# An Improved ID based Proxy Signature Scheme based on Elliptic Curve Cryptography

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Abstract-Proxy signature schemes allow the original signer of a message to delegate his signing capability to a proxy signer to generate a valid proxy signature on behalf of the original signer. One such scheme is proposed by Zhang and Kim which is based on Elliptic Curve Cryptography and Identity based Signature. However, Zhang's scheme requires secure channel for transmission of private key, has no provision of private key revocation and signature verification by any user. In this paper, we propose an improved ID based proxy signature scheme based on bilinear pairing. The scheme employs Knapsack algorithm for key distribution which eliminates the need for secure channel for sending the private keys from Private key generator (PKG) to respective users. The scheme also supports private key revocation by concatenating time parameter with public key of proxy signers. The signature can be verified only by a designated verifier. It is shown that the proposed proxy signature scheme satisfies all security requirements. Finally, the proposed proxy signature scheme is compared with that of Zhang and Kim's scheme and is shown to have merits over the latter one. Therefore, the proposed scheme can be a potential candidate for implementation of future proxy signature schemes.

Keywords–Proxy Signature Scheme; ID based Cryptography; designated verifier scheme; ECC; Knapsack Algorithm;

### I. INTRODUCTION

Digital Signatures are used in a wide variety of modern cryptographic systems that support data integrity and authentication. In public key cryptography, prior to any communication, each user should obtain a certificate from the Certificate Authority validating their public-private key pair. Proxy signatures schemes are one of the variety of digital signatures. Proxy signature schemes are required when the original signer is not available for some duration due to some reasons. Proxy signature scheme can be of two types depending on the signing authority. In full delegation scheme, signing rights are given permanently to the proxy signer. In partial delegation, signing right is delegated for a fixed period of time. The period of delegation and the type of messages that can be signed is usually specified by message warrant issued by the original signer at the time of delegation of signing authority.

In public key cryptography, the users must obtain their public-private key pair from the Certificate Authority prior to message communication [1]. In case of ID based cryptography, a trusted third party called as Private Key Generator (PKG) generates public-private key pair for the signers and transmits it to them via secure channel [2] [3]. In the recent proposals of proxy signatures, the public key of the signers is based on their popular public IDs (such as email id, telephone number etc).

Mambo [4] has described a proxy signature scheme based on Discrete Logarithm Problem (DLP). Recently, an improved proxy signature scheme based on RSA algorithm was proposed by Akanksha et. al [5] in Secureware 2015. The first ID based proxy signature scheme was proposed by Zhang and Kim, which requires a secure channel for transmission of private keys to the respective signers. Zhang and Kim [6] have described an identity based proxy signature scheme based on Elliptic Curve Cryptography. SK Hafizul [7] has described a designated verifier proxy signature scheme. This scheme [6] requires a secure channel for transmission of private key from PKG to user. It has no provision for private key revocation and the signature can be verified by any unknown verifier. The proposed proxy signature scheme attempts to overcome the drawbacks of this particular scheme.

In this paper, we have proposed an ID based proxy signature scheme that eliminates the requirement of a secure channel for transmission of secret key from PKG to signer. It also allows for changing the private key from time to time or when it is compromised to avoid its misuse for a long time. The proposed signature scheme has a designated verifier for verification of the signature created by the signer.

The rest of the paper is organized as follows. Section 2 decribes the Zhang and Kim's scheme. In Section 3, an improved ID based proxy signature scheme is proposed with its security analysis presented in Section 4. Finally, the paper is concluded in Section 5.

