Combining Finite Element and Multibody Modeling Techniques for Time-Efficient Simulation of Nonlinear Gear Dynamics

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***Abstract*—Dynamics and vibroacoustics of mechanical transmissions are perceived as one of the major concerns in the contemporary geared driveline design. These characteristics influence durability and efficiency of the system, as well as its quality. However, due to strong local effects and system nonlinearities, computer analyses of gears can be extremely time consuming, and hence, in the industrial practice, are carried out only to some limited extent. This paper describes a time-efficient multibody approach to this problem, which allows for fast analyses of high fidelity, complex numerical gearbox models. The approach presented in the paper consists of two phases: firstly, static transmission error curves, describing the gear meshing stiffness, are derived by analyzing high fidelity Finite Element models of the gears; subsequently, time-efficient multibody simulations of gear dynamics under different operating conditions are achieved by using a specifically defined user force element that allows to take the meshing stiffness variability into account thanks to the previously calculated static transmission curves.**

***Keywords*–Gears; Mechanical Transmissions; Transmission Error; Dynamics; Nonlinear Statics.**

I. INTRODUCTION

The design of a drivetrain based on a gearbox requires understanding of a number of different mechanical engineering fields, among which material strength, fatigue, noise and vibration (NV) or manufacturing issues are particularly relevant. The contemporary methods used to support mechanical design process take advantage from numerical tools, which allow to prepare and test a number of alternatives in relatively short time and without the need of preparing expensive physical prototypes. However, in the case of the mechanical transmissions, to obtain reliable and accurate results, it is required to use high fidelity, very detailed computational models. This drives a need for accurate experimental campaigns, which could be used for model validation purposes. A description of a modern gear test rig and a technique for measuring the transmission error between two meshing gears can be found in Ref. [1]. The difficulty in gear numerical modelling is driven by the fact that the phenomena observed on the surface of gear teeth are often local (e.g., Hertzian stress) and nonlinear (e.g., contact force moving along the tooth profile). The former problem is caused by high forces acting on a very small, curved contact surface of the teeth. High, Hertzian stress develops locally and causes local material deformation, which influences the teeth contact mechanics. The nonlinearities instead, can have various sources, among which the most significant are connected with the number of teeth staying in contact during gear rotation (expressed by the so-called contact ratio number - see Figure 1) and by nonlinear deflection of a tooth, which is caused by contact force traveling along its profile, which normally has uneven thickness from the bottom, to the top. In particular, three-dimensional dynamic analyses aimed at assessing drivetrain NV characteristics are often very time consuming and require significant computational power. To overcome this problem, it is common to describe the analyzed gears by simplified, one-dimensional or planar models [2] [3] [4], which can cover the general behavior of the system, representing roughly its deflection caused by loading forces. In the same time, however, they neglect important phenomena influencing its performance: misalignments, shuttling or teeth microgeometry modifications. These factors contribute considerably to dynamic loading of the gears, causing gear excitation and, as a consequence, system vibrations.

The term of misalignment refers to inaccuracies in gear relative positioning, which result in unsymmetrical load distribution on teeth surfaces. These positional errors can be caused by lack of parallelism between the shaft rotation axes, deflection of the mechanism components or manufacturing errors (e.g., gear or shaft eccentricity). Shuttling, which is a fluctuation in the axial position of the resultant contact force (i.e., along the rotation axis), leads to oscillations on bearing forces and dynamic moments in the plane of action (i.e., the plane spread over the contact force vectors, along the axial direction). Shuttling happens intrinsically for gears with inclined teeth (i.e., helical gears), due to the traveling contact areas, from one corner of the tooth surface to the opposite, when the gears spin. Other reason can be due to shifts in contact stress distribution caused by gear misalignment [5]. Microgeometry modifications (e.g., tip relief, crowning) are applied often to the gears in order to improve the interaction between the teeth and to decrease local stress concentration on the teeth edges [6]. In Ref. [7], the authors analyzed the dynamic characteristics of a gear pair, which was subjected to excitation caused by external forces and geometry inaccuracy. This paper provides a wide theoretical insight into the gear dynamics problem, proposing an analytical approach which can be used to solve it. The authors described also numerical calculations, which were aimed at understanding the influence of forcing parameters on the analyzed system. However, the proposed method does not rely on MB modelling technique, severely limiting its applicability in the industrial practice.

