

# Pedestrian Activity Simulation in Shopping Environments

## An Irregular Network Approach

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**Abstract**—Micro-scale agent-based modeling can be used for the simulation of pedestrian movement for low and high density scenarios and of the effect of changes in an environment. Such models can also be used for pedestrian dynamics in city centers to show the design effects in the shopping environment. This paper focuses on the generation of the movement network and the underlying behavioral rules that conducts the activation of pedestrians on the network representing a shopping environment. The store visits will be realized by Monte Carlo simulation.

**Keywords**- Monte Carlo Simulation; Activity Agenda; Agent Based Simulation Modeling; Pedestrian Dynamics; Irregular Network .

### I. INTRODUCTION

Agent-based modeling is a computational methodology that allows us to create, analyze, and experiment with artificial virtual worlds populated by agents. A specific research area is micro-scale agent-based modeling that can be used for the simulation of pedestrian movement for low and high density scenarios and for the effect of changes in the environment. Such kind of models can also be used for pedestrian dynamics in city centers to show the design effects in the shopping environment. In this context, Ali and Moulin [1] describe their multi-agent simulation prototype of customers' shopping behavior in a mall. The basis of this paper is an agent based model to simulate pedestrian dynamic destination, route and scheduling behavior. This agent based model is under development, where the simulation of movement patterns is embedded in a more comprehensive model of activity behavior.

Representation is a substantial issue in simulating pedestrian dynamics. One can distinguish the representation of the pedestrian environment and the representation of pedestrians. In the domain of a city center, representation of a pedestrian environment includes the geometry of the shopping environment such as stores and streets, the network as a cellular grid, and pedestrian objects. Pedestrian representation includes socioeconomic characteristics, speed, goals, familiarity with the environment, and an agenda of activities. This activity agenda includes planned activities and can be rescheduled by unplanned activities. It is assumed that pedestrians perceive their environment and that they are supposed to carry out a set of activities. For completing an activity, pedestrians spend time in stores. As a consequence,

time duration influences their movement behavior over the network.

In this way, we want to achieve the full potential of an agent based model. Also, that it involves explicitly the modeling of the dynamics of the individual pedestrians. Such models are called agent-based simulation models [2]. In our case, the pedestrians are the agents in the agent-based simulation model.

Although a 3D presentation of pedestrian movement is the ultimate goal, it is nevertheless meaningful to test the underlying principles in an appropriate 2D representation of pedestrians and their environment. Repast Symphony [3] and NetLogo [4] can be used as modeling and simulation toolkit because they provide a suitable simulation framework that supports skeletons of agents and their environment, and their interoperability (e.g., Geographic Information System, also called GIS). In our approach, we use Repast as guideline for the theoretical framework of the simulation process. On the other hand, we will use NetLogo for the actual simulation because it easily allows the empirical testing of the principles of the simulation approach. Also, we will use shape-file information of the environment and a network structure for visualizing the 2D environment.

The realization of pedestrian movement uses a choice network approach. Herein, the network is an irregular lattice of cells and the choice of movement direction is determined by activation of pedestrians' activities. The domain of the agent-based modeling approach is pedestrian behavior in a shopping environment and the choice mechanism that are involved. It shows some similarity with other models that investigate pedestrian movement with fine-scale considerations and pedestrians' shop-around behavior (e.g., [5]), or principles of bounded rationality [6]. In tackling the combined 'Multi Agent System (MAS) – Cellular Automata (CA)' approach, the inspiring 'situated cellular agents' approach [7] is worth mentioning. Rooted on basic principles of CA, this approach takes into account the heterogeneity of agents and provides interaction between agents locally and at-a-distance interaction; also, the notion of perception and action is included in affective agents [8].

Our model distinguishes itself from other similar models because store visit is included in the pedestrian movement behavior in the shopping environments. To our knowledge, this has not been done in this way.

This paper shows some similarities with previously published works (e.g., [9][10]), but this paper makes it consistent with each other. Moreover, improvements have been made, in particular the mathematical underpinnings of the irregular network approach.

This paper discusses successively pedestrian agent and its environment in Section II, the engineering basis of the simulation process in Section III, the simulation of store visits in Section IV, and the simulation of pedestrian activity and pedestrian movement in Section V, called simulation pedestrian dynamics. A discussion about the conclusions and future directions will conclude this paper in Section VI.

