

## Simulation Model of a Bus Line in Changing Traffic Conditions

Marek Bauer

Cracow University of Technology  
Chair of Transport Systems  
Cracow, Poland  
mbauer@pk.edu.pl

**Abstract**—This paper presents the author's stochastic model of bus line operation, taking into account the variability of traffic conditions, as a result of disturbing factors. General structure of the model bases on graph and events theory. In this approach, bus line has been described by bus stops and sections between these stops, and the most essential traffic processes: running following sections, alighting and boarding passengers at bus stops, waiting for the possibility to departure. In the mathematical description of the simulation model, a dynamical system of matrix-vector equations is used. The fundamental element of presented model is the state vector of the system. It includes all important information about operation of each bus line: variables allowing to define location of each bus on a line in time and space, variables describing the occupancy of all buses and values of scheduled ride times which enable to evaluate punctuality. All parameters of the model have been determined with statistical and stochastic methods using, on the basis of extensive and identification own research including various types of sections (with or without separated bus lanes) and diversified location and types of bus stops. Presented simulation model can find wide application among others: to create feasibility studies for investments in public transport, evaluation of solutions streamlining bus traffic, planning bus routes and developing timetables.

*Keywords- urban transport; bus lane; simulation*

### I. INTRODUCTION

Operation of public transport requires permanent quality control of provided services. It is only with complete knowledge about the state of public transport, when efficient continuous improvement activities can be carried out. For this reason, public transport quality research should be conducted in a manner possibly continuous, in order to enable evaluation of not only current quality indicators but also efficiency of measures taken, such as: separated bus lanes and priorities in traffic lights.

Bus service is far more prone to traffic disorders than rail transport due to moving, mostly, in the stream of other vehicles. Currently, in Polish cities, one observe the increasing influence of overcrowded streets on deterioration of bus traffic conditions leading even to total breakdown of punctuality and regularity. The speeds achieved by buses vary, even within the same traffic route and the same time period. There are many factors influencing travel times of

buses (e.g., [1], [2]), like the infrastructure of streets, intersections, and bus stops, the traffic conditions and organization of traffic, as well as the motoric and behavioral conditions.

General rules of bus public transport modeling and optimization were shown in publications [1], [3] and [4]. There are many publications with micro-simulation point of view. In [5], Mahmoud and Hine described a multi-criteria evaluation of user perception towards bus services and measures the gap in the perceptions held by current and potential users. Paper [6] illustrates a new method of calibration of bus performance parameters in the microscopic scale. Liu and Sinha [7] considered the problem of reliability of an urban bus network using a dynamic micro-simulation model framework. All above approaches are characterized by high level of accuracy.

However, micro-simulation models are very problematic due to the large number of inputs. For example – in Polish conditions, measurements in public transport consist of occupancies and stop-to-stop travel times registration. However, traffic volume measurements are usually not conducted at the same time. The range and the quality of the available information are usually not sufficient to be used in micro scale models for large areas of the city. On the other hand, the macro-simulation models have too general character. However, there is a gap between the highly accurate micro-simulation models and models in macro scale. Therefore, it was decided to build a model in which the reference point will be the efficiency of a bus line. This model should take into account traffic condition and various kinds of street infrastructure.

The aim of the author's own research is among others to determine:

- What is the influence of disturbing factors onto bus line operation?
- What functional parameters (including: average travel and running speeds and their variation) could be obtained in predetermined conditions under the influence of different external factors?
- What is the efficiency of separated bus lanes?

Answering these questions at the stage of investment planning is the basis of adequate implementation of privileges for bus public transport. A tool facilitating evaluation of potential investment effects presents a

**simulation model of a bus line operation** in differentiated traffic conditions. This model could be applied to the analysis of each bus line or public transport corridor, operating in urban conditions, taking into consideration streets' infrastructure and traffic conditions. The model can be helpful for time-table's better designing, in scheduling procedures, in planning and designing bus routes, and in network analysis (with the four steps approach [8]), in estimating input data for macro-simulation models.