Other factors which should not be neglected when gears are under NV analysis are the variable meshing stiffness and the Transmission Error (TE). The former describes fluctuation

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of gear pair stiffness and is caused by uneven load distribution among the teeth during the gear rotational movement (see Figure 1). It is governed by the gear contact ratio and is load dependent. Nonlinear stiffness encountered in mechanical transmissions was studied by using a piecewise representation in Ref. [8], which describes the analytical and numerical approaches towards the solution for such problems. The TE is defined as a deviation from a perfect motion of gears, which can be caused by different reasons: misalignments, teeth deflection under load, manufacturing inaccuracies and others. All these factors constitute for mechanical transmission complex dynamic behavior and should be taken into account during the design phase. One approach to solve this problem is based on the Finite Element Method (FEM), which is capable of solving accurately even the most complicated design cases. However, its major drawback, i.e., the simulation time, precludes its application to demanding problems analyzed in the time domain. In Ref. [9], the authors took the endeavor to simulate gear interaction by using comprehensive FEM approach and to tune specially prepared one degree-of-freedom (DoF) models, to be able to represent its complex nonlinear behavior. As they reported in the paper, they were able to obtain either good correlation with the results in frequency domain or satisfying meshing stiffness representation, but not both simultaneously. Similar studies were described in Ref. [10], in which the authors took the efforts aimed at correlating the results from FEM-based and analytical, one DoF dynamic models with the experiments.

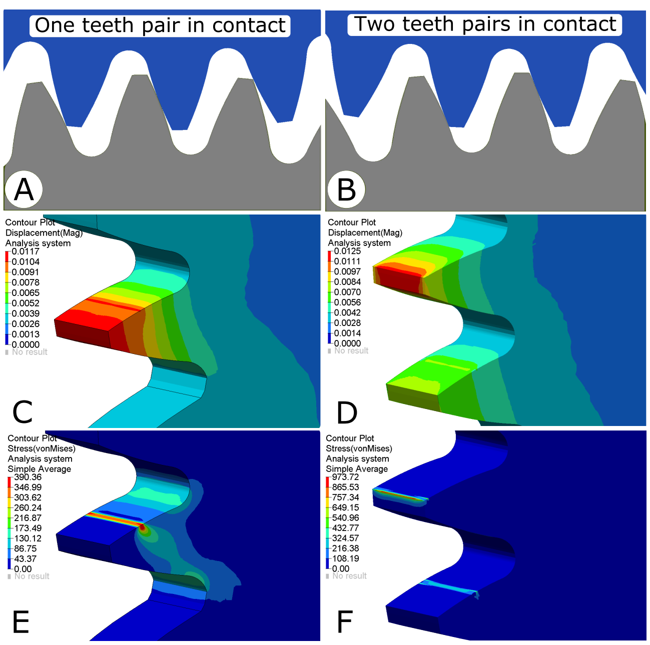

Figure 1. Analysis of gear rotation angle, for which one (A) and two (B) pairs of teeth stay in contact. Figures C and D show the teeth deflection pattern, while local Hertzian stress is presented in figures E and F.

In this paper we present a numerical study on the dynamics of a spur gear pair, which was carried out on a multibody (MB) model by implementation of a methodology reported in Ref. [11] and [12]. The utilized approach is capable of analyzing complicated geared drivelines under variable loads and operating conditions within computational time, which is orders of magnitude shorter than in the case of the adequate FEM-based runs.