## II. PEDESTRIAN AGENT AND ENVIRONMENT

The basis for a pedestrian agent structure includes methods like *perceive*, *interpret* and *updEnv*, where the pedestrian agent has its own control. The behavior represents the set of possible attitudes.

The environment consists of streets, a set of stores, and pedestrians represented by agents. Streets are presented as an irregular lattice of cells (cellular network), which is used to simulate agent movement. A pedestrian agent moves with his own behavior and personal characteristics. At each time, there is an update of pedestrian agents' positions (*updEnv*). In fact, each cell in the cellular network can be considered as an information container object; it contains information about the area size, street or store characteristics, and agents' positions. We regard a restricted environment of a pedestrian agent in the cellular network. The cellular network provides percepts (*perceive*) and the pedestrian agent performs actions in them (*interpret*).

The pedestrian behavioral aspects are shown in Fig. 1. Perceptual fields, which guide which stores a pedestrian agent will perceive, may vary according to the agent's awareness threshold, which in turn may depend on his motivational state, age, etc., and the signaling intensity of the store or establishment, which is assumed a function of distance and appealing architecture.

Perceptual Field → Activation of the Agent → Completing an Activity

Figure 1. Pedestrian behavioral aspects.

When stores are signaled and become included in agent's perceptual field, the agent has to decide whether or not to act and visit the store. This is called the activation of the agent. We assume that activation is defined and depends among others on agent's personal characteristics, motivation, familiarity with a store, suitability to conduct a visit, and the agent's consideration set.

Estimation results of these behavioral principles are not part of this paper and are described in [11].

## III. ENGINEERING BASIS OF THE SIMULATION PROCESS

This section provides some understanding in the engineering basis of the simulation of pedestrian movement. The principles are rooted in agent-based modeling and

simulation, which is currently a fundamental tool for predicting the behavior of complex systems. As mentioned before, we use the principles of Repast Symphony creating the model [12].

The model structure in Repast Symphony is based on contexts and projections. The core data structure is called a context that represents from a modeling perspective, an abstract population: the objects in these populations are referred as agents.

The context provides the main infrastructure to define a population and the interactions of that population. An abstract environment is created in which agents exist at a given point in the simulation. The context also holds its own internal state for maintaining the collection of agents; this state can consist of multiple types of data. That provides agents with information about the world in which they interact. In addition, data fields can be maintained by the context; herein a data field is a n-dimensional field of values with which the agents in a context interact. These data fields can be directly associated with a physical space wherein the field is generic, whereby each value is derived from a set of coordinates.

Projections take the population as defined in a context. They impose a new structure on it, and the structure defines and imposes relationships on the population; therefore an agent population is realized once a projection is applied to it.

A feature of Repast Symphony is the ability to integrate GIS data directly to the simulation; it provides a set of classes that allow shapefiles to be displayed. A shapefile is a storage format for storing geometric and associated attribute information. For example, shapefiles can be provided by QuantumGIS [13]. A GIS contains multiple layers of data; each layer is made up of a number of elements. Each feature in the layer has aspects to it; its geographical coordinates (but it could be also a polygon, a polyline or polypoint) and the data associated with it [14]. GIS store data about layers in database files, with each record in the file referring to a feature in GIS. Actually, integration with GIS means shapefile integration. Agents are created using these data, and the simulation process provides the population by the context creator.

Agents, can be created, recreated and destroyed at each simulation step. They will be created by "Introduce Agents", and the update of all agents in the environment occurs in the "Agent Loop". Fig. 2 shows the Agent Loop of the context creator.

The interaction with the environment is provided by the shapefile containing GIS data and the other one for the generated network from this GIS data. The context needs this GIS data for the data fields which provides the information from the environment.

In our approach, the environment consists of polygons representing the network of shops and streets. In QuantumGIS, feature data will be connected to cells of the network and layers will be created.

