Modelling Drivers' Route Choice Behaviour through Possibility Theory Using Driving Simulator

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***Abstract*— This paper presents a modelling approach based on the Possibility Theory to reproduce drivers' choice behaviour under Advanced Traveller Information Systems (ATIS). The Possibility Theory is introduced to model uncertainty embedded in human perception of information through a fuzzy data fusion technique. Drivers' choice models are often developed and calibrated by using, among other, Stated Preferences (SP) surveys. An experiment is presented, aimed at setting up an SP-tool based on driving simulator developed at the Technical University of Bari. The obtained results are analysed in order to compare the outcomes of the proposed model with preferences stated in the experiment.**

Keywords- modelling; route choice behaviour; possibility theory; data fusion; stated preference; driving simulator

I. INTRODUCTION

The study of travellers' behaviour in Advanced Travellers Information Systems (ATIS) contexts is a crucial task in order to properly simulate phenomena like compliance with information, route choices in presence of information, etc. Several researchers have dealt with conceptual models of drivers' behaviour under information provision. Basic idea in these models is that each driver updates his/her knowledge of costs of alternatives using provided information. Then, he/she compares the updated costs of alternatives and chooses, among them, the best one from his point of view. However, since both knowledge of alternatives and information are rarely perfect, uncertainty affects single person's decision. Handling uncertainty is therefore an important issue for these models. Approaches followed by different scientists to face this issue can be arranged into two main groups, according to how uncertainty has been modelled. In the first group, the approach followed generally uses randomness to represent uncertainty. For this kind of models, unavailability of full numerical data could limit the model reliability; in fact, these models are generally unable to handle non-numerical values of parameters. On the contrary, models included in the second group can easily model un-certainty through verbal, incomplete or imprecise data using the concepts of Fuzzy Logic. In fact, the fundamental concepts of Fuzzy Sets Theory, linguistic variables, approximate reasoning, and computing with words introduced by Zadeh have more understanding for uncertainty, imprecision, and linguistically articulated observations. These concepts support "the brain's crucial ability to manipulate perceptions-perceptions of distance, size, weight, colour, speed, time, direction, force, number, truth, likelihood, and other characteristics of physical and mental objects. A basic difference between perceptions and measurements is that, in general, measurements are crisp whereas perceptions are "fuzzy" [1]. First, Teodorović and Kikuchi [2] proposed a route choice model based on Fuzzy Set Theory. In order to estimate models of the travellers' behaviour, observation of reactions is needed. The most adopted approach for collecting data is the Stated Preferences (SP) one. Two main types of tools for SP in ATIS contexts are the most popular: driving-simulators (DSs) and travel-simulators (TSs). Both methods are computer-based. DSs are characterised by a greater realism, provided that the respondents are asked to drive in order to implement their travel choices, as it happens in the real world. In TSs, travel choices are entered after having received a description of travel alternatives and of associated characteristics, without any driving. In most cases, data have been collected by using TSs, as for instance in [3][4][5][6][7]; while only a limited number of studies have been carried out by adopting DSs [8][9][10][11][12]. In this work, an experiment has been carried out at the Technical University of Bari. Other than the assessment of the internal consistency of the experiment allows for validating the simulation environments, thus showing that more trials and experiments worth to be implemented. The network reproduced in the virtual experiment refer to real one in Bari (Italy). Moreover, the respondents recruited for the experiments were travellers familiar with the networks. The network has been proposed to respondents in the simulations in a double configuration, with and without ATIS. In turns, the configuration without ATIS was presented to respondents with some variants, reproducing different congestion levels and travel times, accordingly with their statistical distribution in the real world. The paper is structured as follows. In the next section, the possibility theory and the data fusion analytical formulation are presented. In section 3, the experiment designed to acquire information about drivers' choice behaviour through a driving simulator is described. In section 4, results of the proposed model are carried out and, in the last section, conclusions are reported.

II. MODELLING PERCEPTION AND DATA FUSION

An informative system, like the ATIS, may provide information to users before they begin the trip (pre-trip

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information) or while they are moving (en-route information). In the first case, static choice models are involved; in the second case, dynamic ones. In both cases, travellers combine information with their previous experience to obtain a prediction about the cost of each path and to choose the best one.