#### II. ZHANG AND KIM'S SCHEME

Let PKG be the private key generator. It generates publicprivate key pairs for the original and proxy signer. Let, Alice (A) be the alias name for original signer and Bob (B) be the alias name for proxy signer. Let  $Z_p$  be a field of order p. Let P be an element of  $Z_p$  having order p. Let p be a primitive element of  $Z_p$ .  $G_p$  be an additive cyclic subgroup of  $Z_p$  generated by P and  $G_M$  be a multiplicative group obtained by bilinear pairing of  $G_p$  and  $e: G_p \times G_p \to G_M$  be a bilinear map that maps an element in  $G_p$  to an element in  $G_M$ . Table 1 summarizes the list of conventions and notation used in paper. The scheme advances as follows:

1) Setup Phase

In setup phase, Private Key Generator(PKG) generates its own public/private key pair. Let  $P_{pub}$  be

SYMBOL         SIGNIFICANCE           A         Original Signer	
A     Original Signer       B     Proxy Signer	
C Verifier	
$C_{1,i}, C_{2,i}$ Encrypted $r_i$ $G_p$ Additive group over $Z_p$	
F CI F	
$G_M$ Multiplicative Group obtained by Bilinear Mapping of $G_p$	
$H_1, H_2$ Publicly Known Hash function $ID$ Identity of the user e.g. email.	
$P$ Generator element of $Z_p$	
PKG Private Key Generator	
Ppub         Public key of PKG	
$Q_A, S_A$ Public-private key pair of original signer	
$Q_B, S_B$ Public-private key pair of proxy signer	
$Q_C, S_C$ Public-private key pair of verifier	
$Q_W, S_W$ Public-private key pair of proxy signer in proposed scheme to	sign
any message	
$Q'_i$ Intermediate Public Key of the User i in the proposed scheme $S'_1$ Signature of original signer on message warrant $m_w$ in proposed scheme	
$S'_1$ Signature of original signer on message warrant $m_w$ in prop	posed
scheme	
Signature of proxy signer on message m in proposed scheme	
$U_A, c_A$ Signature of original signer on message warrant mw in Zang and F	Kim's
scheme	
$U_B, c_B$ Signature of proxy signer on message m in Zang and Kim's sch	neme
$Z_P$ [0,p-1]	
$Z_p *$ (1,p-1]	
$Z \in_R [1, p-1]$ Random Number (Nounce) selected from $Z_P$	
$e$ Bilinear map which maps an element in $G_M$ to an element in $G_M$	
k <sub>A</sub> Random number generated by original signer in Zang and F	Kim's
scheme	
$k_B$ Random number generated by proxy signer in Zang and Kim's sci	heme
$l$ Bitwise length of private key $S_i$ of user i in proposed scheme	
m <sub>w</sub> Message warrant	
$p$ Number of elements in field $Z_p$	
r <sub>i</sub> Point on elliptic curve randomly selected by user i for Knap	psack
algorithm	
s Master key or secret key of PKG	
$t_i$ Time for which the generated public key in proposed scheme is	valid

TABLE I. LIST OF SYMBOLS

the PKG's public key that is generated using PKG's master key s as follows:

- a. Let  $G_P$  be an additive cyclic subgroup of  $Z_p$  and  $G_M$  be a multiplicative cyclic group obtained by bilinear mapping of  $G_p$  each of prime order p.
- b. Let P be the generator element of  $G_p$
- Define a bilinear map  $e: G_p \times G_p \to G_M$ . c.
- PKG selects a random number  $s \in_R Z_n^*$  and d.
- PKG calculates its own public key Ppub as e. follows
- f.  $P_{pub} = sP$

The system public parameters are params = $(G_p, G_M, e, p, P, Ppub, H_1, H_2)$ , where  $H_1$  and  $H_2$ are publicly known hash functions.

#### 2) **Extract Phase**

In Extract phase, PKG calculates public and private key pairs  $(Q_A, S_A)$  and  $(Q_B, S_B)$  based on  $ID_A$ and  $ID_B$  for original and proxy signer respectively. Let ID be the public identity of the user such as telephone number or email id, etc.