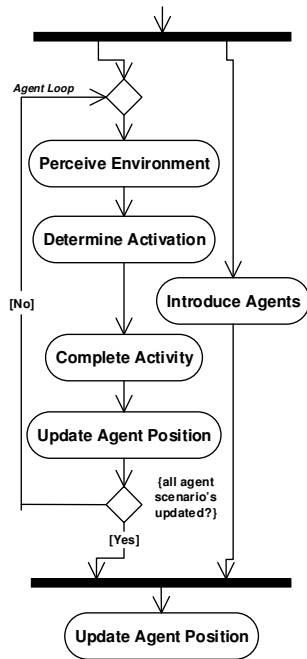


Figure 2. Activity diagram part of the agent loop of the *context* creator.

After that, the GIS database will be provided that can be used in NetLogo for environmental information for pedestrian agents for perceiving this environment.

#### IV. SIMULATION OF STORE VISITS

Borgers and Timmermans [15] used Monte Carlo simulation for the incorporation of the numbers of stops and the sequence of planned stops/purposes, because in their opinion the concept of multi-stop, multi-purpose behavior is relevant for understanding pedestrian behavior. According to this line of thought, we assume an activity agenda includes a number of planned and unplanned store visits that can also be considered as a number of non-impulse and impulse store visits.

Every pedestrian agent receives at its introduction in the simulation a pedestrian scenario. This pedestrian scenario includes besides general characteristics like gender, age, companionship, also, familiarity with the city center, motivation, time budget, and activity agenda. After a store visit, the activity agenda will be rescheduled. The number of planned and unplanned store visits is determined by a Monte Carlo simulation.

For this purpose, data from visitors to the city center of Eindhoven are gathered by interviewing them about their motivation and the stores they visited. For this survey about pedestrian behavioral principles, data were collected from 405 respondents. Visitors are also asked about successful visits and which of them were planned and unplanned. Data about their activity agendas were collected from 770 respondents.

These routes provide the collected data. The findings from this collected data of the number of planned and unplanned visit stops show a skewed distribution. The skewed distributions are different depending of gender, age category and motivation. We often need a skewed distribution where probability densities below and above the mean are distributed differently. In this case, we assume, by analogy with multiple stops, a Gamma distribution.

The probability density function of the Gamma distribution is given by:

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \quad (1)$$

where,  $k$  is the shape parameter ( $k > 0$ ) and  $\theta$  is the scale parameter ( $\theta > 0$ ).

Both  $k$  and  $\theta$  will be positive; they are derived from the skewed normal distribution of the number of (planned) stops from their data collection depending on gender, age category and motivation. The shape parameter  $k$  is derived from the skewness of the skewed normal distribution and the scale parameter  $\theta$  is derived from the mean and  $k$ . The discussion about the values of the parameters and the related data collection has been described elsewhere [16].

The Gamma inverse function  $G(p)$ , which is the inverse cumulative distribution function, is given by (2). Given a random number  $p$  from a uniform distribution in the interval  $(0, 1)$ , the value of  $G(p)$  has a Gamma distribution with parameters  $k$  and  $\theta$ . That means, given a number  $p$  on the  $x$ -axis provides the number of stops on the  $y$ -axis; where real values are rounded to integer values.

$$G(p) = F^{-1}(p; k, \theta) = \{x: F(x; k, \theta) = p\} \quad (2)$$

where

$$p = F(x; k, \theta) = \frac{1}{\theta^k \Gamma(k)} \int_0^x t^{k-1} e^{-t/\theta} dt$$

For example, Fig. 3 shows the  $G(p)$  distribution for the *goal oriented* orientation with respect to the number of (planned) stops. Also, there is a distinction for gender (male, female) and age ( $<55$ ,  $\geq 55$  years).

The number of unplanned stops can be derived from the calculation of the number of stops minus the number of planned stops.

Time duration is the time spent by a pedestrian in a store. For the simulation run, the time duration is also determined by a Monte Carlo simulation.

The findings from the collected data of the duration of a visit to a store indicate that this duration meets the Weibull distribution, and that this duration is dependent of the store category as well as the priority of the store. Fig. 4 shows the activity diagram of completing an activity in which time duration will be determined.