To incorporate information on system conditions in the choice process, we assume that drivers:

- have some experience about the attributes of the transportation system;
- use information to update his experience;
- choose an alternative according to his updated experience.

Since the drivers' knowledge about the transportation system could be imprecise or approximate, it can be expressed in the same way we used for perceived information. So, both drivers' knowledge and information can be expressed in terms of Possibility, like in Figure 1.

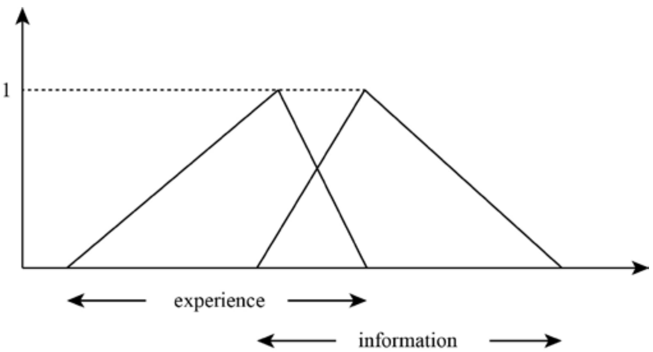

Figure 1. Possibility distributions of experience and information

To update knowledge of the system, drivers aggregate data coming both from their experience and from current information. However, aggregation could not be always meaningful, since data coming from different sources can be far from each other, and thus not compatible. Therefore, a suitable aggregation function should include also a measure of compatibility.

To measure compatibility, Yager and Kelman [13] proposed the relationship $R: X^2 \rightarrow I = [0,1]$ such that:

- $\forall x \in X$, $R(x,x) = 1$
- $\forall (x,y) \in X^2$, $R(x,y) = R(y,x)$
- For a given x, $R(x,y)$ is a convex fuzzy set.

A suitable expression for a compatibility function R defined in X^2 is :

$$R(x_1, x_2) = \begin{cases} 0 & \text{if } |x_1 - x_2| > k \\ 1 - \frac{1}{k}|x_1 - x_2| & \text{if } |x_1 - x_2| \le k \end{cases} \quad (1)$$

where k must be carefully selected case by case, to obtain a proper compatibility measure. Extension to X^n of R is also possible, through the relation:

$$\mathbf{R}(x_1, \ldots x_n) = \min_{i,j=1,\ldots n} R(x_i, x_j) \quad (2)$$

In this paper, we have used for data fusion the *Ordered Weighted Average* (OWA) operator and the compatibility function R, defined in (1).

Given a set $A = \{a_1, a_2, \ldots a_n\}$ and a fusion function F, an OWA operator is a weighting vector $W = [w_1, \ldots w_n]$ such that:

- $w_i \in [0,1]$;
- $\Sigma_i w_i = 1$;
- $F(a_1, a_2, \ldots a_n) = \Sigma_i b_j w_j$

in which b_j is the j-th largest element of A. By adjusting the weighting vector, we can represent different drivers' attitudes: when W favours the smaller valued arguments in the aggregation process it reflects an *aggressive* driver, otherwise it reflects a *cautious* driver.

O'Hagan [14] suggested a method to calculate the weights w_i ($i = 1, \ldots n$) through the following simple mathematical programming problem:

$$\text{Maximize} \quad -\sum_{i=1}^{n} w_i \ln w_i$$

$$\text{subject to} \quad \begin{cases} \sum_{i=1}^{n} w_i \, h_n(i) = \beta \\ \sum_{i} w_i = 1 \\ w_i \ge 0 \quad \forall i \end{cases} \quad (3)$$

where $h_n(i) = \frac{n - i}{n - 1}$, and $\beta \in [0,1]$ is a coefficient representing, in our case, drivers' cautiousness. Note that, if fusion involves only two sets, then $h_2(1) = 1$, $h_2(2) = 0$. Thus, from the constraints of previous program (Eq. 3):