In section 2, the general structure of the model is described, whilst in Section 3, the mathematical model based on the system matrix equations was shown. Section 4 explains two important models' elements: bus running time between following stops and initial conditions for every course on the bus line. In Section 5, an example of possible simulation results is presented. Finally, Section 6 presents the conclusion of the paper.

## II. GENERAL STRUCTURE OF THE MODEL

It has been initially assumed, that the model will reflect processes occurring on a bus line, on meso scale. It will help obtain output data more precise than in the case of commonly applied macro-simulation models. At the same time, the model does not require such a great volume of input data as in micro-simulation models. Previously, this kind of approach was used in [9], where mathematical description of an urban bus route in peak hour traffic was presented.

The structure of the bus line operation model is based on graphs and events theories. The graph theory gives a possibility of selecting basic elements of any bus line. Whereas, the theory of events is very useful in description of all the special events taking place during bus line operation.

According to the graph theory, any bus line can be described as a simple digraph structure (Figure 1), where the set of vertexes consists of bus stops on the line, and set of edges consists of sections between these stops.

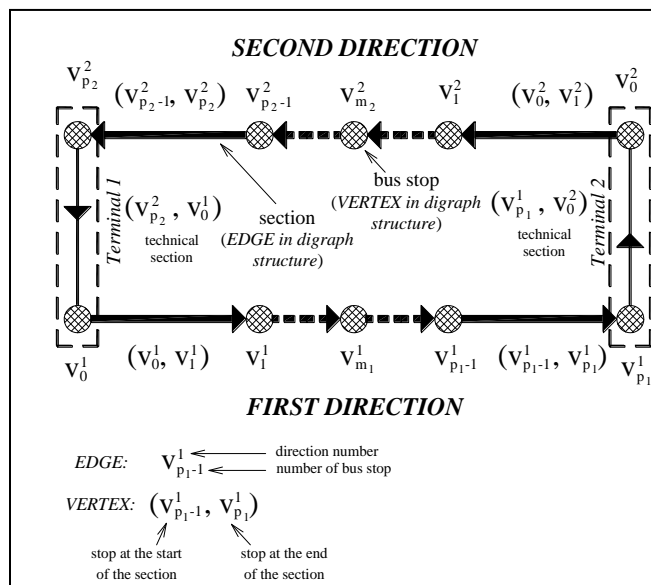


Figure 1. General structure of the model of bus line operation.

Majority of bus lines in European cities (excluding circle lines) operates in two directions. Last stop in first direction is situated on the same terminal as the first stop of second direction. Sections between these stops have technical character, and are also taken into consideration.

The description of model's elements is shown in Table I.

TABLE I. SETS OF ELEMENTS IN DIGRAPH STRUCTURE

Name of set	Description
Set of bus stops	$V^p(D) = \{v_m; m = 0, \dots, p\}$
Set of stop-to-stop sections	$A^o(D) = \{(v_m, v_{m+1}); v_m \in V^p(D); m = 0, \dots, p-1\}$
Set of technical sections	$A^T(D) = \{(v_p, v_0); v_0, v_p \in V^p(D)\}$

In this moment, the bus line can be treated as completion of individual modules: "between stops section" – "stop", located at the end of this section. This approach could be used in stop-to-stop travel time analysis. On the basis of times of departure from following stops, one is able to establish the time span between individual stops, forming the basis for constructing schedules and also the values of stop-to-stop speeds. As well, punctuality and regularity indicators can be calculated. It is however not possible, to indicate reasons for current and potential disorders on the bus line, as the information about the time between departures does not give any insight into the structure of travel time between following stops – how much time of it is the running time and how much is dwell time. As a result, it is difficult to estimate, if extended time of module completion on a line is caused by difficult traffic conditions on a section, or if it is the effect of exceeding traffic capacity of a stop, or increased numbers of alighting and boarding passengers. In the same way, positive effects of privileges for buses (e.g., separated bus lanes) are not easy to extract.

For describing the traffic processes occurring on the bus line, the methodology of discrete events has been applied. The essence of which is the assumption that times of subsequent events are predictable on the basis of previous events occurrence. The simulator does not take into account the state of a model between subsequent events, it reacts to specific events occurring subsequently and the model status remains unchanged – until it changes, as a result of previous event. Simulation watch moves until the next event from the events' list occurs and then operations provided for this event are performed.