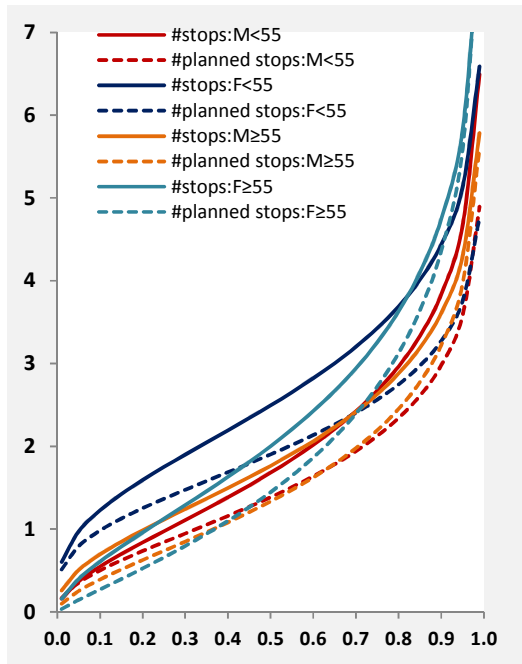


Figure 3.  $G(p)$  for Goal Oriented motivation, different gender and age category; Number of (planned) Stops vs. Probability.

The Weibull probability function is given by:

$$f(x; k; \lambda; \theta) = \frac{k}{\lambda} \left(\frac{x-\theta}{\lambda}\right)^{k-1} e^{-\left(\frac{x-\theta}{\lambda}\right)^k} \quad (3)$$

where,  $k$  is the shape parameter ( $k > 0$ ),  $\lambda$  is the scale parameter, and  $\theta$  is the location of the distribution.

Table I shows the values of the parameters of the Weibull distribution for the different store categories. The specific store categories include among others, jewelry, bell companies, candy shop, etc. If  $\theta = 0$  then we have to do with the 2-Parameter Weibull distribution.

The percent point function  $G(p)$ , which is the inverse cumulative distribution function, is given by:

$$G(p) = \theta + \lambda(-\ln(1 - p))^{1/k} \quad (4)$$

Given a random number  $p$  from a uniform distribution in the interval  $(0,1)$ , the value of  $G(p)$  has a Weibull distribution with parameters  $k$ ,  $\lambda$ , and  $\theta$ .

TABLE I. WEIBULL PARAMETERS FOR STORE CATEGORIES

Category	Description	Priority (%)	$k$	$\lambda$	$\theta$
1	Clothes-1	$\geq 1$	1.00	1.00	0.46
1	Clothes-2	$\geq 0.5 < 1$	1.22	0.99	0
1	Clothes-3	$< 0.5$	1.80	0.80	0
2	Shoes		1.10	0.62	0
3	Body & health		1.65	0.75	0
4	Department store		1.47	0.83	0
5	Specific-1	$< 0.7$	1.22	0.88	0
5	Specific-2	$\geq 0.7$	1.32	0.48	0

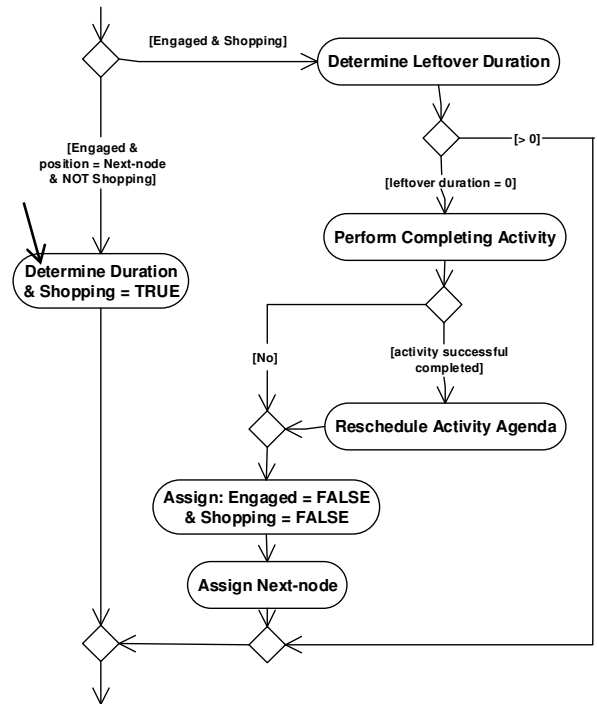


Figure 4. Activity diagram of completing an activity.

That means, a random number drawn from  $(0,1)$  provides the probability  $p$  which in turn provides the time duration. Fig. 5 shows the percent point function.