$$w_1 = \beta, \quad (4)$$

$$w_2 = 1 - \beta. \quad (5)$$

The basic hypothesis we have made in this work to set up a value of β, is that drivers' cautiousness is a function of uncertainty related to perceived information. Let us explain this last concept through an example. Assume that the shorter one of two alternative paths is temporarily closed by barriers. In this case, information that path is closed is not uncertain, that is $U(I) = 0$, and drivers must choose the longer path. This means that the OWA operator should favour the largest value, that is $w_1 = 1$, and consequently $\beta = 1$ from (4). Conversely, if instead of barriers there is an informative system giving very vague information about the condition of the path, uncertainty $U(I)$ is very large, and drivers should prefer to rely on their own experience. In this case, the OWA operator favours the smallest value, that is w_2 approaches 1 and thus, from (5), β approaches 0. From this example, it appears that the parameter β can be interpreted also as drivers' compliance with information. In fact, $\beta = 1$ means that the driver is totally compliant with information, $\beta = 0$ means the opposite.

Experimental studies have been carried out in last years by some researchers to find out a value of drivers' compliance. Different values, ranging from 0.2 to 0.7, have been found, mainly due to the fact that β is affected by the level of uncertainty imbedded in information. In this study we have assumed that:

- drivers' compliance with information decreases with increasing of uncertainty. This means that the relative elasticity of compliance with respect to uncertainty is negative. In analytical terms: $\frac{d\beta/\beta}{dU(I)/U(I)}<0$;
- the increase of compliance with additional information is greater in case of ignorance than in case of complete knowledge. That is, the relative elasticity is a function of uncertainty itself.

On the basis of these hypotheses, the following linear relationship between relative elasticity uncertainty level has been carried out:

$$\frac{d\beta/\beta}{dU(I)/U(I)} = -\gamma\cdot Y(I) \qquad (6)$$

and hence:

$$\beta = \frac{1}{e^{\gamma U(I)}} \qquad (7)$$

where γ is a parameter to be calibrated, which takes into account individuals' attributes like age and gender. When n different sources provide information I_i (i=1,…n) with uncertainty $U(I_i)$, compliance rate is calculated as

$$\beta = \min_{i=1,\dots n} \frac{1}{e^{\gamma U(I_i)}} \qquad (8)$$

Now, to incorporate the compatibility concept in the fusion function, we follow the method suggested in [13]. Therefore, let

- A_i (i= 1,…n) be a collection of fuzzy sets. Recall that fuzzy sets and Possibility distributions can be represented in the same way;
- $B = F(A_i)$ be the result of aggregation;
- $A_{i\alpha}=[\, l_{i\alpha}, r_{i\alpha}]$ be the α-cut associated with A_i;
- $l^*_\alpha = \max_i[l_{i\alpha}]$ be the largest lower bound of any α-cut;
- $r^*_\alpha = \min_i[r_{i\alpha}]$ be the smallest upper bound of any α-cut;
- $U^*_\alpha = \inf\{x \mid R(l^*_\alpha,x) \geq \alpha\}$ be the smallest value compatible with l^*_α at level α;
- $V^*_\alpha = \sup\{x \mid R(r^*_\alpha,x) \geq \alpha\}$ be the largest value compatible with r^*_α, at level α.

Provided that $U^*_\alpha \leq r^*_\alpha$ and $V^*_\alpha \geq l^*_\alpha$, the α-cut of B can be calculated as:

$$B_\alpha = [F(d_{1\alpha},\dots d_{n\alpha}), F(e_{1\alpha},\dots e_{n\alpha})] \qquad (9)$$

where:

$$d_{i\alpha} = \begin{cases} l_{i\alpha} & \text{if } l_{i\alpha} \geq U^*_\alpha \\ U^*_\alpha & \text{otherwise} \end{cases} \qquad e_{i\alpha} = \begin{cases} r_{i\alpha} & \text{if } r_{i\alpha} \geq V^*_\alpha \\ V^*_\alpha & \text{otherwise} \end{cases} \qquad (10)$$

The information fusion model incorporates important aspects, such as:

- dynamic nature of information integration. The perceived cost of an alternative is influenced by the user's previous experience and memory;
- accuracy of the informative system. The more accurate information is, the more important is the effect on the drivers' perception;
- non-linear relationship between information and perception.

