Train Timetable Optimization for Parallel Single-track Sections During Track Closure

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***Abstract*—In parallel single-track sections, which are common in high-speed railways in Europe, when one of the tracks is closed due to vehicle failure or maintenance work, the other track can be used for travel in both directions to avoid significant delays or cancellation of train operation. However, for bidirectional operation on some sections of a parallel single-track line, it is necessary to adjust the original train timetable; usually, this adjustment causes some trains to be delayed. Therefore, in this study, we propose a mathematical model for train timetable planning when performing bidirectional operations on some parallel single-track sections to minimize the total delay of each train.**

Keywords-Parallel single-track; Track closure; Train timetable; Maintenance; Optimization.

I. INTRODUCTION

In a railway operation site, there are cases in which tracks for some sections have to be closed due to vehicle or signal failure. In addition, there are cases where it is necessary to close a track for maintenance work. If a track closure occurs during business hours, it might have a severe negative impact on train operation, and in some circumstances, it may become necessary to cancel train operations all over. However, on a parallel single-track, in which single tracks capable of bidirectional operation are laid in parallel, even if one track is closed, it is possible to avoid significant train delays and cancellation of trains. For example, tracks of many European high-speed railways such as Train a Grande Vitesse (TGV) [1] and InterCity Express (ICE) [2] have parallel single-track sections; a cross section is provided approximately every 50 km. Therefore, even if one of the tracks is closed, bidirectional operation can be performed on other tracks.

When performing two-way operation on some sections of a parallel single-track line, it is usually necessary to adjust the normal train operation timetable (hereafter, simply referred to as the original timetable). During this time, if the delay of each train increases due to changes in the travel time, it disturbs passengers, respectively as well as rolling stock schedule and crew schedule. Therefore, it is desirable that the travel time adjustment, when performing bidirectional operation, is planned well to minimize the delay.

Since track closure has significant impact on train operation, several studies for managing or planning a timetable during track closure have been carried out to date [3]-[9]. For example, [8] proposed the methodology to plan timetables in consideration of passenger services during track closure and [9] proposed the methodology to plan simultaneously the period of track closure and traffic flow. In these studies, it is acceptable to cancel train services during track closure. However, in practice, it may be required not to cancel train service and to plan a timetable sustaining the capacity in the original timetable during track closure. In this study, we propose a mathematical model for planning train timetable when performing bidirectional operation due to track closure to minimize total delay of each train without cancelling train services. We carry out simulations to verify the usefulness of the proposed mathematical model.

This paper is organized as follows. Section II explains the problem definition of train timetable planning during track closure. Section III presents a mathematical formulation for the problem. Section IV provide simulation results for several track closure periods. We conclude the paper in Section V.

II. TRAIN TIMETABLE PLANNING DURING TRACK CLOSURE

This section describes a problem definition of a train scheduling problem during track closure.

A. Bidirectional Operation on a Parallel Single-track Section

In parallel single-track sections capable of bidirectional operation, normal operation is carried out on two independent train routes with trains having different traveling directions. However, when a certain section of one route is closed, trains of that route are moved to the other route via a crossing line just before the closed section; they travel on that route and move to their original route via a crossing line at the end of the track closure. That is, when a certain section of one route is closed, two-way operation on the other route should be planned. Figure 1 shows an example of two independent

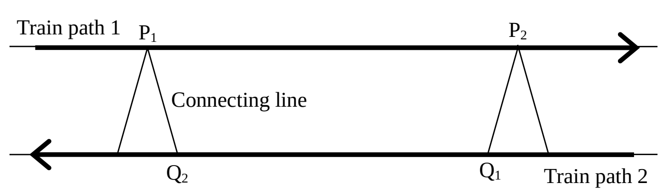

Figure 1. Train paths on a parallel single-track section

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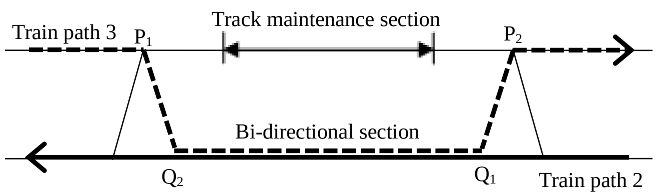