- Let  $ID_i$  is the public ID of i where  $i \in$ a. (A, B)
- given identity ID of b. For the а signer(telephone number, email id, etc), PKG computes the public key  $Q_i$  as for ID as follows:

 $Q_i = H_2(ID)$ 

The private key  $S_i$  is calculated by PKG as  $S_i = sQ_{ID}$ 

where s is the private key of PKG  $s \in \mathbb{Z}_p$ 

Then,  $Q_{ID_i}$  is the public key of i where  $i \in (A, B)$  $S_{ID_i}$  is the private key of original signer where  $i \in$ (A, B)

PKG sends  $S_A$  and  $S_B$  to A and B respectively on secure channel.

Note that  $ID_A$  and  $ID_B$  i.e. IDs of original and proxy signers are publicly known

Since  $H_2$  is public function, anyone can calculate  $Q_A$ and  $Q_B$ 

#### 3) **Proxy Key Generation**

To delegate his signing capability to a proxy signer, the original signer A makes signed warrant  $m_w$  that consists of public IDs of A and B, type of messages that can be signed by proxy signer (B) and validity period of proxy signer's signatures.

To delegate the signing capacity to the proxy signer, the original signer (Alice) makes the signed warrant  $m_w$  consisting of public IDs of original and proxy signer, type of messages that can be signed and valid time period for proxy signature. The proxy key  $S_{Bm}$ is generated by Bob as follows:

A randomly selects  $k \in_R Z_p^*$  and computes a.  $r_A = e\left(P, P\right)^k$ 

$$c_A = C(I, I)$$

$$c_A = H_1(m_w \parallel r_A)$$

$$U_A = c_A S_A + kP$$

A then sends  $(m_w, c_A, U_A)$  to B on secure b. channel.

Note that  $S_A$  and P lie on elliptic curve on  $Z_p$  and  $c_A$  and  $k_A$  are scalar quantities. and  $r_A$  is not sent explicitly from A to B

On receiving the above information from A, c. proxy signer B computes the following:

 $r_A = e\left(U_A, P\right) e\left(Q_A P_{pub}\right)^{-c_A}$ 

and accepts the signature to be valid if and only if

$$c_A = H_1\left(m_w \parallel r_A\right)$$

This validates that B has received information from A only(authentication).

d. If the signature on message warrant is valid, B computes his private proxy key as follows:  $S_{Bm} = c_A S_B + U_A$  where  $S_{Bm}$  is a modified proxy key created by proxy signer using the original proxy key sent by PKG to user.

#### 4) **Proxy Signature Generation**

The message m is signed by proxy signer B using his proxy key  $S_{Bm}$  as follows:

- Proxy signer B selects a random number a.  $k_B \epsilon Z_p *$
- B computes  $r_B = e (P, P)^{k_B}$ b.
- B computes the proxy signature on message c. m using his proxy signature key  $S_B$  as follows:

$$c_B = H_1 (m \parallel r_B)$$
$$U_B = c_B S_{Bm} + k_B P$$

d. B broadcasts  $(m, c_B, U_B)$ . where m is the message,  $r_B$  is an intermediate value and  $(c_B, U_B)$  is the signature of B on message m. The signature generated by this scheme is proxy protected as it can be created by the proxy signer only.

#### **Verification Phase** 5)

Any verifier can verify signature on message m to be valid as follows:

Verifier computes a)

$$r_B = e \left( U_B, P \right) \left( e \left( Q_A + Q_B, P_{pub} \right)^{H_1(m_w \| r_A)} r_A \right)^{-c_A}$$

b) Verifier accepts signature to be valid on message m if and only if  $c_B = H_1 \left( m \parallel r_B \right)$ 

## A. Security Analysis of Zhang and Kim's scheme

The security analysis of Zhang and Kim's scheme is as follows:

- Secure channel is needed for transmission of secret 1) key from PKG to original signer A and proxy signer Β.
- If the private keys of original signer A and proxy 2) signer B has been compromised, even then since people use their popular public IDs as public key, the system is no longer secure.
- 3) Validity of generated signature can be verified by anyone which may not be desirable in some situations.

