Dynamic Calibration of Force Transducers Using Sinusoidal Excitations

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Abstract— **The primary calibration of force transducers using sinusoidal excitations with electrodynamic shaker systems will be described. First a view comment concerning the importance of dynamic force measurements will be given. That will be followed by a mathematical description of the basics of dynamic measurements based on linear differential equations. Some useful approximations are given to average measured data. The technical equipment will be introduced together with a discussion concerning the traceability as well as the uncertainty consideration. Finally an exemplary calibration performed on a strain gage transducer will be presented.**

Keywords: dynamic force, force calibration, laser vibrometer, acceleration measurement

I. INTRODUCTION

In the last few decades very precise static force measurements were developed and are now routinely used for calibration services in many national metrology institutes (NMI's) around the world. The force scale which is covered nowadays reaches from µN-MN [1-2]. Thereby, relative measurement uncertainties down to $2{\cdot}10^{-5}$ are obtained using deadweight machines, which are the best standard to realize a traceable force. The force, F, is just the product of the SI base unit mass, m, and the gravitational acceleration, g, following Newton's law, $F=m{\cdot}a$, with the acceleration, $a=g$.

Besides the precise realization of a force in a standard machine, there must be selected force transducer available which can be used as a transfer standard to give the primary calibration to the secondarily calibration laboratories and industry. The crucial fact is now that often these static calibrated force transducers are used in dynamic applications. That is the reason why more and more NMI's have established procedures for a dynamic calibration of force transducers and also other sensors.

Currently in the European Metrology Research Programme (EMRP) one promoted research topic is the: "Traceable Dynamic Measurement of Mechanical Quantities", which includes, apart from a work package about dynamic force, also work packages about dynamic pressure, dynamic torque, the electrical characterization of measuring amplifiers and mathematical and statistical methods and modelling [3].

Similar to the static calibration philosophy primary calibrations have to be provided which guarantee traceability to the SI base units and also transfer transducers (reference standards) to transfer these calibrations e.g. to an industrial application. This transfer turned out to be the most complicated task because of the crucial influence of environmental conditions present in certain applications. Mostly the transducers are clamped from both sides which lead to sensitivity losses due to the dynamics of these connections which are more or less not infinitely stiff. On the other hand the resonant frequency often shifts down to lower frequencies which can also drastically change the sensitivity. The problem can be solved to a certain extent by modelling the whole construction including all relevant parameters. For that reason it is also important to determine the force transducer parameters like stiffness and damping which can be obtained during a dynamic calibration. This article describes one possibility for a primary dynamic force calibration using sinusoidal excitations. The whole procedure as well as most of the set-ups where developed over two decades and are extensively described in [4]. Other methods as well as analysis procedures for dynamic force calibration are described in [5-9].

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II. MATHEMATICAL DESCRIPTION

To obtain an analytical "handle" for the description of the dynamic behaviour of a dynamically excited force transducer, the well-known spring-mass-damper model can be applied. In figure 1 one can see a simplified picture of a force transducer which is equipped with a test mass, m_t. The connection of that mass to the transducer is modelled by a certain stiffness, k_c, and a damping constant, b_c. The transducer itself consists of a bottom mass, m_b, and a head mass, m_i. Both masses are also connected by a spring with stiffness, k_f, and a corresponding dumping constant, b_f. The coordinates in space of all three masses are then given by the vector (x_t, x_i, x_b), if only a vertical movement is considered. A periodical force acts from the bottom on the mass, m_b, (see Fig.1). This force is generated by an electrodynamic shaker system. The acceleration of the top mass, $\ddot{x}_t$, the acceleration on the shaker table, $\ddot{x}_b$, and the force transducer electrical signal are measured during the calibration procedure. This transducer signal is directly proportional to the material tension/compression and can be described in the model by the difference of the spring coordinates x_i-x_b .