On every stop-to-stop section, a large group of possible events influence onto travel time. They are connected with any possible stoppings during the line operation at the stops areas and at the intersections, e.g., the: moment of stopping at the inlet of intersection (because of red signal), the moment of stopping just before the stop (because of busy stop positions), the moments of start and end of alighting and boarding passengers, the moments of start and end of opening door, the moment of physical departure from the stop. They have fundamental influence on the state of the bus line, but their number is individual for any course on the line.

Only few of them are obligatory, while the rest might occur or not.

In the presented simulation model, it is assumed that only the most important discrete events should be taken into consideration (Figure 2). Three possible events have strong influence onto current state of the line: moment of start alighting and boarding passengers, moment of end alighting and boarding and moment of departure from stop.

These three discrete events always have a place (except request stops), and start three of the most significant processes on every stop-to-stop section:

- **Running time** – defined as time interval between the moment the bus is starting from one stop and stopping at the next one.
- **Alighting and boarding time** – defined as time period between the moment of start and the moment of end of alighting and boarding passengers, even if door is still open.
- **Time of waiting for possibility to departure** – defined as time between the moment of end of alighting and boarding passengers to the moment of physical departure from the stop.

According to Figure 2, the stopping time consists of alighting and boarding time and time of waiting for possibility to departure, because of impossibility of start its movement.

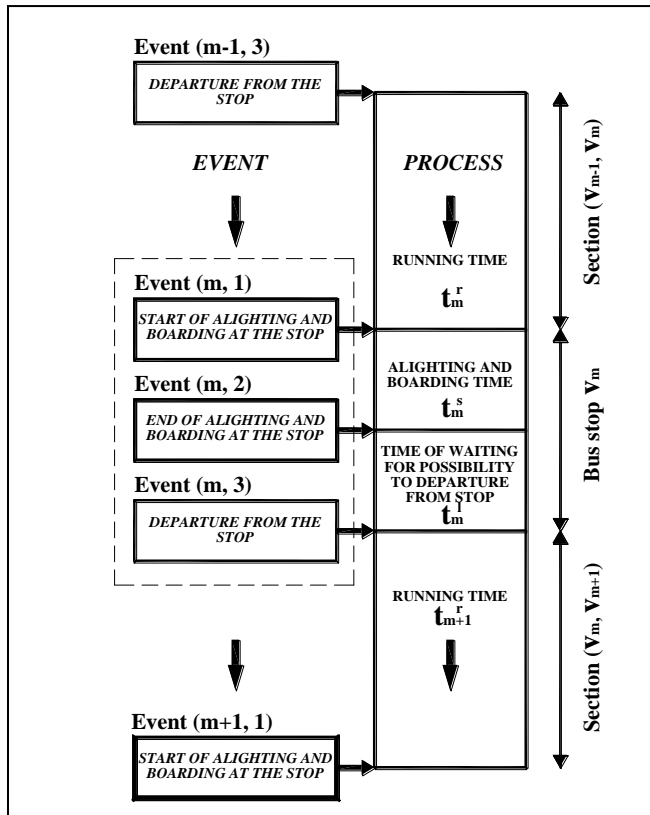


Figure 2. Possible events during one stop-to-stop operation.

All remaining events were aggregated to these three processes. The biggest simplification concerns running time, where the number of possible influences is the largest. But, because of the planning character of the model, and its usefulness – this simplification was assumed. If necessary, this approach can be completed, as in [10].

On the first and last stop (on terminals) – only one event is important. In case of beginning stop (number “0”), only the moment of departure is important (event 3), in case of the last stop on the line – only the moment of start of alighting and boarding is considered (event 1). Moment of departure from first stop can be estimated as semi-random variable.

The last part of the structure of the model – is a group of buses operating on the line. Final description on every event on the bus line has the form  $(k, j, m, i)$ , where:

- $k$  – course number (1, 2, 3, ...; even numbers – first direction, odd numbers – second direction),
- $j$  – individual number of the bus on the line ( $j = 1, 2, \dots$ ),
- $m$  – number of stop ( $m = 0, 1, 2, \dots, p$ ),
- $i$  – number of event ( $i = 1, 2, 3$ ).