Figure 2. Change of the route of train path 1 due to track maintenance resulting in track closure

routes on a parallel single-track section, showing route 1 from left to right and route 2 from right to left in the case of normal operation. Further, P_1 and P_2 indicate the connection points of a crossing line on route 1, and Q_1 and Q_2 the connection points of a crossing line on route 2. Figure 2 is an example of a case where a track section between point P_1 and point P_2 is closed due to maintenance work; bidirectional operation between point Q_1 and point Q_2 is also shown as route 1 in Figure 1, which is then changed to route 3 in Figure 2 (trains on route 3 travel in the order $P_1 \rightarrow Q_2 \rightarrow Q_1 \rightarrow P_2$).

It is impossible for two trains with different traveling directions to simultaneously travel on a section on which bidirectional operation can be performed (hereafter, simply referred to as a bidirectional section). In other words, trains with the same travelling direction can continue running on a bidirectional section; however, if two trains with different traveling directions travel on a bidirectional section, train on one route needs to wait for the train on the other route to pass. Therefore, it is necessary for the train timetable on a bidirectional section to be planned by considering the trains with different traveling directions.

B. *Train Scheduling Problem During Track Closure*

There is a concern that passenger dissatisfaction will increase due to an increase in the congestion rate of trains due to longer operation intervals if trains are cancelled when performing a bidirectional operation. In addition, if the order of trains is changed, it will interfere with the vehicle operation schedule and the crew schedule. Moreover, if the operation time of each train is advanced, passengers may not be able to get on the scheduled trains. Therefore, the following assumptions with respect to a train schedule are made when bidirectional operation is performed due to track closure.

1. Trains will not be canceled.
2. The train operation sequence will be maintained as per the original timetable.
3. The operation time of each train will be the same as or later than the original timetable.

Further, the following assumptions are made with respect to track closure section and period (start time to end time of track closure).

4. Only one track is closed on a certain section.
5. The track closure section is between the crossing that is before the section closed for maintenance and the next crossing (section between points P_1 and P_2 in Figure 2).
6. A train will not be in the track closure section during the track closure period.

From assumption 6, for a route with a closed track section, a train traveling on the track closure section during the track closure period will be moved to the other track in the bidirectional section. Then, the following assumptions are made for routes 1, 2, and 3 in Figures 1 and 2.

7. Trains traveling on the track closure section on route 1 during the track closure period as per the original timetable travel on route 3 until the track closure ends and on route 1 after the track closure ends.
8. Trains traveling on the track closure section on route 2 during the track closure period as per the original timetable always travel on the same route.

Moreover, to smoothing the delay time from the original timetable of each train, it is necessary to plan such that the delay of each train is within a certain limit. Then, the following assumption is made.

9. The delay time of each train has an upper limit.

The problem to be addressed in this study is to plan the train timetable when performing bidirectional operation during track closure to minimize the total delay of each train under the above conditions. For this, the following times, shown in Figure 2, need to be planned.

- The time when trains traveling on the track closure section on route 1 during the track closure period as per the original timetable pass point P_1
- The time when trains traveling on the track closure section on route 2 during the track closure period as per the original timetable pass point Q_1

As described above, the timetable of the bidirectional section must be planned considering train schedules of routes with different directions simultaneously. On the contrary, a train timetable, not on a bidirectional section, can independently adjust train schedules of one-directional routes. Therefore, we assume that, after planning a train schedule for a bidirectional section, the timetable for other sections can be adjusted for each route for synchronization. In the following, we also assume that the track closure section and track closure period are given.

III. Formulation Of A Train Timetable Planning Problem

This section describes a mathematical formulation of a train timetable planning problem defined in Section II.