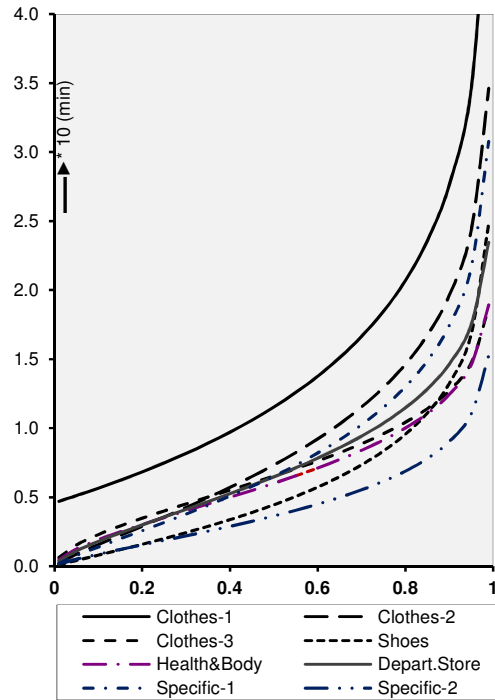


Figure 5. Percent point function at different store category

The findings about time duration can be found in [17].

## V. SIMULATION PEDESTRIAN DYNAMICS

As mentioned in Section III, the environment consists of polygons representing the network of shops (stores) and streets. Polygons are used to indicate borders, functional areas like walkways. Adjacent cells means that connections are possible, for instance pedestrian agents can move from one cell to an adjacent cell. If cells are not strictly adjacent, no movement from each other is possible.

Each cell in the network has a node. Therefore, the network consists of  $N$  nodes and  $L$  links. A subset of these  $N$  nodes are linked to  $J$  shops, and a subset  $E$  of these  $N$  nodes represents the entry/departure points of the simulation system. The link between a street-cell and a shop-cell will be established by the adjacent part of the polygon representing the shop and the street-cell. Effectively, pedestrian agents are situated in the cells of the network, namely a street-cell or a shop-cell and bounded to a node on the underlying representation, and pedestrian agents can move on the implicit generated network to other nodes if they are linked together. The network is irregular because a clear border between a shop-cell and adjacent cell is desired.

The test ground is the inner-city center of Eindhoven. We will perform the simulation on a segment of a section of this city center. Fig. 6 shows a segment of the city center and the associated full network with their links.

As mentioned before, we use a network of irregular cells. Fig. 6 (right) shows such a network that consists of street-cells, which includes the inner lane and two outer lanes, and shop-cells with one cell for each shop.

In research areas like geo-computation, land-use change, and urban planning, one can find extensions on the traditional formalization of CA to include an irregular spatial structure [18][19][20]. These models are based on uniform CA transition rules.

Tomassini [21] pointed out that standard lattice cellular automata and random Boolean networks can be extended to a

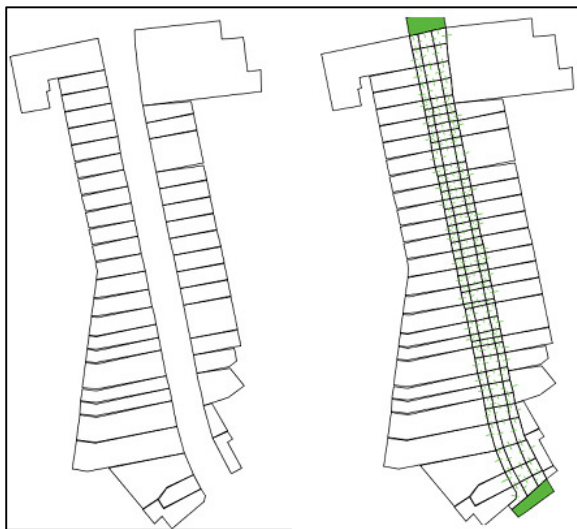


Figure 6. Main street of the city center (left) and the full layers (right).

wider class of generalized automata networks that can be built on any connected graph. In this approach, there is no uniform transition function; the change is in the local transition function.

We have no uniform lattice of cells and no uniform transition rules, because in our approach, a street-cell  $i$  in the network is bounded to the stay of a maximum number of pedestrian agents related to node  $i$ .