The system depicted in figure 1 can be finally modelled by the following system of linear differential equations with constant coefficients:

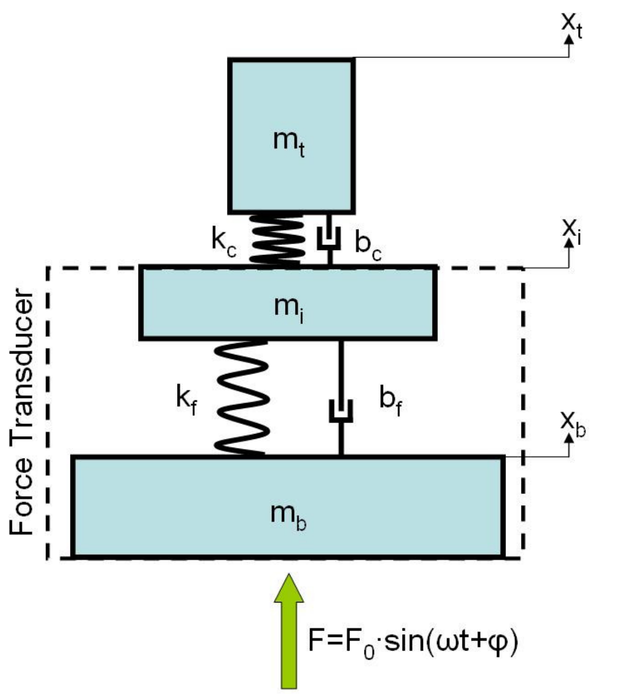

Figure 1. Schematic model of a force transducer equipped with a top mass. The transducer itself can be seen as a spring-mass-damper, which consists of two masses connected by a spring. The adaptation of the test mass is also modelled by a damped spring, whose stiffness is mostly much stronger than that of the transducer.

$$
\begin{aligned}
m_t\ddot{x}_t &= -k_c(x_t - x_i) - b_c(\dot{x}_t - \dot{x}_i) \\
m_i\ddot{x}_i &= k_c(x_t - x_i) + b_c(\dot{x}_t - \dot{x}_i) - k_f(x_i - x_b) - b_f(\dot{x}_i - \dot{x}_b) \\
m_b\ddot{x}_b &= k_f(x_i - x_b) + b_f(\dot{x}_i - \dot{x}_b) + F
\end{aligned} \tag{1}
$$

It should be noted that the system can be simplified if the coupling of the top mass has practically no influence on the dynamic process. This would correspond to the special case, $k_c \rightarrow \infty$, $b_c \rightarrow 0$, and the top mass as well as the head mass of the transducer can be summarized as one mass body. In the calibration process the dynamic sensitivity, which is the ratio between the measured force transducer signal and the acting dynamic force, is measured as follows.

$$
\left|\frac{x_i - x_b}{x_t}\right| = \frac{m_t + m_i}{k_f} \cdot \sqrt{\frac{\left(1 - 2\omega^2 \frac{\mu}{k_c}\left(1 - \frac{b_c^2}{2\mu k_c}\right) + \left(\omega^2 \frac{\mu}{k_c}\right)^2\right)}{\left(1 + \left(\omega \frac{b_f}{k_f}\right)^2\right) \cdot \left(1 + \left(\omega \frac{b_c}{k_c}\right)^2\right)^2}} \tag{2}
$$

In equation 2 the reduced mass $\mu=(m_t \cdot m_i)/(m_t+m_i)$ was introduced. This equation can be drastically simplified if the top mass coupling is neglected and one applies a Taylor series development of the second order for the frequency ω:

$$
\left|\frac{x_i - x_b}{x_t}\right| \approx \frac{m_t + m_i}{k_f} \cdot \left(1 - \frac{b_f}{k_f}\omega^2\right) = p_1 \cdot \left(1 - p_2 \cdot \omega^2\right) \tag{3}
$$

This equation is very convenient for fitting purposes to approximate the measured sensitivities just by the two parameters, p_1 and p_2.

The measured sensitivity is calculated from the ratio of the transducer signal, U_f, and the acting dynamic force:

$$
S_f = \frac{U_f}{(m_t + m_i) \cdot \ddot{x}_t \cdot K_{corr}} \approx S_{f0} \cdot (1 - p\omega^2) \tag{4}
$$

The parameter, K_{corr}, takes into account the vertical acceleration gradient over the mass body. Finite element simulations have shown that the individual mass points of the mass body have slightly different accelerations in the vertical direction [4]. This correction factor can be neglected, if quite small masses are used (only a few

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centimetres in height). The factor S_{f0} is the static sensitivity obtained for the limiting case $\omega=0$, whereby $p=p_1$. As one can see from approximation (4), the sensitivity drops down quadratically with increasing the frequency ω.