Set (2, 3, 10, 3) means that bus number 3 during course number 2 has just departed from the stop number 10.

Was also assumed, that model should have stochastic character. Full description of the model’s structure is presented in [11].

### III. THE MATRIX MODEL

For reasons of utility of presented model, it is critical to capture dynamics of individual processes occurring on a line, in particular the changes of location of subsequent buses in time and space. Vehicle movement and connected with it, the passenger relocations (described with events), result in changes of the system state, the system being a bus line. Earlier described elementary occurrences happening during each module (between stops section – bus stop) are the grounds for developing a matrix system.

Knowledge of the system state in a current moment and of the past system inputs (especially those of random character), allows to specify the system state at later moments. Dynamics of the model is taken into account by changes of traffic conditions on sections between bus stops, random changeability of passengers’ streams at bus stops and thanks to including the influence of time the buses spend at the final terminals on starting moments, and also duration of subsequent courses.

#### A. Matrix equations

The proposed model of a bus line operation can be expressed as dynamic vector and matrix system of discrete events, as following:

$$x_{k,j,m+1} = A^r \cdot x_{k,j,m,3} + B_1^r \cdot u_{k,j,m+1}^{rw} + B_2^r \cdot u_{k,j,m+1}^m + C^r \cdot w_{k,j,m+1}^r \quad (1)$$

$$y_{k,j,m+1} = D^r \cdot x_{k,j,m+1} + E_1^r \cdot x_{k,j,m,3} + E_2^r \cdot x_{k-n,j-l,m+1} + E_3^r \cdot x_{k,j,0,3} \quad (2)$$

$$x_{k,j,m+1,2} = A_1^s \cdot (x_{k,j,m+1,1})^2 + A_2^s \cdot x_{k,j,m+1,1} + B^s \cdot u_{k,j,m+1}^s + C^s \cdot w_{k,j,m+1}^s + v_{k,j,m+1}^s \quad (3)$$

$$y_{k,j,m+1,2} = D^s \cdot x_{k,j,m+1,2} + E_1^s \cdot x_{k,j,m+1,1} + E_2^s \cdot x_{k,j,0,3} \quad (4)$$

$$x_{k,j,m+1,3} = A^l \cdot x_{k,j,m+1,2} + B^l \cdot u_{k,j,m+1}^l + C^l \cdot w_{k,j,m+1}^l \quad (5)$$

$$y_{k,j,m+1,3} = D^l \cdot x_{k,j,m+1,3} + E_1^l \cdot x_{k,j,m+1,2} + E_2^l \cdot x_{k-n,j,m+1,3} + E_3^l \cdot x_{k,j,0,3} \quad (6)$$

The occurrence of each event on a line (taking into consideration every bus operating on the bus line ) is defined by a system of two equations:

- equation of the system state (i.e., (1), (3), (5)),
- equation of system outputs (i.e., (2), (4) and (6)).

The description of all model vectors is given below.

### B. Vectors

The most vital element of the system is the state vector defined only at discrete moments of time, responding to occurrence of subsequent events on a line (Table II). It consists of current time of simulation (for every vehicle on the line in every determined moment of time), occupancy of vehicle, current distance from starting stop (every vehicle on the line) and scheduled, cumulated travel time – needed for punctuality indicators.

Vectors of deterministic system inputs are assigned to basic movement processes occurring on a line. Two of them include information influencing travel of the section between bus stops, with and without bus lane – these are section lengths and the number of crossroads located on the sections. The vector of deterministic inputs influencing the passenger exchange includes average intensities of alighting and boarding passenger streams, while vector of inputs connected with bus waiting for the possibility to leave the bus stop focuses average standard time of waiting, average length of extending the waiting time and time of covering a section according to a time-table.

Vectors of random system inputs represent uncertainty at the system input and contain random component of the time of covering the section between bus stops, of passengers exchanges and of waiting for the possibility to leave the bus stop, as well as random components of the number of passengers alighting and boarding at the bus stops.