A. *Terminology*

The following terms are defined with respect to Figures. 1 and 2. For an original timetable of route 1, let $H_1 = \{1,2,\ldots,\mu\}$ be the set of all trains traveling on the track closure section (between points P_1 and P_2) during the track closure period. For an original timetable of route 2, let $H_2 = \{\mu+1, \mu+2, \ldots, \mu+\nu\}$ be the set of all trains traveling on the bidirectional section (between points Q_1 and Q_2) during the track closure

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period. We also let $H = H_1 \cup H_2$. Note that the operation sequence, according to the original timetable, of trains included in H_1 and H_2 is $1 \rightarrow 2 \rightarrow \ldots \rightarrow \mu$ and $\mu + 1 \rightarrow \mu + 2 \rightarrow \ldots \rightarrow \mu + \nu$, respectively. For example, the track closure period is assumed to be from 13:10 to 15:10 and the time required for trains on route 1 to travel on the track closure section and the time required for trains on route 2 to travel on the bidirectional section is 1 hour. In this case, if trains passing through point P_1 at the following times exist in the original timetable of route 1, then trains passing point P_1 at the times indicated by the solid underline are included in H_1.

11:30, 12:00, <u>12:30</u>, <u>13:00</u>, <u>13:30</u>, <u>14:00</u>, <u>14:30</u>, <u>15:00</u>, 15:30, 16:00

Moreover, if trains passing through point Q_1 at the following times exist in the original timetable of route 2, then trains passing point Q_1 at the times indicated by the dotted underline are included in H_2.

11:35, 12:05, <u>12:35</u>, <u>13:05</u>, <u>13:35</u>, <u>14:05</u>, <u>14:35</u>, <u>15:05</u>, 15:35, 16:05

It should be noted that trains passing through point P_1 (respectively, point Q_1) before the start time of the track closure period are also included in H_1 (respectively, H_2).

In Section II, when the track between points P_1 and P_2 of route 1 is closed, we set up a problem to determine the times for passing through point P_1 for trains included in H_1 and times for passing through point Q_1 for trains included in H_2 to minimize the total delay of all trains. In the following, we formulate this problem into a mathematical programming problem. Here, from assumptions 7 and 8 in Section II, trains in H_1 are scheduled to travel on route 3 until the end time of track closure and are scheduled to travel on route 1 after the end time of track closure. On the other hand, trains in H_2 are always scheduled to travel on route 2.

The following times defined below are assumed to be given.

- ϖ_η : The time required for train $\eta \in H_1$ to travel from point P_1 to point P_2 on route 3 or for train $\eta \in H_2$ to travel from point Q_1 to point Q_2 of route 2 (traveling time)
- ω_η : The time required for switching the travel route from route 3 to route 2 for train $\eta \in H_1$ and from route 2 to route 3 for train $\eta \in H_2$ (route switching time)
- α_η : The time interval from the time train $\eta\,(\neq \mu) \in H_1$ passes through point P_1 to the time when the following train $\eta + 1 \in H_1$ passes through point P_1 or from the time when train $\eta\,(\neq \mu + \nu) \in H_2$ passes through point Q_2 to the time when the following train $\eta + 1 \in H_2$ passes through point Q_2 (minimum headway)
- δ_η : The upper limit of the permissible delay time of train $\eta \in H$ (permissible delay time)

Here, it is assumed that the times defined above are in minutes and are all non-zero. In addition, it is assumed that traveling time ϖ_η is defined considering the speed regulation in a section including switches, reduced speed operation on a maintenance section, etc., and that minimum headway α_η is defined considering the running speed of each train, various restrictions on train operation, etc.

In the original timetable of trains included in H, train 1 passes through point P_1 first, and train $\mu + 1$ passes through point Q_1 first. If we let σ' be the first time when train 1 passes point P_1 and the time when train $\mu + 1$ passes through point Q_1 in the original timetable, we only need to take time σ' and later into consideration in our problem. Therefore, we define $T_1 = \{1, 2, \ldots, \kappa\}$ as the set of times with one-minute intervals from σ' to the end time of the track closure. Here, the first element of T_1 represents time σ' and the κth element represents the end time of track closure. For example, if time σ' is 12:00 and the end time of the track closure is 15:00, then the elements 1, 2 and 3 of T_1 will be 12:00, 12:01, and 12:02, respectively, and element $\kappa = 181$ of T_1 indicates the end time of track closure, i.e., 15:00. We also define the following.

- σ_η: The time in T_1 when train $\eta \in H_1$ passes through point P_1 in the original timetable of route 1 or when train $\eta \in H_2$ passes point Q_1 in the original timetable of route 2.