The parameter β itself is function of information, so that the updated cost is a non-linear function of information.

Possibility is a useful concept in representing decision-maker's uncertainty about the attributes of individual alternatives, but cannot be used directly by analysts; for this reason, a conversion to Probability values on the basis of a justifiable principle is needed.

To pass from Possibility to Probability we use the probabilistic normalization ($\sum_i p_i = 1$), along with the Principle of Uncertainty Invariance, systematized by Klir and Wang [15]. This principle specifies that uncertainty in a given situation should be the same, whatever is the mathematical framework used to describe that situation.

Under the requirement of normalization and uncertainty equivalence, we should use a transformation having two free coefficients. Thus, according to Geer and Klir [16], we use the log-interval scale transformations having the form:

$$\Pi_i = \beta\cdot(p_i)\ \alpha \qquad (11)$$

where Π_i is Possibility and p_i Probability of the i-th alternative; α and β are positive constants.

From (11) we obtain: $p_i = (\Pi_i/\beta)^{1/\alpha}$ and, applying the probabilistic normalization, $\beta=\left(\sum_i \Pi_i^{1/\alpha}\right)^{\alpha}$ whence, setting $\varepsilon = 1/\alpha$:

$$p_i = \Pi_i^{\varepsilon} \Big/ \left(\sum_i \Pi_i^{\varepsilon}\right) \qquad (12)$$

To calculate ε, we use the Principle of Uncertainty Invariance. Given an ordered Possibility distribution $\{\Pi_1, \Pi_2,\dots \Pi_i, \Pi_{i+1},\dots \Pi_n\}$ for which is always the case that $\Pi_i \geq \Pi_{i+1}$, the possibilistic counterpart of the probabilistic uncertainty, called U-Uncertainty, is given by the following function:

$$U = \sum_{i=1}^{n} \left(\Pi_i - \Pi_{i+1}\right) \ \log_2 i \qquad (13)$$

According to the Principle of Uncertainty Invariance, information I and uncertainty U must have the same value:

$$-\sum_{i=1}^{n} p_i \ \log_2 p_i = -\sum_{i=1}^{n} \frac{\Pi_i^\varepsilon}{\sum_{j=1}^{n}\Pi_j^\varepsilon} \ \log_2 \frac{\Pi_i^\varepsilon}{\sum_{j=1}^{n}\Pi_j^\varepsilon} = \sum_{i=1}^{n} (\Pi_i - \Pi_{i+1}) \ \log_2 i \qquad (14)$$

where $\Pi_{n+1} = 0$ by definition. Numerical solution of (14) is always possible, except when $\Pi_i = K\ \forall i$, $K\in[0, 1]$. However, in this case, from (12) we can easily obtain $p_i = 1/n\ \forall i$.

III. DESIGN OF THE EXPERIMENT

In order to carry out the SP experiments, a PC-based driving simulator of Technical University of Bari has been adopted (Figure 2).

The UC-win/Road driving simulator software was used. This software is developed by FORUM8, a Japanese company. UC-win/Road is plugin-based, allowing to extend software functionalities by using the UC-win/Road SDK Framework that allows for Delphi code. In our case, a plugin was created for data acquisition during driver's simulation, allowing to record for successive analyses (and in CSV format) data related to speed, position, steering, etc. In particular, we have employed data related to position in post-processing in order to observe route choices made by respondents. The simulation system works on a single computer provided with NVidia Graphic Card (1Gb of graphic memory) and a Quad-Core CPU, which guarantees very good real-time rendering and computation performances. The simulation is based on a steering wheel (Logitech™ MOMO Racing Force Feedback Wheel), able to provide force feedback, as well as six programmable buttons (ignition, horn, turn signals, etc.), sequential stick shifters and paddle shifters. A 22" wide-screen monitor was used in order to have a good field of view, also showing internal car cockpit with tachometer and speedometer. Environmental sounds are reproduced to create a more realistic situation.