Pedestrian behavior and pedestrian profile characteristics result in pedestrian activity and choice moments. That results in position change from a current position in cell  $i$  linked to node  $i$  to a next position in cell  $j$  linked to node  $j$ . Strictly speaking, our approach is not CA-based; nevertheless at each time step there is an update of the network according to 'network rules'.

A pedestrian agent will be introduced in the simulation by setting an entry position, which will be done by Monte Carlo method. Also, a pedestrian agent receives a pedestrian profile including pedestrian characteristics, desired speed, activity agenda, etc. At a certain point of time, the network of shop-cells and street-cells is populated with pedestrian agents. At this point of time, all pedestrian agents have their current-node and according cell position. For each pedestrian agent, the to-node will be derived from the execution of pedestrian's activity. The following 'network characteristics' applies:

Let  $L$  consists of a lattice of cells, representing the irregular network of shop-cells and street-cells, with size  $N$  which is the number of cells.

The state of cell  $i$  is determined by its utilization, which depends of the number of pedestrian agents in cell  $i$  and the maximum possible number of pedestrian agents in cell  $i$ . Therefore,

the utilization of cell  $i$  is defined by  $u_{i,k_i}^{m_i}$  with  $i=1,\dots,N$ ,  $m_i$  = maximum of pedestrian agents in cell  $i$ ,  $m_i \in \mathbb{N}$ , and  $k_i$  = number of pedestrian agents in cell  $i$ ,  $k_i \in [0, m_i]$ .

A configuration of  $L$  at time  $t$  is defined as

$Q(t) = (u_{1,k_1}^{m_1}(t), u_{2,k_2}^{m_2}(t), \dots, u_{N,k_N}^{m_N}(t))$ , where  $u_{i,k_i}^{m_i}(t) \in [u_{i,0}^{m_i}, u_{i,m_i}^{m_i}]$  is the state of cell  $i$  at time  $t$ .

The progression of  $L$  in time is then given by the network update function, also called the evolution operator,  $\Theta$

$$\Theta: Q(t) \rightarrow Q(t+1), t=0,1, \dots \quad (5)$$

The network update function contains the *walk-to-node* operation for each pedestrian agent in the network and includes the following rule:

```
If pedestrian-agent pauses or
cell linked to to-node is full-occupied
Then wait {to-node ← current-node}
Else walk-to to-node
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Fig. 7 shows a population of pedestrian agents in the previous mentioned segment of the city center at a certain moment (different colors means opposite direction).



Figure 7. Main street of the city center populated with pedestrian agents.

Striking is the apparent agents overflow at one of the shops. That has to do with the high priority of the shop, and that this shop has a lower floor. This is in contrast to the other shops in this street segment which only have a ground floor.

## VI. DISCUSSION AND FUTURE DIRECTIONS

In this paper, we presented a simulation platform for performing pedestrian movement simulation in a shopping environment. Pedestrian movement depends on pedestrian behavior, which depends of behavioral principles like perception, activation and completing an activity. The outcome of pedestrian activation is pedestrian movement on the network of shops and streets. In our approach, pedestrians are represented by agents and the network of shops and streets are represented by a lattice of irregular cells; each represented by their node. Pedestrian agents move from node to node and are situated into the cells related to those nodes. They are situated randomly in those cells, but if the area is occupied they cannot move to that cell. This reduces the complexity of the simulation by ignoring collisions and with that collision detection. This approach makes the simulation feasible because computer power is less binding.

Also, activity agenda's for pedestrian agents could be incorporated, including planned and unplanned shop stops resulting in store visits with time duration of the store visit.

In the next phase, the simulation in a 2D environment must be validated. In first instance we look at face validity,

because it appears to be a reasonable imitation of a real-world shopping environment to people who are familiar with the real shopping environment. The validation test will consist of comparing outputs from the system under consideration to model outputs for the same set of input conditions.

Future developments should make the pedestrian agent model suitable for a 3D environment with lifelike virtual persons. In that case, the pedestrian agent movement will be realized from cell point to cell point considering collision detection. Finally, this will result in a virtual environment of a real situation, populated with virtual visitors and a real visitor (user) moving amongst these virtual visitors. In that case, a user can assess an environment that has high reality content. For city managers or designer, it is possible to gain deeper insight into pedestrian activity behavior in city centers, even for those that do not exist yet.

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