For example, if σ' is 12:00 and the time when train $\eta \in H_1$ passes through point P_1 in the original timetable is 13:10, then $\sigma_\eta = 71$.

In the original timetable of trains included in H, train μ passes through point P_1 last and train $\mu + \nu$ passes through point Q_1 last. On the other hand, from assumption 9 in Section II, the delay of train $\eta \in H$ must be less than or equal to δ_η. Therefore, if we let τ' (note that it is the time in T_1) to be the later time between $\sigma_\mu + \delta_\mu$ and $\sigma_{\mu+\nu} + \delta_{\mu+\nu}$, in addition to T_1 previously defined, we only need to take the time interval from $\kappa+1$ to τ' into consideration for our problem. However, depending on the problem, there exist cases where time τ' is earlier than time $\kappa+1$. Therefore, we also set $\lambda = \max\{\tau', \kappa+1\}$. Moreover, let $T_2 = \{\kappa+1, \kappa+2, \ldots, \lambda\}$ be the set of times of one-minute intervals from time $\kappa+1$ to λ, and $T = T_1 \cup T_2$.

B. *Definition of the Train Scheduling Problem*

Let $A = H \times T$ be the set of direct products of the set of trains H and the set of times T; the subsets of A that restrict the set of train H to H_1 and H_2 are defined as follows.

$$A_1 = \{(\eta, \tau) \in A \mid \eta \in H_1,\ \tau \in T\}$$

$$A_2 = \{(\eta, \tau) \in A \mid \eta \in H_2,\ \tau \in T\}$$

Furthermore, variable $\xi_{\eta\tau}$ is assigned to element (η, τ) of A. Variables $\xi_{\eta\tau}$, $(\eta, \tau) \in A$ is 1 if the train $\eta \in H_1$ ($\eta \in H_2$) passes through point P_1 (Q_1) at time $\tau \in T$ and is 0 otherwise. Here, the total number of variables $\xi_{\eta\tau}$ is $|A| = (\mu + \nu) \times \lambda$.

As mentioned previously, trains in the original timetable were assumed to not be canceled. Therefore, train $\eta \in H$ must be planned to pass through point P_1 or point Q_1 at any time between 1 and λ. That is, variables $\xi_{\eta\tau}$ must satisfy the following.

$$\sum_{1 \le \tau \le \lambda} \xi_{\eta\tau} = 1,\ \eta \in H \tag{1}$$

In addition, it was assumed that the operation time of each train must be the same as or later than the original operation time and that the delay of each train should not exceed the permissible delay. Therefore, train $\eta \in H$ must be planned to pass through point P_1 or point Q_1 at any time from σ_η to $\sigma_\eta + \delta_\eta$. That is, let $\beta_\eta = \sigma_\eta + \delta_\eta$, $\eta \in H$, variables $\xi_{\eta\tau}$ must satisfy the following.

$$\sum_{\sigma_\eta \le \tau \le \beta_\eta} \xi_{\eta\tau} = 1,\ \eta \in H \tag{2}$$

Furthermore, the train operation sequence is assumed to be maintained as per the original timetable. Therefore, all trains operated before train $\eta \in H_1\ (\eta \neq 1)$ must pass through point P_1 before time τ in order for train η to pass through point P_1 at time τ. Similarly, all trains operated before train $\eta \in H_2\ (\eta \neq \mu+1)$ must pass through point Q_1 before time τ for train η to pass through point Q_1 at time τ. Here, for the first train in H_1 and first train $\mu+1$ in H_2, there is no need to consider the above constraints. Moreover, because only those trains can pass through point P_1 or point Q_1 at time 1, it is not necessary for trains passing through point P_1 or point Q_1 to consider the above constraints. Now, we define the subset of A_1 obtained by subtracting the elements for the case $\eta = 1$ and the case $\tau = 1$ and the subset of A_2 obtained by subtracting the elements for the case $\eta = \mu + 1$ and the case $\tau = 1$ as follows.

$$A'_1 = \{(\eta, \tau) \in A_1 \mid \eta \neq 1,\ \tau \neq 1\}$$
$$A'_2 = \{(\eta, \tau) \in A_2 \mid \eta \neq \mu + 1,\ \tau \neq 1\}$$