Random components for the duration of processes are established as random variables from normal distribution of zero average values and variances defined for particular types, respectively, buses, stops and between stops sections. The random components of the number of passengers alighting and boarding have been modeled as random variables established from the Poisson distribution.

TABLE II. INPUT VECTORS IN PRESENTED SIMULATION MODEL OF BUS LINE OPERATION

Name of the vector	Vector	Description
State vector	$\mathbf{x}_{k,j,m,i} = \begin{bmatrix} T_{k,j,m,i} \\ P_{k,j,m,i} \\ L_{k,j,m,i} \\ R_{k,j,m,i} \end{bmatrix}$	Current simulation time
		Current number of passengers inside vehicle
		Current length
		Schedule travel time
Vector of deterministic system inputs (section with separated bus lane)	$\mathbf{u}_{k,j,m+1}^{rw} = \begin{bmatrix} s_{m+1}^{rw} \\ 0 \\ l_{m+1}^{rw} \\ 0 \end{bmatrix}$	Number of signalized intersections
		-
		Length of section with separated bus lane
		-
Vector of deterministic system inputs (section without bus lane)	$\mathbf{u}_{k,j,m+1}^{rn} = \begin{bmatrix} s_{m+1}^{rn} \\ z_{m+1}^{rn} \\ l_{m+1}^{rn} \\ 0 \end{bmatrix}$	Number of signalized intersections
		Number of non-signalized intersections
		Length of section without bus lane
		-
Vector of deterministic system inputs (alighting and boarding)	$\mathbf{u}_{k,j,m+1}^s = \begin{bmatrix} \lambda_{m+1}^a \\ \lambda_{m+1}^b \\ 0 \\ 0 \end{bmatrix}$	Average intensity of alighting passengers
		Average intensity of boarding passengers
		-
		-
Vector of deterministic system inputs (waiting for possibility to departure)	$\mathbf{u}_{k,j,m+1}^l = \begin{bmatrix} \hat{t}_{k,j,m+1}^{t,z,p} \\ \hat{t}_{k,j,m+1}^{tw,z,p} \\ 0 \\ r_{k,j,m+1} \end{bmatrix}$	Average normal time lost by bus at the stop
		Average extended time lost by bus at the stop
		-
		Scheduled stop-to-stop running time
Vector of random system inputs (section)	$\mathbf{w}_{k,j,m+1}^r = \begin{bmatrix} \hat{t}_{k,j,m+1}^{rw,z^0} \\ \hat{t}_{k,j,m+1}^{rn,z^0} \\ 0 \\ 0 \end{bmatrix}$	Random component of running time (bus lane)
		Random component of running time (common lane)
		-
		-
Vector of random system inputs (alighting and boarding)	$\mathbf{w}_{k,j,m+1}^s = \begin{bmatrix} \hat{t}_{k,j,m+1}^{s,z^A} \\ \hat{a}_{m+1} \\ \hat{b}_{m+1} \\ 0 \end{bmatrix}$	Random component of alighting and boarding time
		Random component of the number alighting passengers
		Random component of the number boarding passengers
		-
Vector of random system inputs (waiting for possibility to departure)	$\mathbf{w}_{k,j,m+1}^l = \begin{bmatrix} \hat{t}_{k,j,m+1}^{t,z,p} \\ \hat{t}_{k,j,m+1}^{tw,z,p} \\ 0 \\ 0 \end{bmatrix}$	Random component of normal time lost at the stop
		Random component of extended time lost at the stop
		-
		-
Vector of constant system inputs	$\mathbf{v}_{k,j,m+1}^s = \begin{bmatrix} v_{k,j}^{s,z^A} \\ 0 \\ 0 \\ 0 \end{bmatrix}$	Constant value
		-
		-
		-

System outputs are tightly linked to the system state. There can be distinguished 11 output values, which are included in three vectors (Table III). These are: drive duration at the moment of beginning passenger exchange, actual time of covering a section, current distance from the starting stop, interval between vehicles at the time of reaching a bus stop, drive duration at the moment of finishing passenger exchange, alighting and boarding time, number of passengers on a bus leaving a bus stop, drive duration at the moment of leaving the bus stop, time of waiting for the possibility to leave the bus stop, deviation from the time table, interval between vehicles at the moment of vehicle leaving the stop. On the basis of system outputs line functioning quality indicators can be determined, including indicators of punctuality and regularity and travel comfort.

