Mixture Based Hybrid Regularization Method for Blind Image Deconvolution

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***Abstract*—Following our recent work on mixed Poisson-White Spike noisy image restoration, we present a multi-convex optimization model to address the fundamental problem of Poisson blind image deconvolution (BID). This problem is encountered in a special application of X-ray radiography in hydro-tests, which also plays an important role in advanced tomographic imaging. We utilize a combined two-dimensional Square Cauchy-Gaussian distribution, whose parameters are totally unknown, to characterize the base structure of the convolution kernel. A new prior density function for the convolution kernel is proposed by integrating the structure density into a Kullback-Leibler divergence. The multi-convex optimization model is derived by a joint maximum a posteriori estimation (MAP) procedure, into which local estimation and expectation maximization algorithm are involved to gain convexity and solvability. To solve the proposed model numerically, a block coefficient descent based algorithm is to be proposed, in which majorization-minimization algorithm and Barzilai-Borwein estimation along with alternating direction minimization of multipliers are utilized to promote the computational efficiency. Numerical results show the effectiveness of our proposed algorithm, as well as its adaptivity.**

Index Terms*—*Poisson blind image deconvolution*; *Square Cauchy distribution*; *Multidirectional fractional-order derivatives*; *Multi-convex*; *Combined 2-dimensional Square Cauchy-Gaussian distribution

I. INTRODUCTION

We consider a photon-limited BID problem encountered in a specific application of flash X/γ-ray radiography (FXR) for hydrodynamic experiments [1] [2], which serves as crucial preprocessing in advanced FXR imaging, especially in 2-dimensional (2D) and 3-dimensional (3D) tomographic reconstruction. Till now, the existing methods of digital radiograph processing are commonly non-blind or based on known Gaussian kernel under the classical Gaussian noise assumption. In applications, however, due to physical and geometrical limitations of the digital radiography system, the acquired data suffer many types of perturbation other than the Gaussian ones. Besides a typically signal-dependent component, i.e., photon shot or Poisson noise [3]–[5], it is noteworthy that heavy-tailed very impulsive components must be taken into account for better understanding both the system blur and the noise as well [6], [7] and [4].

The rest of this submission is organized as follows. In Section 2, we present a new noisy degraded image model and then introduce briefly the modeling procedure of our proposed hybrid regularization model for Poisson BID. In Section 3, we introduce in brief our main idea on how to solve the proposed model numerically. An experimental result is shown to validate our approach. More details on the numerical scheme and additional experimental results will be present on site. Section 4 concludes this submission.

II. PROPOSED MODEL

In applications, due to the type and amount of the contamination, it is difficult to present an accurate noise model, and most studies are built upon a common or oversimplified, *signal-independent* choice, i.e., additive white Gaussian noise. In this study, we pay more attention to two kinds of non-Gaussian perturbation. In fact, Poisson noise can be introduced to the intensity image by the counting process at the scintillator and the CCD array, which is a typical example of *signal-dependent* noise, and more familiar to fluorescence microscopy, positron emission tomography and astronomy. Moreover, a special case of impulsive behavior has to be considered. Indeed, there always exists a large number of radiation particles in high energy radiography, although not irradiated directly, reaching the CCD detector and contaminating the image. This perturbation can also be produced by secondary scattered neutron and gamma radiation hitting the detector even endowed with sophisticated shielding [8]. The struck pixels are displayed as white spikes, which are generally hold much bigger signal counts than those of neighboring pixels and more frequently observed in the dark or extremely low contrast areas.

A highly valued aspect of high energy radiography is the ability to resolve fine details in the high-density object, and resolution depends on the size of the radiographic source spot [9]. Various techniques and definitions have been proposed for measuring and characterizing the spot size at different laboratories, from which a general conclusion can be drawn that heavy-tailed distributions, including two-dimensional Cauchy distribution and Square Cauchy distribution, also known as Quasi-Bennett distribution and Bennett distribution [9], respectively, are more physically realistic than the Gaussian one to characterize the source spot, although their parameters are still very hard to measure or determine in good accuracy. It is shown in [10] that the Cauchy distribution and Gaussian distribution are of Lévy-stable type. On the other end, to model the detector response or simulate X-ray scattering in

X-ray radiography, it is notable that Cauchy distribution can also be deduced theoretically from the assumption of isotropic radiation [11].

In this section, we address the Poisson BID problem in the image space. We adopt the one-dimensional mixture of Poisson and Square Cauchy (MPsC) distributions [4] to characterize the noise in radiographic data, and extend the MPsC denoising method to address the Poisson BID problem. To better condition the deblurring process, our main idea is introducing a combined two-dimensional Square Cauchy-Gaussian distribution with unknown parameters to approximate the basal structure of the unknown blur kernel, and then, integrating it into a Kullback-Leibler (KL) divergence (see [4] and references therein) to construct our prior kernel density function.

