Chaos-Based Communication Systems Based on the Sprott D Attractor

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***Abstract*—We propose a chaos-based communication system employing the symbolic dynamics of the Sprott D chaotic attractor. The chaotic dynamics is modeled by a graph representing the evolution of the chaotic trajectories. Finite-state encoders are employed to map unrestricted binary sequences in the restricted sequences generated by the discretization of the chaotic flow. Finally, the performance of the proposed system is analyzed.**

***Keywords*–Chaos communication; Symbolic dynamics; Topology of three-dimensional chaotic attractors.**

I. Introduction

Chaos-based communication systems are suitable alternatives in modern communications due to some inherent characteristics of the chaotic behavior such as decorrelation, non-periodic behavior, broadband spectrum, energetic efficiency [1]. The use of chaotic dynamical systems has been considered in several different scenarios [2]. In particular, three-dimensional chaotic attractors have been successfully employed in the design of digital communication systems [3].

The chaotic trajectories within three-dimensional chaotic attractors can be discretized by a labeled partition of a Poincaré section. This defines a map between the continuous flow and the symbolic representation of the chaotic system. The symbolic sequences generated by a chaotic attractor are restricted due to the dynamical constraints imposed by the chaotic flow, and these restrictions can be exploited in the design of chaos-based communication systems. The dynamical mechanism of a chaotic attractor is represented by a set $\mathcal{F}$ that represents *words* (or sequences) that never occur in the symbolic representation of the attractor dynamics. The forbidden set $\mathcal{F}$ is used to construct a graph that represents the discrete dynamics of the system and is employed in the design of Finite-State Encoders (FSE), used to map unrestricted into restricted sequences. Moreover, the FSE are used in the decoder to estimate the information sequence.

II. Main Results

To illustrate the proposed methodology, we detail the design of a communication system based on the Sprott D system, defined by

$$\left\{ \begin{array}{lcl} \dot{x}(t) & = & -y(t) \\ \dot{y}(t) & = & x(t) + z(t) \\ \dot{z}(t) & = & x(t)z(t) + ay^2(t) \end{array} \right. \tag{1}$$

where $a = 2.3$ is the control parameter [4]. The Poincaré section for the Sprott D system is a plane perpendicular to the xy plane with $y = 0$. Figure 1 shows the Sprott D attractor

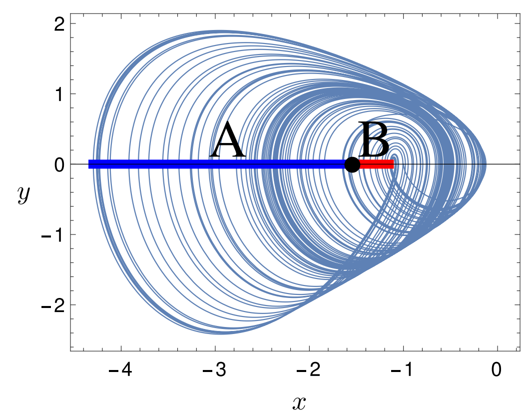

Figure 1. Sprott D attractor and its Poincaré section with a labeled binary partition in projection on the xy plane.

in projection on the xy plane and its Poincaré section with a labeled binary partition over the alphabet $\mathcal{A}_2 = \{\mathrm{A}, \mathrm{B}\}$. The threshold of the partition is defined by the minimum of the Poincaré return map. When a chaotic trajectory crosses the Poincaré section, we assign the label of the region where the crossing occurs in the symbolic evolution of the system. The continuous flow is mapped into symbolic sequences $S_N = s_0 s_1 s_2 \ldots s_{N-1}$, where $s_i \in \mathcal{A}_2$ is the label of the visited region in the Poincaré section in the ith crossing. Therefore, S_N records the order of visitation in the regions of the Poincaré section in N consecutive crossings.

The first step in the proposed methodology is to find the set $\mathcal{F}$ for the Sprott D system. We search in S_N for words with length n that never occur as sub-sequences of S_N. In this work, we consider $n \leq 10$ and in this case the set $\mathcal{F}$ is

$$\mathcal{F} = \{\mathrm{BB}, \mathrm{BAAB}, \mathrm{BAAAAB}\}. \tag{2}$$

Applying the procedures described in [5], we construct a graph that represents the dynamics of a restricted system according to the specification of the set $\mathcal{F}$. Then, we construct the FSE for the Sprott D system employing the method described in [6], which is illustrated in Figure 2. The FSE maps one information bit into two symbols $s_1 s_2$. The next step to design the communication system is to associate the chaotic signals with the state transitions of the FSE.