TABLE III. OUTPUT VECTORS IN PRESENTED SIMULATION MODEL OF BUS LINE OPERATION

Name of the vector	Vector	Description
Output vector (between stops running)	$y_{k,j,m+1}^r = \begin{bmatrix} T_{k,j,m+1}^r \\ t_{k,j,m+1}^r \\ L_{k,j,m+1} \\ h_{k,j,m+1}^r \end{bmatrix}$	Travel time from beginning stop
		Section running time
		Distance from beginning stop
		Between buses interval (before stopping)
Output vector (alighting and boarding)	$y_{k,j,m+1}^s = \begin{bmatrix} T_{k,j,m+1}^s \\ t_{k,j,m+1}^s \\ P_{k,j,m+1} \end{bmatrix}$	Travel time from beginning stop
		Alighting and boarding time
		Occupancy of vehicle
Output vector (waiting for possibility to departure)	$y_{k,j,m+1}^l = \begin{bmatrix} T_{k,j,m+1}^l \\ t_{k,j,m+1}^l \\ d_{k,j,m+1} \\ h_{k,j,m+1} \end{bmatrix}$	Travel time from beginning stop
		Time of waiting for possibility to departure
		Deviation from time table
		Between buses interval (after stopping)

The remaining elements of the model (described in [11]), are: matrixes of status, inputs and outputs containing among others, established on the basis of empirical studies, running time parameters, dwell time and time of waiting for the possibility to leave a bus stop.

#### IV. ESTIMATION OF CHOSEN MODEL COMPONENTS

All matrix elements have been estimated with the help of statistical methods, on the grounds of vast measurements' results of bus operation in four Polish cities – mostly with the use of GPS receivers. A similar method was used in [12]. In total, more than 21 000 departures from the bus stops have been registered. Below, the results of estimation of selected model's parameters are presented.

##### A. Running time

The most significant element of a single module on the bus line is the running time. At the same time it is the most

difficult element to determine, due to the number and influence of distracting factors. Even in very similar traffic conditions, it is possible to obtain very different travel times of subsequent buses of a line. In particular, substantial differences of travel times occur at the sections where buses travel on general access lanes, which are typical for high use of capacity and even exhausting the capacity. In the model, diversity of sections between stops has been taken into account – firstly – in terms of existing or not separate bus lanes, next, in terms of the way to isolating or the degree of using conventional capacity of a lane.

In case of section with separated bus lanes, average running time for individual types of sections between bus stops has been modeled with linear function of multiple regression:

$$\bar{t}_m^r = \beta_l^r \cdot l_m + \beta_s^r \cdot s_m \quad (7)$$

Dependent variables in this model are: the length of the section between following stops ( $l_m$  [km]) and the number of intersections with traffic lights located on this section ( $s_m$  [-]).

Sections with separated bus lanes differ among each other, therefore, they have been diversified in terms of number of cars turning right with bus lane using, and also number of maneuvers connected with access to targets, located by the lane, including parking lots on pavements.

Regression formula in case of section with separated bus lane, without essential influence of turning right vehicles' traffic (type AB) – on the basis of measurements: less than 100 [veh/h] – on the inlet of signalized intersection is as follow:

$$\bar{t}_{m+1}^{r,AB} = 1,14 \cdot l_m + 0,38 \cdot s_m \quad (8)$$

If, on the section is located at least junction on which more than 100 [veh/h] turning right with bus lane using (type AS), then, the average running time can be estimated from the other formula:

$$\bar{t}_{m+1}^{r,AS} = 2,17 \cdot l_m + 0,27 \cdot s_m \quad (9)$$

Whereas, in case of frequent (evaluated subjectively) additional maneuvers on the bus lane (type AP), connected with access to targets, located by the lane (e.g., parking, supplies), average running time can be calculated by the formula:

$$\bar{t}_{m+1}^{r,AP} = 2,87 \cdot l_m + 0,18 \cdot s_m \quad (10)$$

Bus running time on the sections without separated bus lanes can be described by similar regression formula with one additional variable. It is number of intersections  $z_m$  [-], without traffic lights, where buses perform subordinate relations:

$$\bar{t}_m^r = \beta_1^r \cdot l_m + \beta_2^r \cdot s_m + \beta_3^r \cdot z_m \quad (11)$$