The subsequent paragraphs describe the two phases of the proposed simulation approach: firstly, static transmission error (STE) curves, describing the gear meshing stiffness, are derived by analyzing high fidelity finite element (FE) models of the gears; subsequently, time-efficient MB simulations of gear dynamics under different operating conditions are achieved by using a specifically defined user force element that allows to take the meshing stiffness variability into account thanks to the previously calculated static transmission curves. It is shown that the utilized MB technique can be successfully used for dynamic simulation of gear meshing and is able to capture its nonlinear, load-dependent behavior. The described analyses were carried out in velocity run-up conditions, for the models loaded by torque of variable values. Because of the required computational time, the same results would not be attainable using the FEM approach.

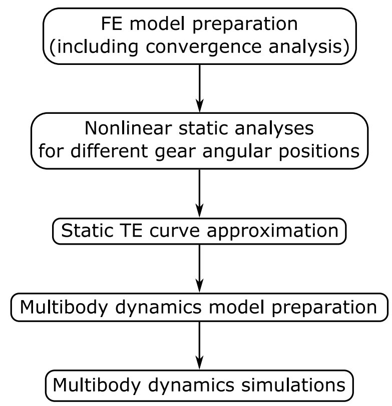

Figure 2. Flowchart of the proposed methodology for gear dynamic simulation.

The paper is structured in the following manner: Section II summarizes the proposed approach, describing in short each activity which must be taken to carry out time-efficient gearbox simulations. Section III provides a detailed description of the prepared FE model and provides a description of the mesh convergence analysis. Section IV presents and compares different methods of calculating the TE curves, allowing for choosing the most efficient approach for STE estimation. Section V describes the implementation of the described methodology in order to understand the dynamic behavior of a spur gear pair in run-up conditions.

II. Gear dynamic simulation methodology

Based on the considerations presented in Section I, we can now describe the methodology for gear dynamic simulations, which is proposed in this paper. For the sake of clarity, the flowchart of the simulation procedure is presented in Figure 2. As mentioned above, the elaborated procedure is based on derivation of a STE curve, which is computed using FEM simulations. As shown in the subsequent paragraphs, nonlinear static analyses are sufficient to describe the desired behavior of the analyzed system. This operation is executed iteratively, for a set of discrete gear angular positions. Following, the STE curve is derived by approximation of the results obtained in the preceding phase. This constitutes an input for the MB simulations, which are carried out on a MB model, prepared using the contact formulation described in Ref. [11] and [12].

MB model prepared in this manner can be used for fast and accurate dynamic simulations of complex gear trains, requiring relatively short computation time.

III. FE MODELING AND ANALYSIS OF THE SIMULATED SYSTEM

The analyses described in this paper were carried out on a pair of meshing, high-precision, identical spur gears, with involute tooth profile, for which Table 1 summarizes the main geometrical data and design specifications.

TABLE I. GEAR DATA AND DESIGN SPECIFICATIONS.

Parameter	*Value*
Number of teeth	57
Normal module	2.60mm
Normal pressure angle	20deg
Tip diameter	154.50mm
Root diameter	141.70mm
Facewidth	23mm
Normal circular tooth thickness at theoretical pitch circle	3.78mm
Contact Ratio	1.45

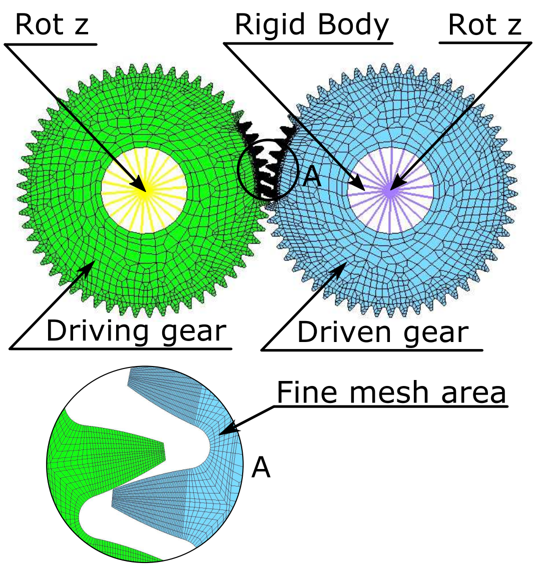

Figure 3. Three-dimensional FE model of a spur gear pair used in the described analyses.

