consideration. Also, the analysis has shown the advantages of the MGOU method in the presence of subcontract optimisation, agent-based approach implementation, and structural uncertainty evaluation. The MGO method's application to a real-world deterministic project scheduling problem was compared with the MS Project and HS methods. The MS project resource reallocation method was found to be unsuitable for the scheduling problem under consideration because of the lack of constraints considered. As a result of the comparison between the MGO and HS methods, an improvement in decision quality under the constraints considered has been achieved using the MGO method.

The disadvantage of the MGO method is the high CPU time, which is 33 times higher than that of the HS method. This fact imposes constraints on the GA generation size (no more than 10 chromosomes) and GA iteration number (no more than 10 generations). Different ways of enhancing the applied GA convergence should be considered in future work to meet the described constraints.

The MGOU method of multi-criteria decision making under uncertainty has been applied to the project scheduling problem aimed at optimising the utilisation of subcontracted workforces and own resources. The decisions found by the MGO method have been analysed under the structural uncertainty of the four additional projects appearance. Inferences have been drawn about the optimal decision flexibility with the environmental condition changes. The results of the experiments have shown the coherence of the use of selected numerical criteria.

The aim of future research is to improve the rate of convergence of the proposed genetic algorithm by applying elitism and taboo algorithms. The dependency between the decision search time and problem dimensions is assumed to be established for the MGOU method. Also, consideration of nonrenewable resource allocation and fuzzy description of uncertainty is planned.

It is planned to extend and apply the developed method for the scheduling of technological logistics in the field of metallurgy. Similar problems have the following features: first, the presence of a plurality of industrial units and vehicles to be scheduled and, secondly, the presence of conflict situations when driving vehicles (cranes and steel teeming ladle cars in the shops). The technological logistics scheduling is complicated by consideration of the production plan for the units of output and the availability of additional technological support operations, which are strictly related to the number of the completed basic technological operations at the industrial unit. It is planned to develop a multiagent simulation model of the industrial unit's work and vehicle movements and optimise the values of the controlled model variables - the route of vehicle movement and the industrial unit's work plan - using a modified genetic algorithm.

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#### REFERENCES

- K. Aksyonov and A. Antonova, "Multiagent genetic optimisation to solve the project scheduling problem," Proc. ICCGI 2013: The Eighth International Multi-Conference on Computing in the Global Information Technology, Nice, France, pp. 237–242, July 2013.
- [2] I. Okada, X. F. Zhang, H. Y. Yang, and S. Fujimura, "A random key-based genetic algorithm approach for resourceconstrained project scheduling problem with multiple modes," Proc. Int. MultiConf. Engineers and Computer Scientists, vol. 1, Hong Kong, pp. 106–111, March 2010.