## **III. PROPOSED SCHEME**

In previous section, the Zang and Kim's ID based proxy signature which did not fulfill all the security requirements. An ID based proxy signature scheme has been proposed that overcomes some of the shortcomings pointed out in the previous section. The given scheme consists of seven phases namely, 1. Setup phase, 2. Public Key Generation phase, 3. Private Key Generation phase, 4. Secret Key Sharing Phase 5. Proxy Key Generation Phase, 6. Proxy signature generation, and 7. Proxy signature verification.

Let PKG be the private key generator. It generates publicprivate key pairs for the original and proxy signer and verifier. Let  $Z_p$  be a field of order p. Let P be an element of  $Z_p$  having order p. Let  $G_p$  be an additive cyclic subgroup of  $Z_p$  generated by P and  $G_M$  be a multiplicative group obtained by bilinear pairing of  $G_p$  and  $e: G_p \times G_p \to G_M$  be a bilinear map that maps an element in  $G_p$  to an element in  $G_M$ .

The various steps involved in the proposed proxy signature scheme are as follows:

#### 1) Setup phase

In this phase, the PKG generates its own public private key pair( $P_{pub}, s$ ) as follows:

- PKG selects an elliptic curve E over  $Z_p$  and a. broadcasts it.
  - PKG randomly selects  $s \in Z_p$  where s is the private key of PKG.
- b. Let P be a point on elliptic curve. PKG generates its public key  $P_{pub}$  as follows:

$$P_{pub} = sP \tag{1}$$

where s is the private key of PKG

PKG then broadcasts  $P_{pub}$  and P.

## **Public Key Generation**

2)

In this phase, PKG generates public keys of original signer A, proxy signer B and verifier C as follows:

PKG calculates intermediate public key  $Q_i'$ using public ID of signer(such as email ID, telephone number etc) and a publicly known hash function  $H_1$ .

 $Q_i' = H_1 \left( ID_i \right)$ 

The intermediate public key  $Q_i'$  is concateb. nated with time parameter  $t_i$  which indicates the validity period of proxy signature key.  $Q_i = Q'_i \parallel t_i$ 

 $Q_i$  is the public key for entity where  $i \in$ (A, B, C)

Note that public key is changed by PKG from time to time so that even if the private key is compromised, it cannot be misused for a longer time.

#### Private key generation phase 3)

PKG computes each user i's private key as follows:  $S_i = sQ_i$ 

where

s is the secret key of PKG

 $Q_i$  is the public key of user i and

 $S_i$  is the secret key of user i

#### 4) Secret Sharing Phase

To obtain its private key, each user i selects a. a random point  $r_i$  on elliptic curve where  $i \in (A, B, C)$ 

Let  $r_i = (r_{ix}, r_{iy})$  where  $r_{ix}$  and  $r_{iy}$  are the x and y coordinates of  $r_i$  respectively.

- b. User then computes  $n_i = |r_{ix} + r_{iy}|$
- User i then selects another random number c.  $k_i \in Z_p$ .
- Each user i then encrypts the point  $r_i$  using d. PKG's public key according to the following equations [8]:  $\hat{C_{1}}_{i} = k_{i}P$

$$C_{1,i}$$
  $n_i = r_i + k_i P_{rul}$ 

 $C_{2,i} = r_i + k_i P_{pub}$ Where  $P_{Pub}$  is the public key of PKG.