The FEM models of a gear pair, which are depicted in Figure 3, were built using the first-order, eight-noded hexagonal elements. The deformation of this type of finite element is described by three translational DoF in each corner node, resulting in 24 DoFs per element. The shape functions describing the stress field in the elements were linear. In order to prevent from hourglass deformations (numerical, erroneous element deformations), the element stress was retrieved by full integration scheme. The model preparation and simulations were performed in Altair HyperWorks software. Two types of analyses were carried out: implicit statics with geometrical nonlinearities with Altair OptiStruct and explicit dynamic simulations with Altair Radioss. Depending on the simulation type, two distinct modeling strategies were used [13].

In the case of the explicit dynamic analyses, the contact between the interacting elements was modeled by using the interface type 24, with node to surface definition and Coulomb friction ($\mu = 0.3$). The kinematic constraints applied to the gear centers allowed only for rotational displacement. The loading torque was generated by a viscous damper connected to the center of rotation (CoR) of the driven gear. A proper rotational velocity of the driving gear CoR was set through the defined boundary conditions.

In the case of the static analyses, geometrical nonlinearities were taken into account. This assured high accuracy of the results. In the case of these computations, the contact between the teeth surfaces was modeled by means of the interface type 7. The loading torque of an appropriate value was applied to the CoR of the driving gear, while the rotation center of the driven gear was fixed in all DoFs.

A. *FE mesh convergence analysis*

In order to reproduce the local phenomena of the simulated system (i.e., deflection and Hertzian stress) while keeping the computational time as low as possible, the model was divided into two areas of coarse and fine mesh. The former was used in the region where there was no interaction between the gear teeth and, hence, no local stress and contact pressure concentrations were present. In this region, the mesh size was set up to 8 mm. In the area in which the two gears interacted with each another, the FE mesh size was decreased. In order to understand the significance of the FE element size, a convergence analysis was carried out, showing the sensitivity of the STE values towards this parameter. Figure 4 depicts the chosen results of these trials: based on these findings it was decided to set the smallest element edge length to 0.09 mm. As depicted in Figure 4, the STE curve obtained for this element size was qualitatively and quantitatively equal to the equivalent calculated using the model described by a finer mesh (element edge length of 0.035mm along a tooth profile).

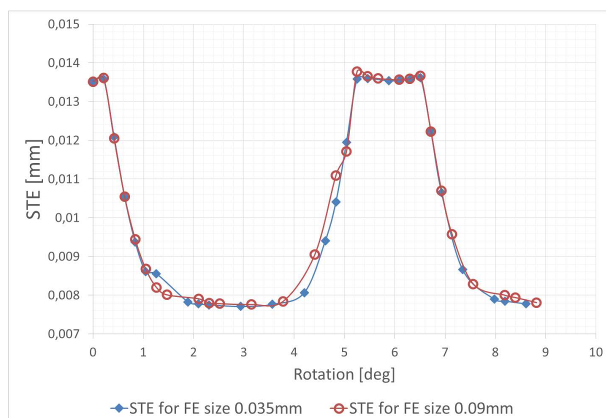

Figure 4. The results of the convergence analysis showing the STE curves obtained for different FE mesh size (torque T=350Nm).

For the NV assessment, the most important outcome of the convergence analysis is the peak-to-peak value of the STE curves and the contact ratio, which is the average number of meshing tooth pairs and can be derived based on the STE curve shape. These two parameters constitute for the internal excitation force of a gearbox, in its operational conditions. As shown in Figure 4, both of these characteristics were covered accurately by the tested models.