Sections without separated bus lanes have been classified in terms of their traffic conditions, assessed by planning method. They are determined on the basis of the traffic ratio of measured hourly traffic volume to estimated value of critical, planning volume, for one direction,  $r$  [-], described in [13]. These critical planning volumes refer to situations in which the car drivers looking for alternative travel paths. The traffic volumes should come from measurements – or in case of new investments – from microscopic analysis. Below, individual formulas for all specified cases were shown.

As very good traffic conditions (defined as type ZA), there were evaluated cases, when traffic ratio  $r$  is smaller than 0,5. In this situation, the bus speeds are high and limited only by drivers:

$$\bar{t}_m^{r,ZA} = 1,77 \cdot l_m + 0,28 \cdot (s_m + z_m) \quad (12)$$

In this case, running times can be shorter even than in case of sections with separated bus lanes (type AB).

Sections with good traffic conditions were defined as sections, where the traffic ratio is smaller than 0,8 (type ZB). In practice, occasional blockings of the buses take place:

$$\bar{t}_m^{r,ZB} = 2,72 \cdot l_m + 0,16 \cdot (s_m + z_m) \quad (13)$$

Medium traffic conditions are most commonly found in big Polish cities (type ZC). In this case (no congestion), bus speeds are strongly limited by other lane users. This time, the traffic ratio is smaller than 1,0.

$$\bar{t}_m^{r,ZC} = 4,08 \cdot l_m + 0,13 \cdot (s_m + z_m) \quad (14)$$

In presented model, the congestion (type ZD) was defined as poor traffic conditions, where traffic ratio is bigger than 1,0, but smaller than 1,2. In this case, many car drivers look for alternative paths, while buses move slowly with frequent stoppings – not only on the inlets of intersections. Therefore, the number of junctions is not relevant. Regression formula has the form:

$$\bar{t}_m^{r,ZD} = 6,47 \cdot l_m \quad (15)$$

The last type of section (ZE) corresponds to a critical traffic conditions, where buses operate in permanent congestion – the traffic ratio is higher than 1,2:

$$\bar{t}_m^{r,ZE} = 8,99 \cdot l_m \quad (16)$$

Random components of travel times on sections between bus stops are generated as random variables from Normal distribution with zero mean and variance defined with the function of section length.

## B. Initial conditions

It is of great importance to establish initial values for simulation of each course on the line, beginning at the first stop (at the terminal). In the model, current time of every course duration depends on the average value and random component of deviation from time table on the last stop, during previous course completed on this terminal. This problem was also considered in [14].

Average value of deviation on the initial stop is determined depending on the length of actual reserve of operational time on the terminal or the volume of delay of reaching the terminal in relation to the time-table moment of scheduled departure during the next course. The value of the deviation random component is estimated with the use of Normal distribution with zero mean and standard deviation estimated on the basis of empirical studies. In order to avoid extreme cases, during simulation the “three sigma” rule has been included.

Occupancy of a vehicle leaving the initial stop equals the number of passengers boarding at this stop. It results from the assumption that the bus approaching the bus stop is empty and it is only possible for the passengers to board. The number of passengers boarding at the initial stop is estimated as a random variable from Poisson distribution, where the parameter is the average value of passengers’ stream intensity and the length of current (simulated) line interval. So, on the first stop on the line, state vector can be thus represented in the form:

$$x_{k,j,0,3} = \begin{bmatrix} R_{k,j,0} - d_{k,j,0} \\ b_{k,j,0} \\ 0 \\ R_{k,j,0} \end{bmatrix}$$

Other model’s elements have been described in [11]. For example, the average alighting and boarding time were modeled by non-linear multiple regression formulas, as the dependence from numbers of alighting and boarding passengers and current occupancy of the bus – for 6 types of vehicles (midi, normal and maxi – with low and high level of floor). Average time of waiting for possibility to departure was varied depending on the bus stop location (near side, far side nearest intersection, on the section) and location of stop positions on the lane or on bay.