Then, the constraint condition on the train operation sequence is expressed as follows.

$$\sum_{1 \le \tau' < \tau} \xi_{\eta'\tau'} \ge \xi_{\eta\tau}, \quad 1 \le \eta' < \eta,\ (\eta, \tau) \in A'_1 \tag{3}$$

$$\sum_{1 \le \tau' < \tau} \xi_{\eta'\tau'} \ge \xi_{\eta\tau}, \quad \mu + 1 \le \eta' < \eta,\ (\eta, \tau) \in A'_2 \tag{4}$$

From the definition of A'_1, A'_2 for any $(\eta, \tau) \in A'_1$ in (3) and any $(\eta, \tau) \in A'_2$ in (4), there exists η' where $1 \le \eta' < \eta$ and η' where $1 \le \eta' < \eta$, respectively. Figure 3 shows an example in which $H_1 = \{1,2,3,4\}$, $T_1 = \{1,2,\ldots,10\}$, and $T_2 = \{11,12,\ldots,15\}$, and the element of row η and column τ represents variable $\xi_{\eta\tau}$. Note that the rows corresponding to H_2 are omitted. In addition, the gray parts indicate variables $\xi_{\eta\tau}$, $(\eta, \tau) \in A'_1$. In Figure 3, in the case in which train 3 passes point P_1 at time 5 is when $\xi_{35} = 1$; the variables surrounding the bold frame must satisfy $\xi_{11} + \xi_{12} + \xi_{13} + \xi_{14} = 1$ and $\xi_{21} + \xi_{22} + \xi_{23} + \xi_{24} = 1$.

		T_1										T_2				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H_1	1	ξ_{11}	ξ_{12}	ξ_{13}	ξ_{14}											
	2	ξ_{21}	ξ_{22}	ξ_{23}	ξ_{24}											
	3					ξ_{35}	= 1									
	4															

Figure 3. Constraints on train operation sequences

Next, we consider the headway between trains. From the definition of minimum headway α_η, when train $\eta\ (\neq \mu) \in H_1$ passes through point P_1 at time $\tau \in T$, the following train $\eta+1 \in H_1$ cannot pass through point P_1 before time $\tau + \alpha_\eta$. Similarly, when train $\eta\ (\neq \mu+\nu) \in H_2$ passes through point Q_1 at time $\tau \in T$, the following train $\eta+1 \in H_2$ cannot pass through point Q_1 before time $\tau + \alpha_\eta$. Note that the last train μ in H_1 and the last train $\mu+\nu$ in H_2 are not included in the problem. We define a subset of A where the elements in case of $\eta = \mu$ and $\eta = \mu + \nu$ are excluded as follows.

$$A'' = \{(\eta, \tau) \in A \mid \eta \neq \mu,\ \mu + \nu\}$$

Moreover, depending on time τ and minimum headway α_η, $\tau + \alpha_\eta$ may exceed the last time λ in T; we introduce the following:

$$\alpha(\eta,\ \tau) = \min\{\ \tau + \alpha_\eta,\ \lambda + 1\},\quad (\eta,\ \tau) \in A''$$

Then, the constraint conditions on the operation time interval are expressed as follows. In (5), from the definition of A'' and $\alpha_\eta > 0$, there always exist $\eta+1$ and τ' satisfying $\tau \le \tau' < \alpha(\eta,\ \tau)$ for an arbitrary $(\eta,\ \tau) \in A''$.

$$\sum_{\tau \le \tau' < \alpha(\eta,\tau)} \xi_{\eta+1,\tau'} + \xi_{\eta\tau} \le 1,\quad (\eta,\ \tau) \in A'' \tag{5}$$