The TE curves were calculated by subtracting the angle of rotation (AoR) of the driven gear from the AoR of the driving gear, according to (1):

$$TE = (\theta_P + \theta_G)_{r_B} \quad (1)$$

where θ_P and θ_G are the AoR for the driving and of the driven gear respectively, and r_B denotes the radius of the base circle of the gears.

IV. METHODS TO COMPUTE STE CURVES

In order to use the MB approach described in the Introduction section, an STE curve needs to be provided by the user as an input. The generation of this curve can be done by FE approach. In order to understand if the time efficient nonlinear static approach can result in an accurate estimation of the STE curve, it was compared with the detailed explicit dynamic simulation outcomes. The significant drawback of the latter is the computational time, which depends on the FE mesh size and therefore in the case of the described type of simulations it can be very long.

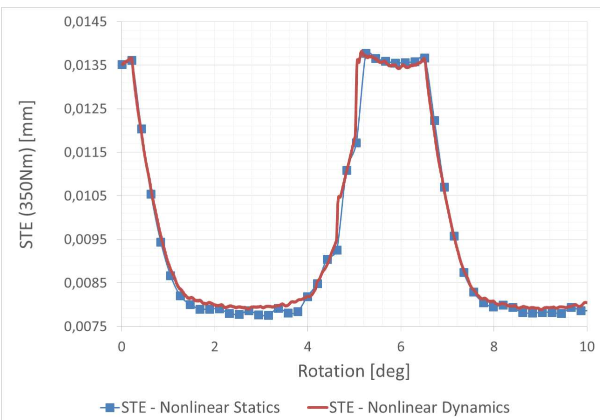

Figure 5. STE for 350Nm torque - comparison between static and dynamic approach.

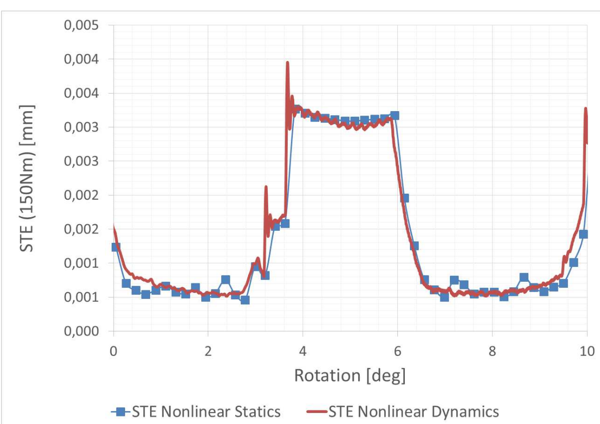

Figure 6. STE for 150Nm torque - comparison between static and dynamic approach.

In the case of the dynamic simulations, the STE curves were obtained by simulating gear rotation with a velocity run-up from zero to a very low, constant rotational speed value (10 rpm), which allowed neglecting inertia contribution to the results. The procedure involving nonlinear static simulations was based on a number of FEM implicit analyses carried out for a discrete rotation angles of the gear pair. The angle step of the driving gear rotation was set to 0.21 deg, which resulted in 30 points per one meshing cycle. This number guaranteed a smooth representation of the STE curve, requiring a relatively low number of simulations, in the same time.

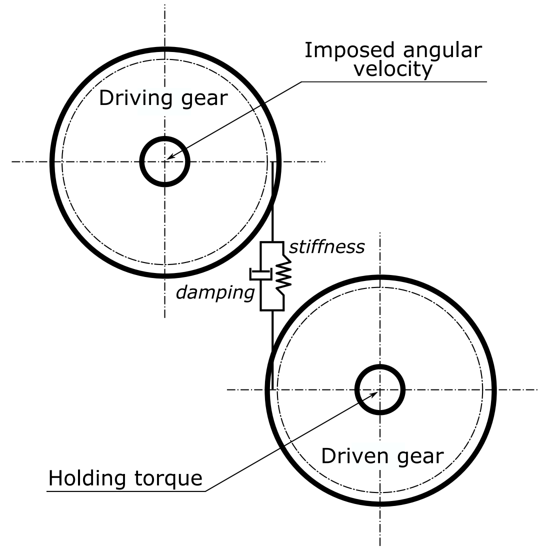

Figure 7. Diagram of a MB model of the simulated gear pair, showing the viscoelastic contact element between the gear teeth (based on [11] and [12]).