During the experiment respondents have been asked for choosing a route among three alternatives. The context is configured in such a way that the choice can be assumed as a (possible) switching from a natural reference alternative. As already discussed, respondents were recruited for the experiment ensuring a familiarity with the experimental context. In fact, the simulated networks were part of a real network in Bari (Figure 3). The choice set can be viewed as composed by a main route (route 1) that connects the considered origin-destination pair. Depending on traffic conditions, the traffic could spill-back up to a later diversion node (detour toward route 2) or even up to an earlier diversion node (detour toward route 3). These three different conditions (straight route, later detour, earlier detour) are conventionally classified here as three different levels of congestion (free-flow/low congestion, intermediate congestion, high congestion).

The experiment has been designed in order to have in most of the times (70%) the system in the intermediate congestion pattern, even if extreme (low and high) congestion levels are less frequent. Before starting the simulation, respondents can adapt themselves with the simulator by driving along each alternative route of the choice set, without ATIS and in free-flow traffic conditions. After this possible training, respondents are asked to make 6 successive trials, grouped in 3 driving sessions. At each session, respondents drive twice. The Variable Message Signs (VMSs) representing the ATIS can be active or not in a random way. The activation of the ATIS is a consequence of an accident occurred, that perturbs the standard traffic pattern to an extent that depends on the accident severity.

Figure 2. Screenshot of the Driving Simulator

Figure 3. Map of the considered network (Bari, Italy)

TABLE I. LOCATION OF VMSS AND MAIN RAMPS

From	To	Distance (m)
Entrance (A)	1st VMS	400
1st VMS	I Diversion, Exit 13A-Mungivacca	300
I Diversion, Exit 13A-Mungivacca	2nd VMS	700
2nd VMS	3rd VMS	1100
3rd VMS	II Diversion, Exit 12-Carrassi	150
II Diversion, Exit 12-Carrassi	Queue/Accident	900
Queue	Exit 11-Poggiofranco	500

The trials are called "*without information*" when the VMSs are not activated; otherwise trials are called "*with information*". Moreover, two messages are provided through ATIS: 'queue' and 'accident', displayed in a random way during all trials. At the first session, respondents can make their choices without information to enforce their perception of the realism of the simulation in terms of consistency with the real network he/she is used to. Then, at a second session, respondents are assisted by information ('queue' or 'accident'). Respondents are provided with information by VMSs located on the main road as reported in Table 1.

The first VMS is 300 meters before the early diversion (Mungivacca, toward route 3), the second is 1250 meters before the late diversion (Carrassi, toward route 2) and the third 150 meters before the same Carrassi diversion node. A queue starts in all cases 900 meters after the later diversion

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node and 500 meters before the exit-ramp of Poggiofranco. Depending on the simulated congestion level, the queue can spill back more or less. In this scenario VMSs display the presence and the position of the queue, but not queue length or estimated queuing time. At the end of each trial, respondent are asked to answer a questionnaire, where they state, as well as other questions, the chosen route, their perception of the delay due to the provided information (minimum, most expected and maximum time value), if any, and the travel time related to their experience on that route. These data are required to define the triangular fuzzy numbers related to their perceptions in order to apply the data fusion technique previously described.

IV. RESULTS

Recruitment was performed at the Technical University of Bari and 10 respondents were randomly selected. Such a small number of respondents is consistent with the pilot nature of the study.

In trials with information, a respondent perceives a delay as he/she reads the message provided by the VMSs. This delay depends not only on subjective factors, but also by the content of the information itself. Table 2 shows the average time in minutes of perceived delay in terms of triangular fuzzy numbers. We can observe that the average time for the message received 'queue' is lower than that for the message 'accident'. This effect can be imagined because the consciousness of the presence of an accident is definitely more effective in terms of perceptions compared to a queue.

Tables 3 and 4 show the route choice percentages stated at the end of the experiment and compared with the outcomes of the proposed model. Without information (Table 3), it appears that respondents tend to follow the fastest route (R1, Poggiofranco). The changing conditions of the traffic and the presence of queue or incident events does not affect the experience of those who tend to choose the fastest route R1 (87%), in normal traffic, 10% of respondents choose the intermediate route R2 and 3% the slowest route R3.