- [3] M. Klimek, "A genetic algorithm for the project scheduling with the resource constraints," Annals UMCS Informatica, vol. 10, issue 1, pp. 117–130, 2010.
- [4] E. Sriprasert and N. Dawood, "Genetic algorithms for multiconstrained scheduling: an application for the construction industry," Proc. CIB W78's 20th International Conf. Construction IT, Construction IT Bridging the Distance, CIB Report 284, Waiheke Island, New Zealand, pp. 341–353, April 2003.
- [5] H. Abdel-Khalek, M. H. Sherif, A. M. el-Lacany, and Y. Abdel-Magd, "Financing – scheduling optimization for construction projects by using genetic algorithms," World Academy of Science, Engineering and Technology, vol. 5, pp. 289–297, 2011.
- [6] M. Karova, J. Petkova, and V. Smarkov, "A genetic algorithm for project planning problem," Proc. Int. Scientific Conf. Computer Science, Krakov, Poland, pp. 647–651, June 2008.
- [7] A. Brezuliani, L. Fira, and M. Fira, "A genetic algorithm approach for scheduling of resources in well-services companies," International Journal of Advanced Research in Artificial Intelligence, vol. 1, no. 5, pp. 1–6, 2012.
- [8] A. Dhingra and P. Chandna, "A bi-criteria M-machine SDST flow shop scheduling using modified heuristic genetic algorithm," International Journal of Engineering, Science and Technology, vol. 2, no. 5, pp. 216–225, 2010.
- [9] F.-C. Yang and W.-T. Wu, "A genetic algorithm based method for creating impartial work schedules for nurses," International Journal of Electronic Business Management, vol. 10, no. 3, pp. 182–193, 2012.
- [10] E. Osaba, R. Carballedo, and F. Diaz, "Simulation tool based on a memetic algorithm to solve a real instance of a dynamic TSP," Proc. IASTED Int. Conf. Applied Simulation and Modelling, Napoli, Italy, pp. 27–33, June 2012.
- [11] J. Xu and C. Feng, "Multimode resource-constrained multiple project scheduling problem under fuzzy random environment and its application to a large scale hydropower construction project," The Scientific World Journal, vol. 2014, 2014. [Online]. Available from: http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3914335/pdf/ TSWJ2014-463692.pdf [Retrieved: May 2014]
- [12] J. He and Z.-P. Wan, "Construction project scheduling problem with uncertain resource constraints," Journal of Construction Engineering and Management ASCE, vol. 15, no.1, pp. 324–326, 2002.
- [13] X. Zhang and X. Chen, "A new uncertain programming model for project scheduling problem," An International Interdisciplinary Journal, vol. 15, no. 10, pp. 1–10, 2012.
- [14] A. Csebfalvi, "A unified model for resource constrained project scheduling problem with uncertain activity durations," International Journal of Optimization in Civil Engineering, vol. 2, no. 3, pp. 341–355, 2012.
- [15] C. Artigues, R. Leus, and F. T. Nobibon, "Robust optimization for resource-constrained project scheduling with uncertain activity durations," Flexible Services and Manufacturing Journal, vol. 25, no. 1–2, pp. 175–205, 2013.
- [16] W.-N. Chen and J. Zhang, "Scheduling multi-mode projects under uncertainty to optimize cash flows: a Monte Carlo ant colony system approach," Journal of Computer Science and Technology, vol. 27, no. 5, pp. 950–965, 2012.
- [17] M. Brcic, D. Kalpik, and K.Fertalj, "Resource constrained project scheduling under uncertainty: a survey," Proc. Central European Conference on Information and Intelligent Systems, Varazdin, Croatia, pp. 401–409, September 2012.
- [18] D. Goldberg, Genetic Algorithms. Addison Wesley, 1989.
- [19] I.G. Chernorutskii, Decision-making methods. St. Petersburg: BHV-Petersburg, 2005.