The comparison between the statically and the dynamically generated STE curves is shown in Figure 5 and Figure 6. Two different loading torque values equal to 150Nm and 350Nm were used in the simulations. It can be seen from the figures that both FEM methods resulted in quantitatively and qualitatively correlated results. The significant difference, however, lied in the computational time needed for the analysis to converge, which was equal to ca. 150 minutes and 3330 minutes for nonlinear static and dynamic analysis, respectively.

The obtained results allowed to choose the most suitable method for the generation of the STE curves, which were needed for the subsequent MB analyses, carried out with the approach proposed in Ref. [11] and [12]. Since the approach based on nonlinear static FEM calculations was significantly more time efficient, it was selected to generate the input data for the MB simulation.

V. MULTIBODY GEAR ANALYSIS

The implementation of the multibody technique requires that the gear geometrical description (i.e., number of teeth, diameter, pressure angle) is provided simultaneously with the data describing STE, spatial misalignments, torque variability and microgeometry modifications. All of this information influences the dynamic response of the system, allowing computing the dynamic transmission error (DTE). The presented in Figure 7 and Figure 8 MB model was built in the Siemens LMS Virtual.Lab 13.1 environment [14], in a manner which was consistent with the FEM representation of the gear pair described above. The constraints used in the MB model allowed rotations of the gear centers of rotation. The loading torque was applied to the driven gear, while the rotational velocity was imposed for the driving gear. Moreover, as shown in Figure 7, the implemented MB gear contact element assumed viscoelastic behavior of interacting teeth.

The DTE curves presented in Figure 9 and Figure 10 were obtained under rotational speed linear run-up conditions, from 0 rpm to 3500 rpm and for two values of the loading torque: 150Nm and 350Nm. The simulated operating time was set to 70 seconds. Since the two gears had a number of teeth equal to 57, the type of excitation due to meshing stiffness variation along one meshing cycle could be considered as a multi-harmonic frequency sweep. With the maximum rotational velocity, it generated an excitation fundamental frequency of 3325 Hz.

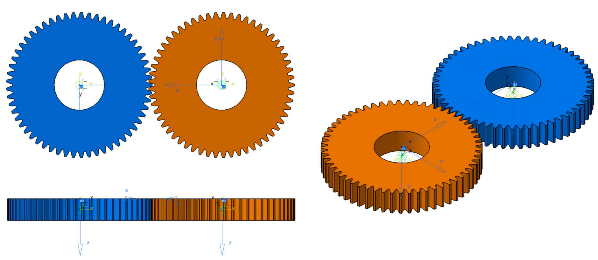

Figure 8. Gears model in multibody dynaminics simulation environment.

By looking at the envelopes of the obtained DTE curves, some local amplitude amplifications can be seen. These corresponded with the gear pair resonance, excited by the TE, due to gear meshing stiffness variability. In the case of the system loaded by the 150Nm torque (Figure 9), the most significant DTE amplifications were found at 116Hz, 438Hz, 597Hz, 907Hz and 1804Hz, while for the same mechanical transmission loaded by the 350Nm torque (Figure 10), the DTE amplifications were found at 122Hz, 479Hz, 650Hz, 980Hz and 1958Hz. The differences between these values are summarized in Table II.

Indeed, in the utilized modeling method, the gear meshing stiffness variation is derived from the STE curve supported by the user and shows a stiffening behavior which is typical of contact problems [15]. Based on its periodic, time-varying characteristics it was possible to cover the nonlinearities present in the described system. In the discussed analyses, the STE curves depicted in Figure 5 and Figure 6 were used.