Besides the amplitude of the sensitivity according to equation (4), also the phase shift between the acceleration x_t and the force signal U_f can be derived by the model:

$$\varphi(\omega) = \tan^{-1}\left[-\frac{b_f}{k_f}\omega\cdot\left(\frac{1+\omega^2\cdot f_1(1/k_c,b_f,\mu)}{1+\omega^4\cdot f_2(1/k_c,b_f,\mu)+\omega^2\cdot f_3(1/k_c,b_f,\mu)}\right)\right]$$

$$\varphi(\omega) \approx \tan^{-1}\left(-\frac{b_f}{k_f}\omega\right) \approx -\frac{b_f}{k_f}\omega \tag{5}$$

The quite complicated equation (5) contains functions f_1-f_3 which are all proportional to $1/k_c$, so that these terms can be neglected for the limiting case of infinite coupling stiffness of the top mass. In addition the arcus-tangent function can be approximated by a Taylor series of the first order for the frequency ω, which leads to a linear phase shift between the acceleration- and force transducer signal.

III. MEASUREMENT SET-UP

The essential prerequisites for a primary sinusoidal force calibration are seen in figure 2. First of all one needs an exciter. At PTB we have three electromagnetic shaker systems, a small one for forces up to 100 N and 10 Hz until 2 kHz, a medium one up to 800 N for 10 Hz till 3 kHz and a large shaker up to forces of 10 kN and frequencies of 10 Hz to 2 kHz. The shakers consist of two parts, the vibration exciter itself and a power amplifier. The kind of excitation is determined by the chosen signal created by a function generator. This signal directly modulates the current signal which drives the coil of the shaker armature.The acceleration of the top mass can be measured principally in two different ways, either by a primary method using a laser Doppler vibrometer or by accelerometers.

Usually the vibrometer is used, which consists of a laser head providing a 632.8 nm red laser beam and a certain controller. There are two kinds of laser heads, one with a fixed beam and the other one with a scanning opportunity. The scanning vibrometer is able to scan surfaces in an angle region of ±25° in the x- and y- directions. This offers the

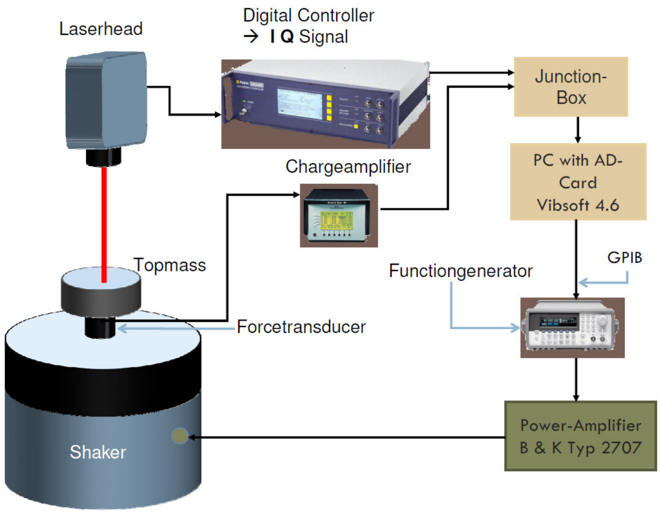

Figure 2. Schematic measurement set-up for the sinusoidal force calibration

possibility to investigate surface vibrations. The modular controller consists of different digital processing units, two velocity decoders, a displacement decoder and a digital quadrature decoder. In summary a frequency range from 0-2.5 MHz with a maximum velocity of 10 m/s and a resolution of 0.02 µm/s can be realized. The signal processing inside the decoders is fully digital, the output is provided as an analogue signal. For precise calibrations the digital quadrature encoder is used in connection with certain software which calculates the displacement according to the arcus-tangent procedure. The analogue IQ output signals are bundled together with the transducer output signal and the acceleration signal from the shaker armature in a junction box which is then cabled to a 5 MHz PC sampling card.