In the presence of information (Table 4), the choice behaviour varies according to the message displayed on the VMSs. The resulting choice behaviour is consistent with the perceptions of time delays derived from the information provided. A greater perceived delay leads to an increased tendency to abandon the preferred path, turning over other available alternatives. As previously described, the model is characterized by the presence of two parameters k and γ (Eq. 1 and 8). These parameters are used to calibrate the compliance with the information system in relation to the provided information. The aim is to find the optimal values of these parameters for each respondent in order to better reproduce the observed behaviour. For sake of simplicity, we have calculated the optimal values of these parameters using Genetic Algorithms (GA) to minimize the root mean square error (RMSE) between the percentages of observed choices and those predicted by the model.

TABLE II. PERCEIVED DELAY TIME (FUZZY VALUES IN MINUTES) IN TRIALS WITH INFORMATION

Respondent	Message 'queue'			Message 'accident'		
	T_{min}	T	T_{max}	T_{min}	T	T_{max}
1	5.5	6.5	8.5	10	15	17
2	10	15	25	10	15	30
3	0	0.5	1	2	3	5
4	4.5	6	10.5	1	2	4
5	2	3	4	5	7	10
6	0	0	0	11.5	15	20
7	2.5	3.5	4	5	8	10
8	1	2	4	3.5	5.5	8
9	3	5	10	7.5	12.5	17.5
10	5	6	10	5	7	11.5
Average	**3.35**	**4.75**	**7.70**	**6.05**	**9.00**	**13.30**

TABLE III. COMPARISON OF ROUTE CHOICE PREFERENCES IN TRIALS WITHOUT INFORMATION

Route	Choice %	
	Observed	Predicted
R1	86.7	90.5
R2	10.0	6.8
R3	3.3	2.7
RMSE	**2.89**	

TABLE IV. COMPARISON OF ROUTE CHOICE PREFERENCES IN TRIALS WITH INFORMATION

Route	Choice %			
	Message 'queue'		Message 'accident'	
	Observed	Predicted	Observed	Predicted
R1	37.50	46.90	0.00	21.65
R2	62.50	47.22	71.43	69.54
R3	0.00	5.88	28.57	8.81
RMSE	**10.90**		**16.96**	

Tables 3 and 4 also show the result of the comparison with observed preferences respectively for trials without and with information. In Table 3, the RMSE value (2.89%) shows the effectiveness of the proposed model to reproduce drivers' choice behaviour in absence of information.

In presence of information, it is important the role of the parameters k and γ, whose optimum values have been obtained through GA. Table 4 summarizes the results obtained relating to the ability of the model to reproduce the choice behavior. In the case of 'queue' message, the RMSE value is 10.90%, lower than the case of 'accident' message, 16.96%. Thus, the proposed model is able to reproduce almost correctly the overall behavior and, therefore, the effect of information on users' choices.

V. CONCLUSIONS

In this paper, the emphasis was on capturing the reasoning process of drivers making en-route choices in presence of traffic information. The influence of uncertainty in updating the knowledge of attributes of a transportation system, like expected travel time on a path, has been modelled using the concept of compatibility between previous knowledge and current information. The presented model points out the relevant role of the Possibility Theory in calculating uncertainty and thus drivers' compliance level with released information.

A modelling framework, which represents the uncertainties embedded in the perception of travel attributes, has been developed through the Possibility Theory. The model allows the quantitative calculation of users' compliance with information, and thus a realistic updating of expected travel time.

To test the model, an SP experiment has been designed using a Driving Simulation Software for generating a virtual scenario of the city of Bari (Italy). A sample of 10 respondents has been identified to drive using a steering wheel and pedals. The scenario has been proposed in different conditions by combining traffic levels, presence and message type using VMSs. Data acquired through questionnaires have been used to parameterize the proposed model in order to reproduce the resulting driving choice behaviour and perceptions.