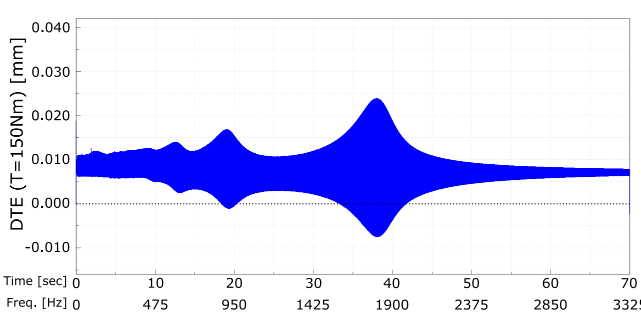

Figure 9. DTE calculated in run-up conditions, for spur gears pair loaded by 150Nm torque.

The obtained results prove the capabilities of the used three-dimensional MB modeling methodology, based on the FE gear contact formulation described in Section III. The depicted in Figure 9 and Figure 10 gear pair behavior could not be captured by consideration of a standard one-dimensional or planar, linear representation of interacting teeth, described in the introductory paragraph - Section I. This is because these simplified models neglect the contact stiffness variability along the tooth profile. On the other hand, if applied, more comprehensive FEM analyses could have resulted in the same results, as it was shown in Ref. [2]. However, in the presented case, the computational time was significantly lower compared with a possible dynamic FEM calculations (e.g., shown in Section III), and was equal to ca. 36 minutes on a standard desktop computer: i7@3.2GHz CPU, 32GB RAM.

TABLE II. THE DTE AMPLIFICATIONS IDENTIFIED FOR THE ANALYSED SPUR GEAR PAIR IN RUN-UP CONDITIONS.

DTE amplification id	*Loading torque* 150Nm	350Nm	*Difference* Δ	Δ[%]
1	116	122	6	5.17
2	438	479	41	9.36
3	597	650	54	9.05
4	907	980	72	7.94
5	1804	1958	153	8.48

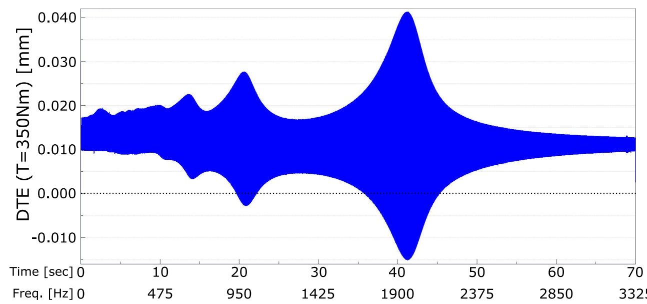

Figure 10. DTE calculated in run-up conditions, for spur gears pair loaded by 350Nm torque.

VI. CONCLUSION AND FUTURE WORK

It was shown in the paper that it is possible to analyze the nonlinear behavior of a meshing gear pair, using a simplified, yet efficient MB analysis technique. By application of this method, the time needed for a comprehensive analysis of a driveline system is orders of magnitude lower, when compared with the equivalent dynamic FEM calculations. This allows to support the industrial design process with an accurate and time-efficient simulation tool, which would make feasible the assessment of different gear design variants during the virtual prototyping phase. Moreover, the proposed methodology for the DTE estimation can be effectively employed for the purposes of driveline structural optimization, which requires a number of iteratively carried out simulations and hence, can be very time consuming. Because of the numerical complexity, this would not be possible using the classical FEM-based approach. The activity scheduled as the nearest future step of the research work presented in this paper is aimed at improving gearbox noise and vibration characteristics, by application of teeth profile microgeometry modifications. By implementation of the described methodology and because of its time-efficiency, the optimization approach can be more comprehensive, including different load cases and operational conditions. This will ease convergence to a global optimum of the imposed problem.

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