IV. TRACEABILITY AND UNCERTAINTY CONSIDERATION

The sinusoidal calibration of force transducers is a primary calibration method which means that all measured quantities are traceable to the SI base units and all measuring equipment used is calibrated using certain standards, which are well established procedures. The calibration of the weights used as top masses is done according to the international recommendation OIML R 111-1 [10]. According to this document the top masses can be classified at least as Class M_1, which leads to a maximum error for a 1 kg weight of 50 mg or a relative standard measurement uncertainty of $5\cdot10^{-5}$. Apart from the mass determination, the acceleration measurement is the most important part of the calibration. In figure 3 the traceability chains are shown for different ways

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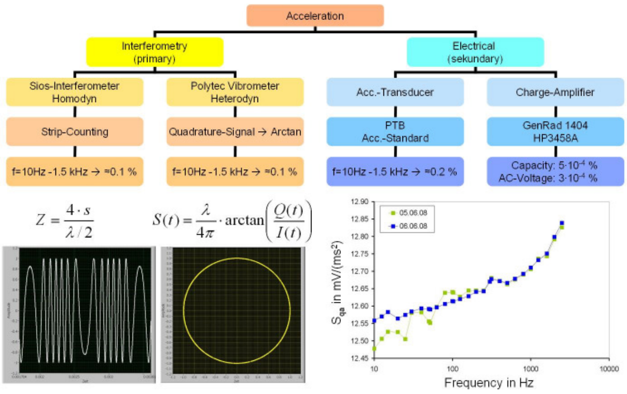

Figure 3. The traceability chains of the acceleration measurement are shown for a primary and secondary method. Left hand side shows the primary method using laser interferometers/vibrometers, the right hand side show the traceability using accelerometers with certain conditioning amplifiers.

of acceleration measurement. There are, in principle, two ways, the primary method using interferometers/vibrometers and the secondary method based on a certain electrical chain. The vibrometer measurement can differ according to the involved overlap of certain laser beams in the homodyne or heterodyne interferometers. Both instruments are based on modified Mach-Zehnder interferometers. In the heterodyne interferometer, the measuring laser beam is spitted whereby one part is additionally mixed with a high frequency using a Bragg cell, usually 40 MHz, to provide the Doppler encoding. The homodyne interferometers are used for larger displacements which can be determined by counting the interference maxima, which is also known as the fringe counting method. The displacement is, thereby, only a function of the laser wave length and the number of fringes. Fringe counting can be performed with very precise instruments, like the high performance counter Fluke PM6681. According to the fringe counting equation, see figure 3, the uncertainty is very small. The relative counting error of the Fluke counter is at 1 kHz, ΔCounts/Counts $\approx 5 \cdot 10^{-8}$ and the relative wavelength error is in the order of $\Delta\lambda/\lambda \approx 10^{-6}$. The main error is made if the displacement comes in the order of $\lambda/2$, because this is the resolution limit of the fringe counting. Keeping in mind a lower limit for the displacement of 400 µm, one can obtain uncertainties of 0.1% in the range from 10 Hz to 1.5 kHz.

As mentioned above, the heterodyne technique is based on the arcus-tangent calculation of the quadrature signal. If one plots the IQ measuring points in a 2 dimensional grid, as seen in figure 3, one obtains a circle in the most perfect case. The fully digital quadrature encoding avoids all errors made in former times by analogue filters and mixing devices. By default the whole electrical chain of the vibrometer controller is calibrated by the manufacturer through coupling of very precise known artificial displacement signals directly in the controller. These measurements result in uncertainties which are below 0.1% [11]. On the other hand, the vibrometer used for the sinusoidal force calibration was calibrated against the national acceleration standard. Thereby, the acceleration values obtained by the measuring program had deviations from the standard set-up of 0.02-0.04%. With a clear conscience one can obtain an uncertainty at least of 0.1% for the frequency range of 10Hz-1.5 kHz with the heterodyne method.