## V. SIMULATION RESULTS

Numerical execution of the model represents author’s simulation software, called “AUTOBUS” – developed in the Mathematica 6.0 environment. It has been created on the basis of demonstrated mathematical, stochastic model. The simulation model has been verified by comparing the results from simulations of two urban lines with the results of independent measurements.

Functioning of the simulation model has been presented on the example of existing Cracow’s bus line No. 130. The line was modeled by taking into consideration real traffic conditions and true values of passengers’ streams. Calculations were conducted for afternoon peak hour, in

order to obtain conclusive results of such a simulation – 20 hours were executed and compared. As a result, the set of all variables defined in output vectors was obtained. Additionally, “AUTOBUS” offers set of statistics, for example: confidence intervals for all calculated variables. Figure 3 shows the travel time on the line 130 (only one direction), while Figure 4 shows the deviations from time table.

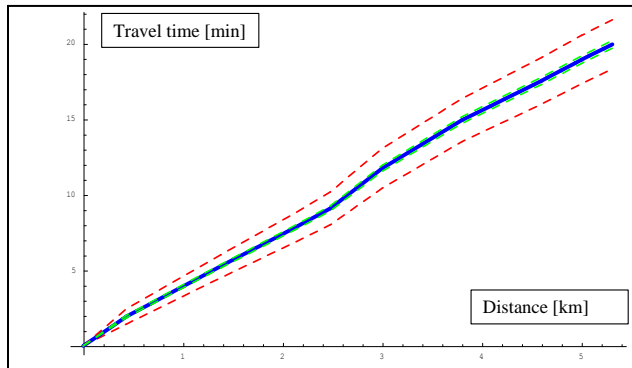


Figure 3. Example of simulation results –travel time on bus line 130 (blue: mean value of travel time, green – confidence interval for average travel time, red – shortest and longest travel time).

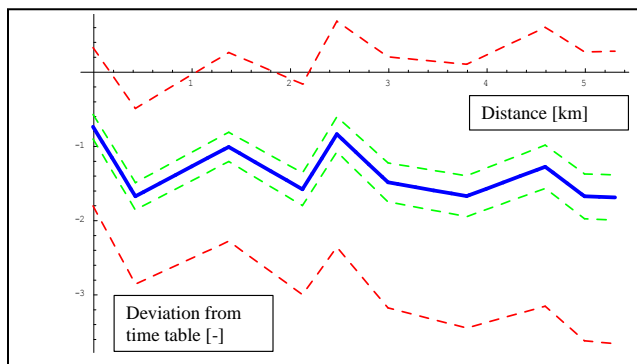


Figure 4. Example of simulation results –deviations from time table on bus line 130 (blue: mean value of deviation, green – confidence interval for average deviation, red – slowest and highest deviation from time table).

Similarly, in this way, the changing passengers' streams can be shown, separately for every bus and for whole bus line.

## VI. CONCLUSIONS

A realization of the bus line operations is assumed to be a sequence of running times between following stops and times spent by buses at the stops. In this paper, a single bus line model in meso scale was presented, which could help to close the gap between existing micro and macro simulation models. This model will make easier the comprehensive analysis of any bus line operating in urban area. It could be used in:

- Scheduling procedures, especially in case of new lines, as a first approach, before starting and during first phase of operation (designed schedules have to be verified by measurements).
- Network analysis of public transport, in estimating input data for macrosimulation models of bus networks (e.g., VISUM software). Model could be useful also for better network calibration.
- Feasibility studies of new network elements, when a more detailed approach is not required.

In the further approach, additional elements will be attached to the model. For better stopping time description also the moments of stoppings just before the stop will be considered. The next step should be extending the model of effective priorities in traffic lights. Till this moment, in Polish cities, this kind of improvements for buses was implemented very seldom.

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