The states transitions $s_1 s_2$ of the FSE in Figure 2 correspond to two successive crossings in the Poincaré section

 ISBN: 978-1-61208-716-0

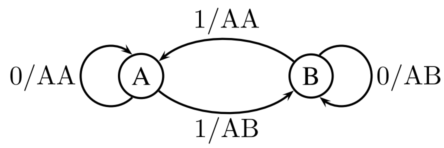

Figure 2. Finite-state encoder for the Sprott D attractor.

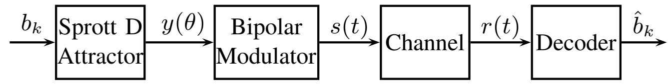

Figure 4. Block diagram of the communication system based on the Sprott D chaotic attractor.

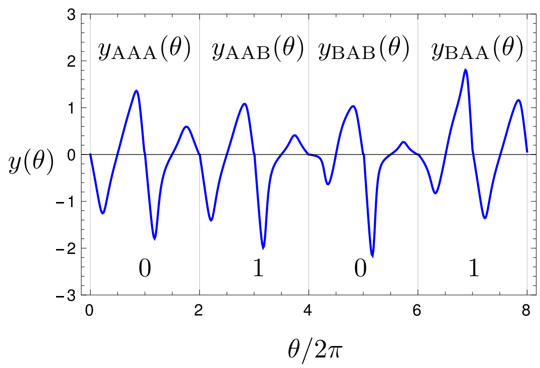

Figure 3. Chaotic signals generated by the Sprott D attractor versus the angle θ to encode the information sequence 0101.

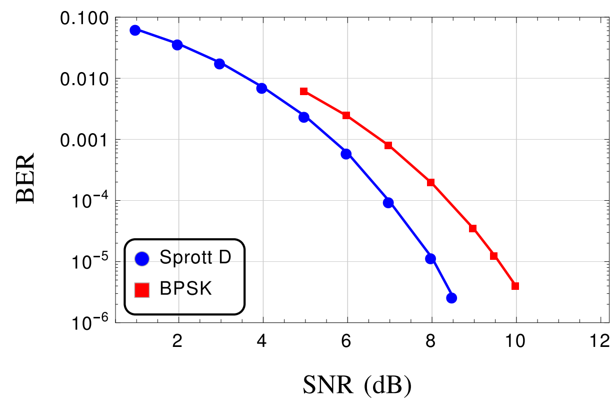

Figure 5. BER for the Sprott D communication system over an AWGN channel. The BER for an uncoded BPSK is also shown.

illustrated in Figure 1. The chaotic signals associated with the transitions are defined as the segments of the chaotic trajectories generated by the variables of the system connecting the corresponding regions of the Poincaré section. Any variable can be used and in this work we employ the variable y. We define the angle θ as the angle between the vector obtained by connecting the center of the attractor to a point of a chaotic trajectory in projection on the xy plane and the Poincaré section [3]. Parameterizing the chaotic signals as $y(\theta)$, the crossings are periodic with angular period 2π. For example, the chaotic signal associated with the transition from state A to state B in the FSE is the segment of trajectory $y(\theta)$ connecting the regions A to A and subsequently connecting the regions A to B, denoted $y_{\mathrm{AAB}}(\theta)$, with angular period 4π. Fig. 3 illustrates a chaotic trajectory versus the angle θ that encodes the information sequence 0101 starting at state A.

The proposed communication system is represented in Figure 4. The state transitions are induced by chaos control [7] according to the information bits $b_0 b_1 b_2 \dots$. The chaotic waveforms $y(\theta)$ are inverted (multiplied by -1) in the bipolar modulator when the information bit is 0 [3]. At each signaling interval T, the signal $y(\theta)$ is mapped into the transmitted signal $s(t)$ with duration T. The signal $s(t)$ is transmitted over an AWGN channel and the received signal is $r(t) = s(t) + n(t)$, where the noise $n(t)$ has uniform power spectral density $N_0/2$. The decoder uses the Viterbi algorithm to estimate the information sequence.

We performed numerical simulations to evaluate the Bit Error Rate (BER) of the proposed communication system. The Signal to Noise Ratio (SNR) is defined as $\mathrm{SNR} = \bar{E}_s/N_0$, where $\bar{E}_s$ is the average energy of the transition curves. Figure 5 shows the performance of the proposed system and the uncoded BPSK system for reference purposes. The proposed system outperforms the BPSK system by approximately 1.8 dB for a BER $= 10^{-5}$.

III. Conclusions

We proposed a methodology to design chaos-based communication systems using a discretization of the chaotic flow generated by the Sprott D chaotic attractor. The discrete chaotic dynamics was used to design an FSE to transmit information when the dynamics is restricted, resulting in better robustness against the channel noise. Therefore, the proposed system is an interesting alternative in chaos-based communications, yielding enhanced performance at the cost of a relatively simple encoding scheme.

Acknowledgment

The authors would like to thank CNPq and FACEPE for the financial support.

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