The effectiveness of the model has been measured evaluating RMSE values between observed and predicted preferences. The proposed model has resulted to be very effective (2.89% RMSE) in absence of information, where no parameterization is needed. Moreover, the model shows very good abilities in reproducing drivers' preferences under information provision (10.9% RMSE for 'queue'; 16.96% RMSE for 'accident').

In a wider framework, the outcomes of this paper can be used to carry out a road pricing system based on information provision. Therefore, a VMS-based Advanced Traveller Information System can be used as a tool for traffic management.

Future developments concern the extension of this experiment to a greater number of respondents in order to better validate the proposed model. Moreover, an improved driving simulator is intended to be used for a better user experience.

REFERENCES

[1] L. Zadeh, "Fuzzy sets as a basis for a theory of possibility", Fuzzy Sets and Systems 1, 1978, pp. 3-28.

[2] D. Teodorović and S. Kikuchi, "Transportation route choice model using fuzzy inference technique", Proceedings of the First international symposium on uncertainty modeling and analysis, College Park, USA, IEEE Computer Society Press, Dec. 1990, pp. 140–145.

[3] M. E. Ben-Akiva, T. Morikawa, and F. Shiroshi, "Analysis of the reliability of preference ranking data", Journal of Business Research 23, 1991, pp. 253- 68.

[4] H. S. Mahmassani and R. C. Jou, "Transferring insights into commuter behaviour dynamics from laboratory experiments to field surveys", Transportation Research Part A, Policy and Practice 34(4), 1998, pp. 243-260.

[5] E. Avineri and J. Prashker, "The impact of Travel Time Information on Travellers' Learning under Uncertainty", Transportation vol. 33 (4), 2006, pp. 393-408.

[6] C. G. Chorus, E. J. E. Molin, T. A. Arentze, S. P. H. Hoogendoorn, J. P. Timmermans, and B. Van Wee, "Validation of a multimodal travel simulator with travel information provision", Transportation Research Part C: Emerging Technologies, Volume 15, Issue 3, 2007, pp. 191-207.

[7] H. L. Chang and P. C. Chen, "The Impact of Uncertain Travel Information on Drivers' Route Choice Behaviour", In TRB 88th Annual Meeting Compendium of Papers DVD, report n. 09, Jan. 2009.

[8] P. Bonsall and T. Parry, "Using an interactive route choice simulator to investigate drivers' compliance with route guidance advice", Transportation Research Record No. 1306, 1991, pp. 59-68.

[9] P. Bonsall, P. Firmin, M. Anderson, I. Palmer, and P. Balmforth, "Validating the results of route choice simulator" Transportation Research C 5(6), 1997, pp. 371-388.

[10] H. N. Koutsopoulos, T. Lotan, and Q. Yang, "A driving simulator and its application for modelling route choice in the presence of information", Transportation Research Part C: Emerging Technologies, Volume 2, Issue 2, 1994, pp. 91–107.

[11] K. V. Katsikopoulos, Y. Duse-Anthony, D. L. Fisher, and S. A. Duffy, "Risk Attitude Reversals in Drivers' Route Choice when Range of Travel Time Information is Provided", Human Factors and Ergonomics Society, Vol.43, No.3, 2002, pp. 466-473.

[12] H. Tian, S. Gao, D. L. Fisher, and B. Post, "A Mixed-Logit Latent-Class Model of Strategic Route Choice Behavior with Real-time Information", Transportation Research Board 91st Annual Meeting, 2012.

[13] R. R. Yager and A. Kelman, "Fusion of Fuzzy Information With Consideration for Compatibility, Partial Aggregation, and Reinforcement", International Journal of Intelligent Systems 15, 1996, pp. 93 -122.

[14] M. O'Hagan, "Using maximum entropy-ordered weighted averaging to construct a fuzzy neuron", Proceedings 24th Annual IEEE Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, 1990, pp. 618-623.

[15] G. J. Klir and Z. Wang, "Fuzzy Measure Theory", Plenum Press, New York, 1992.

[16] J. F. Geer and G. J. Klir, "A mathematical analysis of information-preserving transformations between probabilistic and possibilistic formulations of Uncertainty", Intern. J. of General Systems, Vol. 20, 1992, pp. 143-176.

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