Let $f, u, k, w, b : \Omega \to [0, 1]$ be the recorded noisy blurred image, the source image, the blur kernel, and the additive Square Cauchy noise, and the background, respectively. $\Omega = (-1, 1)^2$ denotes the image domain. In applications, one may assume that b is the mean of a known Poisson distribution. Assuming a convolution blur model for a linear radiographic system, we propose the following noisy degraded image model

$$f = \mathrm{Poisson}(k * u + b) + w \doteq z(u, k) + w, \tag{1}$$

in which z is a realization of Poisson distributed random variable Z with PDF given by

$$P_Z^{U,K}(z; k * u + b) = \frac{(k * u + b)^z e^{-(k*u+b)}}{z!}, \tag{2}$$

k is the blur kernel, $*$ denotes the two-dimensional linear convolution operator, w is a realization of square Cauchy distributed random variable, whose PDF is given by $P_W(w; \sigma_w^2) = \frac{2\sigma_w^3}{\pi(\sigma_w^2 + |w|^2)^2}, \sigma_w^2 > 0, w \in \mathbb{R}$.

Proper formulations of both the latent image prior and the blur prior are crucial to the success of BID methods. In our context, the convolution kernel k is regarded as realization of a random variable $\mathcal{K}$, whose PDF is assumed to be unknown, except it can be characterized by a heavy-tailed mixture of Lévy-stable distributions [9] [10]. For convenience of discussion, in the follow we consider a combined 2-dimensional Square Cauchy-Gaussian distribution with PDF $B_k(\mathrm{x}; \Theta_B) = \sum_{i=1}^{2} \gamma_i p_i(\mathrm{x}; \sigma_i^2), \mathrm{x} \in \Omega$ as prior structure of the kernel, where p_1 and p_2 are defined by

$$p_1(\mathrm{x}; \sigma_1^2) = \frac{\sigma_1^2}{\pi(\sigma_1^2 + |\mathrm{x}|^2)^2}, p_2(\mathrm{x}; \sigma_2^2) = \frac{1}{2\pi\sigma_2^2} \exp\left(-\frac{|\mathrm{x}|^2}{2\sigma_2^2}\right), \tag{3}$$

respectively, the denotation Θ_B represents the set of parameters $\gamma_1, \gamma_2, \sigma_1^2, \sigma_2^2$. $\gamma_i \geq 0$ is a mixture ratio satisfying $\gamma_1 + \gamma_2 = 1$.

We then utilize the KL divergence to measure the difference between the unknown kernel k and the basel structure $B_k(\mathrm{x}; \Theta_B)$, and define a prior constraint on the blur kernel k given by $P_K(k) \propto e^{-J_K(k)}$ with

$$J_K(k)(\mathrm{x}) = k(\mathrm{x})[\ln k(\mathrm{x}) - \ln B_k(\mathrm{x}; \Theta_B) - 1] + B_k(\mathrm{x}; \Theta_B). \tag{4}$$

In terms of the prior constraint on the source image, we employ the combined first order TV and multidirectional FOTV of [12]–[14] to define a gradient's sparsity enhancing PDF of the source image, which is given in the Gibbs form as

$$P_U(u) \propto e^{-J_U(\nabla u, \nabla^\alpha u)}, J_U(\nabla u, \nabla^\alpha u) = g_1|\nabla u| + g_2|\nabla^\alpha u|, \tag{5}$$

where $\alpha \in (1, 2]$ denotes the order of multidirectional fractional-order derivatives in the Grünwald-Letnikov sense, $g_1(\mathrm{x})$ and $g_2(\mathrm{x})$ are defined as the same spatially adaptive functions in [4]. The multidirectional FOTV or FOTV4 [13] [4] is utilized to promote the selectivity of edges, and suppress more perturbation in the image as well.

Under these presumptions, our BID problem is then translated into recovering the latent image u and degradation kernel k from a single observation f with (1), (3), (4) and (5). Introduce denotation $\Theta = \{\sigma_w^2\} \cup \Theta_B$. In our context, we also assume that all the parameters in Θ are unknown.

III. Numerical algorithm in brief

Taking advantage of variational Bayesian and MAP principles, we will propose a multi-convex variational framework for the Poisson BID problem, into which an expectation maximization (EM) scheme is incorporated to estimate directly the parameters of the kernel structure as well.

In brief, we will report the modeling processes of the proposed variational Bayesian framework, in which some modifications, such as local approximation for the blur image and EM estimation for kernel structure parameters, are involved to gain solvability and convexity. Moreover, the convexity of the proposed optimization model is validated. As for the numerical algorithm for our proposed model, a block coefficient descent (BCD) **[15] [16]** based algorithm of the Gauss-Seidel type is to be present, in which majorization-minimization algorithm **[17] [18]** and Barzilai-Borwein estimation [19] along with alternating direction method of multipliers (ADMM) [20] are utilized to promote the computational efficiency. We use a localized structural similarity (SSIM) index to perform quantitative assessment on our methods.