Note that P,  $r_i$ ,  $C_{1,i}$ ,  $C_{2,i}$  and  $P_{pub}$  are points on an elliptic curve over  $Z_p$  and  $k_i$  is a scalar quantity

- User i then sends  $C_{1,i}$  and  $C_{2,i}$  to PKG on e. public channel.
- f. The PKG then decrypts  $C_{1,i}$  and  $C_{2,i}$  and obtain  $r_i$  as follows  $\begin{array}{l} r_i \ = \ C_{2,i} \ - \ s C_{1,i} \ = \ C_{2,i} \ - \ s k_i P \ = \ r_i \ + \\ k_i P_{pub} \ - \ k_i P_{pub} \ = \ r_i \end{array}$

g. PKG then computes 
$$n_i = |r_{ix} + r_{iy}|$$

h. PKG calculates a series  $N_i$  using number  $n_i$ 

$$\begin{split} N_i &= (1, n_i, n_i^2, ....., n_i^j, n_i^{l-1}) \\ \text{where } i \in (A, B, C) \text{ and } j \in (0, 1, 2...., l - 1) \end{split}$$
(1)

where 1 is the bitwise length of the private key.

i. PKG converts  $S_i$  into binary form as  $S_i =$  $(b_{l-1}, b_{l-2}, ..., b_1, b_0)$ Where  $b_{1-1}$  is the Most Significant Bit(MSB) and  $b_0$  is the Least Significant Bit(LSB)

- j. PKG computes  $R_i$  for each user i using KNAPSACK algorithm [9]  $R_i = \sum n_i^j b_j, 0 \le j \le l-1$
- k. Then PKG sends  $R_i$  to the signer on public channel.
- l. Signer i recovers  $S_i = (b_{l-1}, \dots, b_0)$  as follows:

Let  $R_I$  be an intermediate value derived from  $R_i$ 

- I k = 1.
- II  $R'_i = R_i$

III 
$$R_I = R_i' - n_i^{l-k}$$
.

IV If 
$$R_I < 0$$

$$b_{l-k} = 0$$

$$\begin{array}{ll} \text{If } R_I \ge 0 \\ b_{l-k} = 1, \ R'_i = R_I \end{array}$$

$$\mathbf{V} \quad k = k+1$$

VI If  $k \le l$ , go to step III If k > l, then end the process

In this way user i recovers his secret key  $S_i$ .

## 5) **Proxy Key Generation**

b.

Original signer creates a message warrant  $m_w$  specifying public identities of original and proxy signer, validity period of signing of the proxy signature and type of messages that can be signed.

a. The original signer computes

$$S_1 = H_2\left(m_w \parallel S_A\right) \tag{2}$$

Where  $H_2$  is publicly known hash function. and sends  $(m_w, S_1)$  to B on public channel. B computes

$$S_1' = H_2(S_1 \parallel S_B)$$
 (3)

and sends  $(m_w, S_1')$  to PKG on public channel.

c. PKG accepts  $(m_w, S_1')$  if the following equation holds true:

$$S_{1}' = H_{2} \left( H_{2} \left( m_{w} \parallel S_{A} \right) \parallel S_{B} \right) \quad (4)$$

This verification can be done by PKG since  $S_A$  and  $S_B$  are known to PKG only

d. Then, PKG finally computes public key  $(Q_w)$ and private key  $(S_w)$  of proxy signer for signing a message.

$$Q_w = H_1\left(m_w\right) \tag{5}$$

$$S_w = sQ_w \tag{6}$$

- e. PKG then sends  $S_w$  to B on public channel using Knapsack algorithm. PKG also broadcasts the public key  $Q_w$ .
- f. B accepts  $(S_w, Q_w)$  only if the following equation holds true:

$$e(S_w, P) = e(H_1(m_w), P_{pub})$$
(7)

This step ensures data integrity of  $S_w$  and  $Q_w$ .

## 6) **Proxy Signature Generation**

In this phase, proxy signer (B) generates proxy signature on message m in following manner:

a. B computes

b.

$$T = e\left(S_w, Q_C\right) \tag{8}$$

where  $Q_C$  is the public key of verifier.