- [20] M.-Y. Chen, H.-C. Tsai, and E. Sudjono, "Evaluating subcontractor performance using evolutionary fuzzy hybrid neural network," International Journal of Project Management, no. 29, pp. 349–356, 2011.
- [21] K. A. Aksyonov and A. S. Antonova, "Application of simulation and intelligent agents to solve project management problem," International Journal of Computer Science Engineering and Information Technology Research, vol. 3, no. 1, pp. 321–330, 2013. [Online]. Available from: http://www.tjprc.org/view\_archives.php?year=2013&id=14&j type=2&page=3 [Retrieved: May 2014]
- [22] B. Keller and G. Bayraksan, "Scheduling jobs sharing multiple resources under uncertainty: a stochastic programming approach," IE Transactions, vol. 42, no. 1, pp. 16–30, 2009.
- [23] J. Lehman and K. Stanley, "Exploiting open-endedness to solve problems through the search for novelty," Proc. Eleventh Intern. Conf. Artificial Life (ALIFE XI), Cambridge, MA, pp. 329–336, August 2008.
- [24] L. A. Zinchenko, V. M. Kureichik, and V. G. Redko (Eds.), Bionic Information Systems and their Practical Applications, Moscow, FIZMATLIT, 2011.
- [25] W. L. Goffe, G. D. Ferrier, and J. Rogers, "Global optimization of statistical functions with simulated annealing," Journal of Econometrics 60, pp. 65–99, 1994.
- [26] K. Aksyonov et al., "Decision Support Systems Application to Business Processes at Enterprises in Russia," in Efficient Decision Support Systems – Practice and Challenges in Multidisciplinary Domains, C. Jao, Ed. InTech, pp. 83–108, 2011. ISBN: 978-953-307-441-2. [Online]. Available from: http://www.intechopen.com/articles/show/title/decisionsupport-systems-application-to-business-processes-atenterprises-in-russia. [Retrieved: May 2014]
- [27] K. A. Aksyonov and N. V. Goncharova, Multi-agent resource conversion processes dynamic simulation, Yekaterinburg: USTU, 2006.
- [28] K. A. Aksyonov, Research and development of tools for discrete resource conversion processes simulation, DPhil research paper, Ural State Technical University, Yekaterinburg, Russia, 2003.
- [29] A. V. Andreichikov and O. N. Andreichikova, Intelligent information systems. Moscow: Finance and statistics, 2004.
- [30] Modelling system Plant Simulation. The official web site. [Online]. Available from: http://www.plm.automation.siemens.com/en\_us/products/tecn omatix/plant\_design/plant\_simulation.shtml [Retrieved: May 2014]
- [31] W. D. Kelton, J. S. Smith, D. T. Sturrock, and A. Verbraeck, Simio and Simulation: Modeling, Analysis, Applications. New York: McGraw-Hill, 2010.
- [32] Real-time expert system G2. The official web site. [Online]. Available from: http://www.gensym.com/en/product/G2 [Retrieved: May 2014]
- [33] Mutiagent modelling system AnyLogic. The official web site. [Online]. Available from: http://www.anylogic.com [Retrieved: May 2014]
- [34] Mutiagent modelling system RepastJ. The official web site. [Online]. Available from: http://repast.sourceforge.net/index.html [Retrieved: May 2014]
- [35] G. Rzevski, J. Himoff, and P. Skobelev, "MAGENTA technology: a family of multi-agent intelligent schedulers. International conference on multi-agent systems" Proceedings of Workshop on Software Agents in Information Systems and Industrial Applications 2 (SAISIA). Fraunhofer IITB, Germany, 2006. [Online]. Available from:

http://rzevski.net/06%20i-Scheduler%20Family.pdf [Retrieved: May 2014]

- [36] The official UML web site. [Online]. Available from: http://www.uml.org [Retrieved: May 2014]
- [37] SQL Server language reference. [Online]. Available from: http://msdn.microsoft.com/en
  - us/library/ms166026(v=sql.90).aspx [Retrieved: May 2014]
- [38] M. Wooldridge, "Intelligent agent: theory and practice," Knowledge Engineering Review, vol. 10 (2), 1995.
- [39] J.P. Muller and M. Pischel, The Agent Architecture InteRRap: Concept and Application. Sarbrücken: German Research Centre for Artificial Intelligence (DFKI), 1993.
- [40] FIPA ACL Message Structure Specification. [Online]. Available from: http://www.fipa.org/specs/fipa00061/SC00061G.html [Retrieved: May 2014]
- [41] T. L. Saaty, Decision making with dependence and feedback: the analytic network process. University of Pittsburgh: RWS Publications, 1996.
- [42] F. S. Hillier and G. J. Lieberman, Introduction to stochastic models in operations research. New York: McGraw-Hill, 1990.
- [43] A. Chaleshtari and S. Shadrokh, "A branch and bound algorithm for resource constrained project scheduling problem subject to cumulative resources," Proc. International Conference on Information Management, Innovation Management and Industrial Engineering (ICIII), IEEE, vol. 1, Shenzhen, China, pp. 147–152, November 2011.