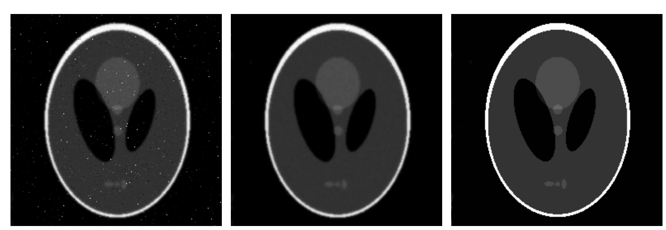

Fig. 1. Left: A synthesized noisy blurred image. Center: A restored image by a comparison algorithm. Right: A restored image using our proposed algorithm.

Numerical results have shown the effectiveness of the proposed algorithm. In Figure 1, we present an example on BID, in which the left image is a synthesized noisy blurred image, obtained by convoluting with a combined Square Cauchy-Gaussian kernel and then adding a mixed Poisson-Cauchy noise (SSIM=0.4581). The center image (SSIM=0.7275) is obtained by a comparison algorithm, or median filter injunction with a shock filter. The right one (SSIM=0.9407)is obtained by our proposed algorithm.

IV. Conclusion

A novel multi-convex optimization framework was proposed for blind deblurring images degraded by Lévy-stable blurs and contaminated by high-level non-Gaussian noises. Numerical results showed the effectiveness of the proposed algorithm on deblurring and denoising simultaneously, as well as adaptivity and quality. Learning based methods may be introduced in some future work to further promote the performance of our proposed method.

References

[1] R. Abraham, M. Bergounioux and E. Trelat, “A penalization approach for tomographic reconstruction of binary axially symmetric objects,” Appl. Math. Optim.. vol. 58, pp. 345–371, 2008.

[2] M. Bergounioux, **et al.**, “Variational methods for tomographic reconstruction with few views,” Milan J. Math.. vol. 86, pp. 157–200, 2018.

[3] D. L. Snyder, A. M. Hammoud and R. L. White, “Image recovery from data acquired with a charge-coupled-device camera,” J. Opt. Soc. Am. A.. vol. 10, pp. 1014–1023, 1993.

[4] S. Wei and L. Kong, “A combined first and fractional order regularization method for mixed Poisson-White Spike noisy image restoration,” Inverse problems and imaging. vol. 18, pp. 38–61, Feb 2024.

[5] T. Le, R. Chartrand and T. J. Asaki, “A variational approach to reconstructing images corrupted by Poisson noise,” J. Math. Imaging Vis.. vol. 27, pp. 257-263, 2007.

[6] C. Ekdahl, “ Characterizing flash-radiography source spots,” J. Opt. Soc. Amer. A. vol. 28, pp. 2501-2509, 2011.

[7] A. S. Carasso, “Direct blind deconvolution,” SIAM J. Appl. Math.. vol. 61, pp. 1980–2007, 2001.

[8] M. Matsubayashi, A. Tsuruno, T. Kodaira and H. Kobayashi, “High resolution static imaging system using a cooled CCD camera,” Nuclear Instruments and Methods in Physics Research A, vol.377, pp. 107–110, 1996.

[9] C. Ekdahl, “Characterizing flash-radiography source spots,” J. Opt. Soc. Amer. A. vol. 28, pp. 2501-2509, 2011.

[10] A. S. Carasso, “Direct blind deconvolution,” SIAM J. Appl. Math.. vol. 61, pp. 1980-2007, 2001.

[11] K. M. Hanson, “ Introduction to Bayesian image analysis,” In: Image Processing, Proc. SPIE 1898 (ed. M. H. Loew), pp. 716–731, 1993.

[12] J. Bai and X.-C. Feng, “ Fractional-Order anisotropic diffusion for image denoising,” IEEE Trans. Image Proc.. vol. 16, pp. 2492–2502, 2007.

[13] Q. Liu and S. Gao, “Directional fractional-order total variation hybrid regularization for image deblurring,” J. Electron. Imaging. vol. 29, pp. 033001, 2020.

[14] J. Zhang and K. Chen,“A total fractional-order variation model for image restoration with nonhomogeneous boundary conditions and its numerical solution,” SIAM J. Imaging Sciences. vol. 8, pp. 2487–2518, 2015.

[15] Y. Xu,“Block Coordinate Descent for regularized multi-convex optimization,” MA thesis, Rice University, Housten, Texas, 2013.

[16] Y. Xu and W. Yin, “A block coordinate descent method for regularized multiconvex optimization with applications to nonnegative tensor factorization and completion,” SIAM J. Imaging Sci.. vol. 6, pp. 1758–1789, 2013.

[17] D. Hunter and K. Lange, “A tutorial on MM algorithms,” The Amer. Statis.. vol. 58, pp. 30–37, 2004.

[18] D. Perrone and P. Favaro, “A logarithmic image prior for blind deconvolution,” Int. J. Comput. Vis.. vol. 117, pp. 159-172, 2016.

[19] O. Burdakov, Y. Dai and N. Huang, “Stabilized Barzilai-Borwein method,” J. Comput. Math.. vol. 37, pp. 916-936, 2019.

[20] D. Gabay,“Applications of the method of multipliers to variational inequalities,” in Augmented Lagrange Methods: Applications to the solution of boundary-valued problems, M. Fortin, R. Glowinski, Eds. North Holland, Amsterdam, 1983, 299-331.