B then computes

$$S_g = H_2\left(m \parallel m_w \parallel T\right) \tag{9}$$

c. B sends  $(m_w, m, S_g)$  to the verifier for verification

## 7) Proxy Signature Verification

To accept the signature is accepted by the verifier by calculating the following:

a. PKG calculates an intermediate value  $\overline{T}$  as follows:

$$\bar{T} = e\left(H_1\left(m_w,\right), S_C\right) \tag{10}$$

Where  $S_C$  is the private key of verifier given by following equation:

$$S_C = sQ_C \tag{11}$$

Where s is the private key of PKG and  $Q_C$  is the public key of verifier.

b. PKG calculates an intermediate variable s' as follows:

$$s' = H_2\left(m \parallel m_w \parallel \bar{T}\right) \tag{12}$$

c. The signature is accepted by PKG if the following equation holds true:  $s' = S_a$ 

As the proxy signer B uses his own private key  $S_W$ , neither the original signer nor PKG can create a valid proxy signature.

Only a designated verifier can verify the proxy signature as the designated verifier's public key  $(Q_C)$  is also involved in creating the signature for message m and it can be verified by the designated verifier only by using his own private key.

### A. An Implementation Example of the Proposed Scheme

The scheme can be implemented using an example given below. The elliptic curve considered is  $E: y^2 = x^3 + 4x + 20$  and the calculations have been done using elliptic curve calculator [10]. The various steps of the proposed scheme can be exemplified as follows:

## a Setup Phase

Let  $E : y^2 = x^3 + 4x + 20$  be an Elliptic Curve defined over  $Z_{29} = (0, 28)$ . Let P=(1,5) be a point on E over  $Z_p$ . We assume that the order of P in 29. Let the private key of PKG, s=3. The public key of PKG  $P_{pub}$  is calculated as follows:  $P_{pub} = sP = 3(1,5) = (20,3)$ 

b **Public Key Generation Phase** Let  $ID_i$  be the publicly known ID of user i. Let  $H_1$  be a hash function that maps  $ID_i$  to a point on E.

$$Q_i' = H_1 \left( ID_i \right)$$

The intermediate public key  $Q_i'$  is concatenated with time parameter  $t_i$  which indicates the validity period of proxy signature key.

 $Q_i = Q'_i \parallel t_i$  $Q_i$  is the public key for entity where  $i \in (A, B, C)$ where  $Q_i$  is the public ID of user i where  $i \in$ (A, B, C)Where A is the original signer, B is the proxy signer and C is the verifier. Let  $Q_A = (20, 3)$ ,  $Q_B = (4, 19)$  and  $Q_C = (15, 27)$ . с **Private Key Generation Phase** Private key of A i.e.  $S_A$  is calculated as follows:  $S_A = sQ_A = S_A = 3(20, 3) = (14, 23)$ Similarly,  $S_B = (17, 19)$  and  $S_C = (19, 13)$ Secret Key Sharing Phase d Let user A selects  $r_A = (3, 1)$ . Therefore  $n_A = |3 + 1| = 4$ The generated series  $N_A = 1, 4, 16, \dots$ A selects a random number  $k_A=2$ A encrypts  $r_A$  as follows:  $C_{1,A} = k_A P = 2(1, 5) = (4, 19)$  $C_{2,A} = r_A + k_A P_{pub} = (3,1) + 2(20,3) = (0,7)$ (4, 19) and (0, 17) is sent by A to PKG instead of (3, 1) on public channel. PKG recovers  $r_A$  as follows:  $r_A = C_{2,A} - sC_{1,A} = (0, 17) - 3(4,19) = (3,1)$ PKG calculates  $n_A = |3+1| = 4$ PKG generates  $N_A = 1, 4, 16,...$ PKG converts  $S_A = (14, 23)$ , the private key of A into binary form (01110, 10111). 14 is encrypted as follows:  $(14)_{10} = (01110)_2 = (0x256) + (1x64) + (1x16) +$ (1x4) + (0x1) = 84.Similarly 23 is encrypted as 277. PKG sends (84, 277) instead of (14, 23) to A on public channel. 84 is decrypted as follows: Let  $R_I$  be an intermediate variable.  $R_I = 84 - 4^4 = -172$  which is negative, hence  $b_4 = 0$ .  $R_I = 84 - 4^3 = 20$  which is positive, hence  $b_3 = 1$ .  $R_I = 20 - 16 = 4$  which is positive, hence  $b_2 = 1$ .  $R_I = 4 - 4^1 = 0$  which is 0, hence  $b_1 = 1$ .  $R_I = 0 - 1 = -1$  which is negative, hence  $b_0 = 0$ . Hence, 84 is decrypted into  $(01110)_2 = (14)_{10}$ . Similarly, 277 is decrypted into  $(10111)_2 = (23)_{10}$ . In this way, A recovers its private key  $S_A = (14, 23)$ . Similarly, B and C receive their private key  $S_B = (17, 17)$ 19) and  $S_C = (19, 13)$ . **Proxy Key Generation** e Original signer selects a message warrant  $m_w = 6$ . The original signer computes  $S_1 = H_2(m_w \parallel S_A) = H_2(3 \parallel (14, 23))$ Where  $H_2$  is publicly known hash function that gives a point  $S_1$  on elliptic curve E. Let  $S_1 = (10, 4)$ and sends (6, (10, 4)) to B on public channel. B computes  $S_1' = S_1' = H_2(S_1 \parallel S_B) = H_2((10, 4) \parallel (17, 19))$ Let  $S_1' = (1, 24)$ 