As mentioned above, trains with different traveling routes cannot travel on a bidirectional section at the same time. Therefore, in cases that train $\eta' \in H_2$ travels after train $\eta \in H_1$ or that train $\eta' \in H_1$ travels after train $\eta \in H_2$ on a bidirectional section, train η' can travel on the bi-directional section only after a train η passes through the bidirectional section. Note that trains included in H_1 and H_2 travel on a bidirectional section until the track closure time ends. Thus, let A_1 and A_2 be limited to the time until track closure ends, and set $A'''_1 = H_1 \times T_1$ and $A'''_2 = H_2 \times T_1$. The time required for train $\eta \in H$ to travel on a bidirectional section is ϖ_η and the time required for switching the travel route after train η passes through the bidirectional section is ω_η. Therefore, if train $\eta \in H_1$ passes through point P_1 at time $\tau \in T_1$, train $\eta' \in H_2$ can pass through point Q_1 only after time $\tau + \varpi_\eta + \omega_\eta$. Similarly, if train $\eta \in H_2$ passes through point Q_1 at time $\tau \in T_1$, train $\eta' \in H_1$ can pass through point P_1 only after time $\tau + \varpi_\eta + \omega_\eta$. Note that depending on traveling time ϖ_η and route switching time ω_η, $\tau + \varpi_\eta + \omega_\eta$ may exceed the last time λ in T; we introduce the following:

$$\varepsilon(\eta,\ \tau) = \min\{\ \tau + \varpi_\eta + \omega_\eta,\ \lambda + 1\},\quad (\eta, \tau) \in A'''_1$$

On the other hand, from assumption 7 in Section II, trains included in H_1 travel on original route 1 as soon as the track closure ends. Therefore, when train $\eta \in H_2$ travels on the bidirectional section, we need to consider the constraint conditions for train $\eta' \in H_1$ mentioned above until time κ when the track closure ends; we do not need to consider it after time $\kappa + 1$. Here, we introduce the following.

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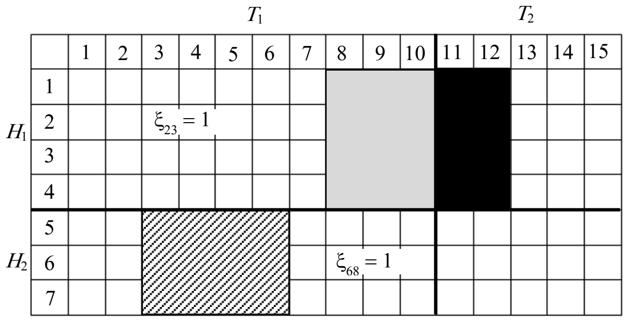

Figure 4. Constraints on switching train paths

$$\phi(\eta,\ \tau) = \min\{\ \tau + \varpi_\eta + \omega_\eta,\ \kappa + 1\},\quad (\eta,\tau)\in A'''_2$$

Then, the constraint conditions for preventing the trains included in H_1 and H_2 from simultaneously traveling on the bidirectional section are expressed as follows.

$$\sum_{\tau\le\tau'<\,\varepsilon(\eta,\ \tau)} \xi_{\eta'\tau'} + \xi_{\eta\tau} \le 1,\quad \eta'\in H_2,\ (\eta,\tau)\in A'''_1 \qquad (6)$$

$$\sum_{\tau\le\tau'<\,\phi(\eta,\ \tau)} \xi_{\eta'\tau'} + \xi_{\eta\tau} \le 1,\quad \eta'\in H_1,\ (\eta,\tau)\in A'''_2 \qquad (7)$$

From $\varpi_\eta,\ \omega_\eta > 0$, there always exist τ' where $\tau\le\tau'<\varepsilon(\eta,\ \tau)$ for any $(\eta,\tau)\in A'''_1$ in (6), and τ' where $\tau\le\tau'<\phi(\eta,\ \tau)$ for any $(\eta,\tau)\in A'''_2$ in (7). Figure 4 shows an example in which $H_2 = \{5,6,7\}$ is added to Figure 3. If we set $\xi_{23} = 1$ when $\varpi_2 = 3$ and $\omega_2 = 1$ in Figure 4, equation (6) shows that all shaded variables must be set to 0. That is, when train 2 passes through point P_1 at time 3, it indicates that trains included in H_2 cannot pass through point Q_1 from time 3 to 6. In addition, if we set $\xi_{68} = 1$ when $\varpi_6 = 4$ and $\omega_6 = 1$ in Figure 4, equation (7) shows that the gray variables must be set to 0. Here, because the trains included in H_1 can travel on original route 1 as soon as the track closure ends, the constraint conditions given by (7) are not imposed for the black variables.

Based on this, the problem defined in this section is formulated into the following mathematical programming problem (P).