The right-hand side of figure 3 shows the conventional method of acceleration measurement using accelerometers in combination with certain conditioning amplifiers. Normally a charge amplifier is used which can be calibrated with a very precise reference capacity and a high accurate voltmeter. The relative standard measurement uncertainty of both devices is a few 10^{-4} % according to the calibration certificate which was obtained by a standard calibration procedure at PTB. For the use in a calibration set-up, one has to consider the whole measuring chain consisting of the accelerometer and its conditioning amplifier. The measuring chain can be included by a calibrated sensitivity factor, S_{qa}, which commonly has an uncertainty around 0.2 %, as illustrated by example in figure 3. According to the charge amplifier calibration for the accelerometer measuring chain also the force transducer can be handled, if a piezoelectric transducer is used.

For the case of transducers based on the strain gage technique, a special calibration device, also called bridge standard, was developed [4]. The bridge standard simulates, in principle, a force transducer and is based on a Wheatstone bridge whose bridge voltage is, as in the real case, provided by the amplifier. In place of a force transducer the bridge standard is connected to the conditioning amplifier. The output to the amplifier is a dynamic bridge detuning which can be steered through an analogue input signal from an arbitrary function generator with voltage amplitudes. Inside the device the input signal is transformed to a mV voltage. The signal which is seen from the amplifier can be measured in addition on an auxiliary output channel.

The total (combined) measurement uncertainty, u_c, of the sinusoidal calibration can then be separated into two main parts, a set-up dependent part, u_s and a part which is obtained by the actual calibration measurement, u_m:

$$u_c = \sqrt{u_s^2 + u_m^2}\,.$$

The part, u_s, is, in principle, a constant given for a certain set up and reflects the smallest achievable measurement uncertainty. This part depends - according to equation 4 on three parts, the uncertainty of the acceleration measurement, the uncertainty of the conditioning amplifier calibration and the uncertainty of the mass determination. Note that the

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Modell Equation:

$S = (U/V)/(m \cdot a)$

S: Sensitivity, *U:* Conditioning amplifier voltage, *V:* Amplification factor, *m:* Topmass, *a*: Acceleration

Uncertainty-Budget:

Quantity	Value	Standard measurment uncertainty	Degree of freedom	Sensitivity coefficient	Uncertainty contribution	Index
U	205.0 mV	0.00488 %	50	$400 \cdot 10^{-9}$	$4.0 \cdot 10^{-9}$ mV/N	0.3 %
V	2500.0	$8.00 \cdot 10^{-6}$ %	50	$-33 \cdot 10^{-9}$	$-6.6 \cdot 10^{-12}$ mV/N	0.0 %
M	10.0 kg	0.0500 %	50	$-8.2 \cdot 10^{-6}$	$-41 \cdot 10^{-9}$ mV/N	30.7 %
a	100.0 m/s²	0.0750 %	50	$-820 \cdot 10^{-9}$	$-62 \cdot 10^{-9}$ mV/N	69.0 %
S	$82.0 \cdot 10^{-6}$ mV/N	0.0903 %	87			

Result:

Sensitivity-Value: $82.00 \cdot 10^{-6}$ mV/N
Relative expanded measurement uncertainty: ±0.18 %
Expansion Factor: 2.0
Coverage: t-Table 95%

Figure 4. There is shown an example of the standard uncertainty evaluation, u_s, of the dynamic sensitivity at a frequency of 400 Hz. The calculation was done according the "Guide to the expression of uncertainty in measurement", GUM [12].

uncertainty of the head mass, m_i, of the transducer is not included in this part, because this mass has to be determined with the aid of the actual measurement. According to the numbers for the certain uncertainties given above, this part results in an uncertainty contribution of 0.1-0.25 %. Figure 4 shows as an example the uncertainty evaluation for u_s which is obtained at 400 Hz.

The part, u_m, includes the uncertainty of the internal mass determination and depends further mainly on the standard deviations of the performed measured sensitivity points. By using the scanning vibrometer for the acceleration measurement on the top mass one can measure up to 100 points, depending on the actual geometry of the weight. Through this opportunity special disturbing influences like rocking modes or mechanical adaptation influences of the transducer can be taken into account. It should be noted that these influences contribute more than other errors made, by e.g., the sine approximation of the measured data or the uncertainties caused by special filter techniques applied in the analysis procedure.