and sends (6, (1, 24)) to PKG on public channel. PKG accepts (6, (1, 24)) if the following equation holds true:  $H_2(H_2(6 \parallel (14, 23)) \parallel (17, 19)) = (1, 24).$ PKG computes public-private key pair  $(S_w, Q_w)$  of proxy signer B as follows:

 $Q_w = H_1(6)$ 

Where  $H_1$  is a publicly known hash function that maps  $m_w$  to a point  $Q_w$  on elliptic curve E

Let  $Q_w$  be (8, 10). The private key  $S_w$  is calculated as follows:

 $S_w = sQ_w = 3(8, 10) = (16, 2).$ 

PKG sends (16, 2) to B on public channel using Knapsack algorithm.

PKG also broadcasts the public key  $Q_w$ .

B accepts (16, 2) only if the following equation holds true:

 $e((16,2),(1,5)) = e(H_1(6),(20,3))$ , where e is a bilinear pairing that maps a pair of elements in additive cyclic group  $G_p$  to an element in multiplicative group  $G_M$ .

The above condition holds true if  $S_w$  is valid.

This step ensures data integrity of  $S_w$  and  $Q_w$ .

## f Proxy Signature Generation

Let the message to be signed by proxy signer be m = 8.

(B) generates proxy signature on message m = 8 in following manner:

B computes

 $T = e(S_w, Q_C) = e((16, 2), (15, 27))$ 

where  $Q_C$  is the public key of verifier. e is a bilinear pairing that maps a pair of elements in additive cyclic group  $G_p$  to an element in multiplicative group  $G_M$ . Let us assume that bilinear pairing e maps (16, 2) and (15, 27) to an element (20, 26).

e((16,2),(15,27)) = (20, 26)

B then computes  $S_g = H_2(8 \parallel 6 \parallel (20, 26))$ Assuming that the hash function gives (13, 6) as output, we obtain the following equation:

 $S_g = H_2(8 \parallel 6 \parallel (20, 26)) = (13, 6)$ B sends (6, 8, (13, 6)) to the verifier for verification.

## g Proxy Signature Verification

To accept the signature is accepted by the verifier by calculating the following:

PKG calculates an intermediate value  $\overline{T}$  as follows:  $\overline{T} = e(H_1(6,), (19, 13))$ 

Let us assume that  $H_1(6) = (2, 6)$ .