(P) Minimize $\sum_{\eta\in H}\sum_{\tau\in T}(\tau - \sigma_\eta)\xi_{\eta\tau}$ (8)

Subject to: Equations (1) - (7)

$$\xi_{\eta\tau} \in \{0, 1\},\quad (\eta,\ \tau)\in A \qquad (9)$$

In this problem (P), equation (8) is the objective function to minimize the sum of the delay of each train. Equation (9) is the binary condition of the decision variable. Here, the problem to minimize the maximum delay of each train, not the total delay time of each train, can be formulated as the following mathematical programming problem (Q) by introducing the variable ψ (≥ 0).

(Q) Minimize ψ

Subject to: Equations (1) - (7), (9)

$$\sum_{\tau\in T}(\tau - \sigma_\eta)\xi_{\eta\tau} \le \psi,\quad \eta\in H \qquad (10)$$

For the problem (P), there may not exist any feasible solution. However, the absence of a feasible solution shows that it is impossible to adjust the original timetable under the given parameter settings. In other words, in this case, it shows that measures such as extending the permissible delay time, shortening the minimum headway, or changing the period track closure are necessary. For train operation during track closure, it is also important to determine whether a feasible solution exists for the problem (P) for a given parameter setting. On the other hand, recent mathematical programming software are becoming faster every year. It is possible to determine the existence of a feasible solution and the calculation of an optimal solution for the problem with actual scale of parameters in relatively short time. Therefore, it is possible to use mathematical programming software such as Gurobi or MATLAB to determine the existence of a feasible solution and the calculation of an optimal solution for problems (P) and (Q).

IV. SIMULATIONS

In this section, we describe the results of a simulation executed to examine the variation in total delay time when the track closure period are 3 hours, 6 hours and 9 hours. In the simulations to be described, we set the parameters as follows.

- The unit time is one minute.
- Trains on the route 1 pass point P_1 at the time 7:31, 8:01, 9:01, 9:31, 10:31, 11:31, 12:31, 13:01, 13:31, 15:01, 15:31, 16:31, 17:31, 18:31, 19:01, 19:31, 20:31, 21:31 as per the original timetable.
- Trains on the route 2 pass point Q_1 at the time 5:59, 7:20, 8:14, 8:50, 9:43, 10:43, 11:50, 12:20, 12:43, 13:50, 14:41, 15:50, 16:20, 16:50, 17:50, 18:20, 18:50, 19:20, 20:20 as per the original timetable.
- We set the input parameters $\varpi_\eta = 20,\ \eta\in H,\ \omega_\eta = 1,\ \eta\in H,\ \alpha_\eta = 2,\ \eta\in H - \{\mu,\ \mu+\nu\},\ \delta_\eta = 30,\ \eta\in H$ respectively.

Table I, Table II and Table III show the results of the track closure periods 3 hours, 6 hours and 9 hours, respectively. For Table I, Table II and Table III to be presented later the following should be noted.

- ω_{start} is the start time of track closure.
- ω_{end} is the end time of track closure.
- $|H_1|$ and $|H_2|$ are the number of trains in H_1 and H_2, respectively.
- $|T|$ is the number of elements in T.
- δ_{total} is the optimal solution value of the problem (P).
- δ_{ave} is the average delay time defined by $\delta_{ave} = \delta_{total} / |H|$.
- *time* is the computational time to judge the existence of a feasible solution of the problem (P) or calculate the optimal solution of the problem (P).
- We implemented simulations using Gurobi Optimizer 8.1.0 on a personal computer having the following specification, Intel® Core™ i7-6700 CPU 3.40GHz, 8.00GB RAM, running Windows 10 Pro.