Experience has shown that the uncertainty part, u_m, is on average below 1 kHz between 0.4-1% and above 1 kHz around 1-2%.

V. MEASUREMENT EXAMPLE

Figure 5 illustrates the output of a special calibration. In this case a 25 kN force transducer based on the strain gage technique was measured with five top masses as seen in the upper panel of figure 4. As seen from the sensitivity plot, all masses are in good agreement below a frequency of 1 kHz.

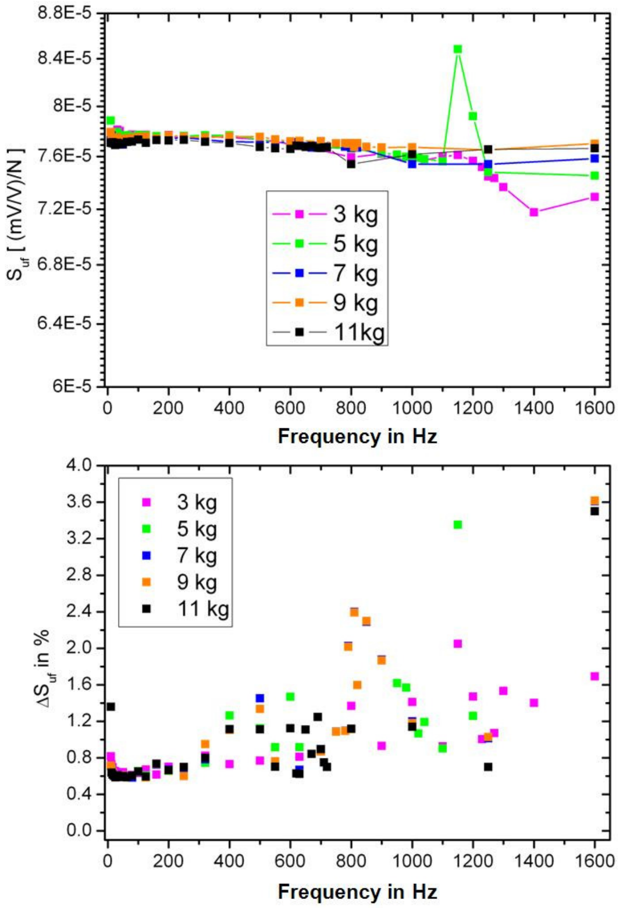

Figure 5: Dynamic sensitivity, upper panel, of a 25 kN strain gage force transducer measured by five top masses. The lower panel shows the uncertainties of the individual measuring points.

This is also confirmed by the combined relative standard measurement uncertainty, given in %, which can be seen in the the lower panel of figure 5. The different top masses cause different resonance frequencies which lay in the range of 700-1400 Hz. Measuring points near the resonance and also beyond naturally have a bigger uncertainty.

To acquire a figure of merit, all the sensitivity curves obtained with the different top masses can be fitted with a function according to equation 4 and the mean value for the obtained parameters can be calculated. Including the uncertainties of the individual points in the fit procedure moreover, leads to a realistic error also for the fit parameters. The averaged fit results, together with the obtained uncertainty range are shown in figure 6. The sensitivity at frequency, f=0, was scaled to 100 % to illustrate the sensitivity drop as a function of frequency in an easy readable way. Thus, it can immediately be seen, that the transducer shows only 96% of its sensitivity at 1.6 kHz.