We also assume the following:

 $\overline{T} = e(H_1(6,), (19, 13)) = (2, 6)$ 

Where (19, 13) is the private key of verifier.

PKG calculates an intermediate variable s' as follows:  $s' = H_2 (6 \parallel 8 \parallel (2, 6))$ 

The signature is accepted by PKG if the following equation holds true:

 $s' = S_q$ 

The above equation holds true if the authorized proxy signer B signs the message m = 8 and designated verifier C verifies the signature.

## B. Security Analysis of the Proposed Scheme

In this section we discuss about the security aspects of the proposed scheme such as trusted PKG, proxy key revocation, designated verifier, proxy protected, unforgability, non repudiation and secure channel. They are as follows:

### 1) Trusted PKG

The security of ID based signatures is based on the fact that PKGs should be trusted. If the PKG is not trusted then the scheme is not secure. However, given a trusted PKG, the scheme is secure.

### 2) Private key revocation

Even if private key of user is compromised, it cannot be misused for a long time as public key is valid only for particular time for which the time parameter  $t_i$  remains unchanged.

### 3) **Designated verifier**

Only designated verifier C can verify the proxy signature which is desirable in some situations. This is done by using the public key of verifier  $Q_C$  in creating the signature  $S_g$  which can be verified only if the verifier has the corresponding private key  $S_C$ . This happens because the designated verifiers public key is also involved in signing the message m and it can be verified by the designated verifier using his own public key.

### 4) Proxy protected

Only the proxy signer should be able to create a valid proxy signature, not the original signer. In this scheme, the secret key of the proxy signer  $S_w$  is calculated by PKG using his own secret key s which cannot be calculated by the original signer due to Elliptic Curve Discrete Logarithm Problem(ECDLP). Hence the proposed scheme is proxy protected.

### 5) Unforgability

Only the proxy signer should be able to create a valid proxy signature. In the proposed scheme, as the proxy signer creates the signature  $S_g$  using his own private key  $S_w$ , no one else can sign on behalf of proxy signer, neither the original signer himself nor a third party.

### 6) **Non-repudiation**

The proxy signer should not be able to deny his signature later on. In this scheme the proxy signer creates signature  $S_g$  by using his private key  $S_w$  and is verified by verifier using proxy signer's public key  $Q_w$  using his public key. Hence, the proxy signer cannot deny his signature.

### 7) Secure channel

In Zang and Kim's scheme, a secure channel is required for transmission of secret key from PKG to signers. In our proposed scheme, the PKG uses KNAPSACK algorithm to encrypt the secret keys and signers use reverse knapsack to extract back the keys. Therefore communication can take place on insecure channel.

TABLE II. COMPARISON BETWEEN ZHANG AND KIM'S SCHEME AND PROPOSED SCHEME

Parameters	Zhang and Kim's Scheme	Proposed Scheme
Number of Steps	5	7
Secure channel requirement	Yes	No
Proxy key revocation	No	Yes
Designated verifier	No	Yes

The proposed scheme eliminates the need for a secure channel for transmission of private key from PKG to signers. It also provides the feature of private key revocation as per the need. The scheme has a provision for designated verifier only. Table 2 summarizes the comparison between Zhang and Kim's scheme and proposed scheme.

### IV. CONCLUSION

In this paper, we have proposed a new ID based proxy signature scheme. The scheme has eliminated the use of secure channel for transmission of private key from PKG to original signer, proxy signer and verifier using KNAPSACK algorithm. This scheme also exhibits Private key revocation feature such that if a private key is exposed, it cannot be used for a long time. This scheme allows the proxy signature to be verified by a designated verifier only. As it satisfies all security requirements, it can be used in future proxy applications. This scheme is designed for a single proxy signer only, which can be extended to multiple proxy signers. However, the proposed needs a trusted PKG. This condition can be removed as part of future work.

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