From Tables I to III, the average delay time for track closure period of 3 hours, 6 hours and 9 hours are respectively 6.24s, 6.38s and 6.32s and there are not significant differences between those. In general, it is more economical for track

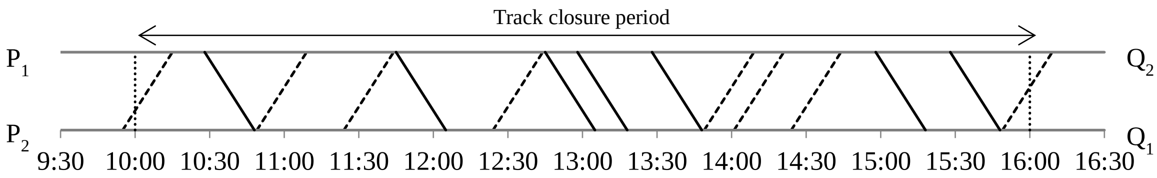

Figure 5. Diagrams for the adjusted train timetable calculated by the proposed methodology

maintenance work to perform a few long-term work rather than many short-term work. Therefore, in the original timetable in this simulation, it is considered to be more economical to plan long-term work rather than short-term work. In this way, the mathematical model proposed in this paper can be used to find a cost-effective maintenance plan. Figure 5 shows the results calculated by the proposed mathematical model in the case track closure period 10:00-16:00 in Table II. In Figure 5, the vertical and horizontal axes means distance (or train position) and time, respectively, diagrams of trains included in H_1 and H_2 are respectively shown by solid lines and dotted lines and the points P_1, P_2, Q_1 and Q_2 have the same meaning as in Figure 1 and Figure 2.

TABLE I. SIMULATION RESULTS FOR TRACK CLOSURE PERIOD OF 3 HOURS

ω_{start}	ω_{end}	$\|H_1\|$	$\|H_2\|$	$\|T\|$	δ_{total}	δ_{ave}	*time*
6:00	9:00	3	1	120	21	5.25	0.21
7:00	10:00	4	3	177	45	6.43	1.13
8:00	11:00	4	3	184	42	6.00	1.28
9:00	12:00	4	4	184	59	7.38	1.70
10:00	13:00	4	4	213	52	6.50	1.94
11:00	14:00	4	3	158	52	7.43	1.05
12:00	15:00	4	4	184	35	4.38	1.64
13:00	16:00	4	4	184	36	4.50	1.72
14:00	17:00	3	4	181	33	4.71	1.27
15:00	18:00	4	3	184	51	7.29	1.30
16:00	19:00	4	4	186	57	7.13	1.64
17:00	20:00	4	4	154	60	7.50	1.40
18:00	21:00	4	5	181	60	6.67	2.18

TABLE II. SIMULATION RESULTS FOR TRACK CLOSURE PERIOD OF 6 HOURS

ω_{start}	ω_{end}	$\|H_1\|$	$\|H_2\|$	$\|T\|$	δ_{total}	δ_{ave}	*time*
6:00	12:00	6	5	274	80	7.27	5.44
7:00	13:00	8	6	360	97	6.93	12.47
8:00	14:00	8	6	364	94	6.71	13.14
9:00	15:00	8	8	390	94	5.88	18.46
10:00	16:00	7	8	367	88	5.87	15.67
11:00	17:00	7	7	338	85	6.07	12.03
12:00	18:00	7	7	338	86	6.14	12.00
13:00	19:00	8	8	390	93	5.81	18.76
14:00	20:00	8	7	361	93	6.20	15.27
15:00	21:00	8	8	364	111	6.94	18.02

TABLE III. SIMULATION RESULTS FOR TRACK CLOSURE PERIOD OF 9 HOURS

ω_{start}	ω_{end}	$\|H_1\|$	$\|H_2\|$	$\|T\|$	δ_{total}	δ_{ave}	*time*
6:00	15:00	10	9	480	115	6.05	35.68
7:00	16:00	11	10	514	133	6.33	49.59
8:00	17:00	11	10	544	127	6.05	54.08
9:00	18:00	11	11	544	145	6.59	59.26
10:00	19:00	11	12	573	145	6.30	69.41
11:00	20:00	11	11	518	145	6.59	54.74
12:00	21:00	11	12	518	146	6.35	61.77

V. CONCLUSION

In this paper, we addressed the problem of how to adjust the original timetable for parallel single-track sections during track closure so as to minimize the sum of the delay of each train by formulating the task as a mathematical programming problem. Then, we verified the effectiveness of our method through simulations. However, there are still several problems that remain. For example, we need to construct a method to recover the delay caused by track closure after track closure ends. We also need to extend our method so that it can be used in the sections including multiple track closure sections.

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