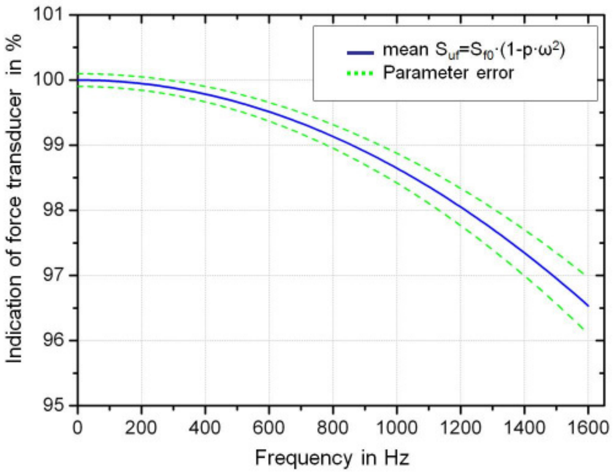

Figure 6: Averaged sensitivity according to the described fit procedure normalized at an indication of 100% given by the transducer at f=0 Hz.

VI. CONCLUSION

The traceable sinusoidal calibration of force transducers was demonstrated. The calibration mainly depends on the acceleration measurement and the calibration of the electrical chain of the conditioning amplifiers used. For the most accurate acceleration measurement, laser Doppler vibrometers can be used which are traced back to the laser wavelength. In the case of piezoelectric force transducers the charge amplifiers can be very precisely calibrated using a reference capacity and a primary calibrated multimeter. For strain gage transducers, a special calibration bridge standard was developed to dynamically calibrate voltage ratios.

In chapter IV it was shown that one can perform the calibration depending on the involved top masses with relative standard measurement uncertainties of ≈0.4-2.0%. The main uncertainty contributions are not caused by the set-up but rather by the mechanical influences like adaptations and the rocking modes of the transducer. These disturbing influences can be detected during a calibration measurement by applying additional sensors like, e.g., triaxial accelerometers. If a certain threshold of transverse acceleration is exceeded, e.g., caused by rocking modes or side resonances of the transducer, the corresponding data will be no further considered. In addition special adapters can be developed to suppress these effects.

REFERENCES

[1] F. Tegtmeier, R. Kumme, M. Seidel, “Improvement of the realization of forces between 2 MN and 5 MN at PTB – the new 5 MN force standard machine”, *XIX IMEKO World Congress*, pp. 186-191, Lisbon, 2009.

[2] Ch. Schlegel, O. Slanina, G. Haucke, R. Kumme, “Construction of a Standard Force Machine for the range of 100 µN – 200 mN”, IMEKO 2010, TC3, pp. 33-36, Pattaya, 2010.

[3] European Metrology Research Programme (EMRP), http://www.emrponline.eu, last access 23.05.2011.

[4] R. Kumme, “Investigation of a primary method for a dynamic calibration of force measuring instruments: a contribution to reduce the measuring uncertainty”, doctoral thesis (in German) , PTB, 1996.

[5] S. Eichstädt, C. Elster, T. J. Esward and J. P. Hessling, Deconvolution filters for the analysis of dynamic measurement processes: a tutorial, 2010, Metrologia, 47, 522-533.

[6] G. Wegener and Th. Bruns, Traceability of torque transducers under rotating and dynamic operating conditions, 2009, Measurement, 42, 1448-1453.

[7] M. Kobusch, Th. Bruns and E. Franke, Challenges in Practical Dynamic Calibration, 2009, Advanced mathematical and computational tools in metrology and testing: AMCTM VIII (2008 Paris France), edited by Franco Pavese, Series on Advances in Mathematics for Applied Sciences - Vol. 78, pp. 204-212, World Scientific Publishing, Singapore.

[8] C. Elster and A. Link, Uncertainty evaluation for dynamic measurements modelled by a linear time-invariant system, 2008, Metrologia, 45, 464-473.

[9] M. Kobusch, “The 250 kN primary shock force calibration device at PTB”, IMEKO 2010, Thailand, Pattaya, November 2010.

[10] International Recommendation, OIML R 111-1, International Organization of Legal Metrology, 2004.

[11] G. Siegmund, “Sources of Measurement Error in Laser Doppler Vibrometers and Proposal for Unified Specifications”, 8th. Int. Conf. on Vibration Measurements by Laser Techniques, Proc. of SPIE Vol. 70980Y, 2008.

[12] ISO/IEC Guide 98-3:2008: Uncertainty of measurement – Part3: Guide to the expression of uncertainty in measurement. ISO, Genf 2008, ISBN92-67-10188-9.

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