Tolerant Control Scheme Applied to an Aerospace Launcher

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***Abstract*—Autonomous aerospace launchers must carry out their missions safely because any accident can lead to important or dramatic consequences. It is essential to develop robust control solutions that guarantee optimal performances even when failures occur during these missions. The main objective of this paper is to present the development of a fault-tolerant control design in the case of an aerospace launcher. The method consists of defining and carrying out effective procedures for early detection of some critical situations and providing an adequate control that maintains the safe behaviour of the launcher. The improved control performance is obtained by using a sliding mode observer for a robust reconstruction of an actuator fault. This reconstruction is used then to generate an added signal in the initial control law that compensates for the effect of the faults. Simulation results will show the efficiency of the proposed method.**

Keywords-aerospace launcher; observer; stability; fault detection; control.

I. INTRODUCTION

The conquest of space is a technological battle that began decades ago, spurring great interest in researchers. Automatic applications in this field play an important role, particularly in modeling, control, and diagnosis aspects.

For the future needs of the CNES (The French Space Agency), it is useful to develop appropriate methodologies for piloting space vehicle launchers.

This work is within the framework of the PERSEUS Project which is a technology development program, undertaken as part of the research and innovation policy of the CNES Launcher Directorate [1]. The PERSEUS project has three objectives: the search for innovation and the development of promising technology applicable to Space transport systems; the undertaking of this work by young people within a university or association context, in order to encourage them to choose a career in space; and finally, the development of a set of ground-based and flight demonstrators in order to draw up a detailed pre-project file of a system for launching nano satellites.

Recently, some research in the Fault Detection and Isolation (FDI) area has led to systems based on the sliding mode idea [2], [3].

Although uncertainties could reduce the effects of faults in the control system and may cause false alarms, undetected faults could cause catastrophic consequences. In this context, Tan and Edwards [4], in 2003, extended their results obtained in 2000 [5] to design a sliding mode observer (SMO) that minimizes the L_2 gain between the uncertainty and the fault reconstruction signal to implement a robust faults reconstruction system.

The objective of the tolerant control system is to keep a safe behaviour for the system even in the presence of faults. Almost all the existing methods in the literature are divided into two classes: passive and active [6]. Passive techniques deal with an expected set of failures on the actuator and lead to a controller design that makes the closed-loop system insensitive to certain faults. Theses methods may lead to a very complex controller especially when the number of possible failures increases. Moreover, when unexpected failures occur, the controller is not capable of stabilizing the system. Active techniques use an FDI system and a control reconfiguration procedure that takes into account the effect of the fault. Different approaches, as model matching and track trajectory have been developed to improve system performances when a fault occurs.

This work improves the results described in an earlier article [7], where only a control scheme was developed on the launcher but where faults were not taken into account. The tolerant control developed here is based on the active technique. The FDI is built on results from [4] and [5] for a robust fault reconstruction. Unlike many previous active schemes found in the literature, the proposed method can be handled directly without completely reconfiguring the controller. A robust actuator fault reconstruction technique is applied to the process, allowing the compensation of the effect of the faults. The synthesis procedure is expressed in Linear Matrix Inequality terms.

Simulation results demonstrate the ability of the proposed fault tolerant scheme to detect actuator failures in real time,

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identify them accurately with low computational overhead, and compensate for those actuator failures to achieve stability of the launcher around zero incidence.

This paper is organised as follows: Section 2 introduces the launcher model and its linear state representation. Section 3 explains the strategy of the detection and control system. Section 4 gives the proposed sliding mode observer, and a robust reconstruction technique for actuator faults is developed in Section 6. Finally, the conclusion is given in Section 7.

II. SPACE LAUNCHER DESCRIPTION

The launcher is assumed to be a rigid structure. Consequently, flexible modes are not considered in the launcher modeling but they may be taken into account as disturbances added to measures.

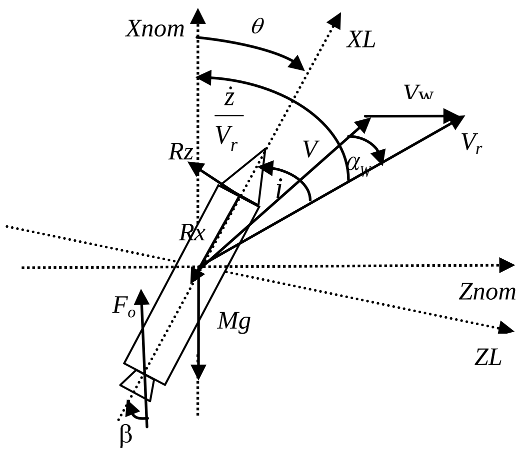

Figure 1. Exterior forces applied on the launcher

V, V_r	: Absolute and relative velocity
V_W	: Wind velocity
$\dot{z}$	: Drift velocity along the pitch axis
i	: Incidence of the vehicle
β	: Thrust deflection angle
θ	: Pitch angle (attitude)
α_w	: Angle between V and V_r
C_x	: Aerodynamic drag coefficient
C_z	: Aerodynamic lift coefficient
R_x	: Drag force
R_z	: Lift force
F_o	: Engine thrust
M	: Instantaneous mass of the launcher
ρ	: Air density
P_{dyn}	: Dynamic pressure
S_{ref}	: Reference surface of the launcher
X_F, X_A	: Distance of the aerodynamic force and the propulsion control to the gravity center

Figure 1 illustrates the exterior forces acting on the launcher system. These forces are as follows:

- The gravity is given by $F_{grav} = M\,g$. (1)

- The dynamic pressure is expressed as $P_{dyn} = (1/2)\rho V_r^2$. (2)

- The aerodynamic force is given by $F_\alpha = (1/2)\,\rho V_r^2\, S_{ref}$. (3)

The aerodynamic force can be decomposed into two perpendicular forces:

- R_Z, the lift, is the component perpendicular to the trajectory; it is the most important force that carries the launcher:

$$R_z = (1/2)\,\rho V_r^2\, S_{ref} C_z \tag{4}$$

- R_x, the trail, is the weakest component and follows an axis parallel to the trajectory; it pulls up the launcher:

$$R_x = (1/2)\,\rho V_r^2\, S_{ref} C_x\,. \tag{5}$$

R_Z , the lift component, and R_X , the trail component are given by:

$$R_z = F_a C_z i \tag{6}$$

and

$$R_x = F_a C_x\,. \tag{7}$$

Considering small angles and applying the dynamic laws leads to the two principal equations modeling the launcher:

$$\begin{cases} \ddot{\theta} = A_6 i + K_1 \beta \\ \ddot{z} = -a_2\theta - a_1 i - (F_o/M)\beta \end{cases} \tag{8}$$

where

$$A_6 = \frac{F_0 C_Z\, X_F}{I_{YY'}},\; K_1 = \frac{F_0 X_A}{I_{YY'}},\; a_1 = \frac{F_0 C_z}{M}, a_2 = \frac{F_0 - R_x}{M}\,. \tag{9}$$

Finally, the launcher model can be represented by the vector equation [7]:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ A_6 & 0 & \dfrac{A_6}{V_r} \\ -(a_1 + a_2) & 0 & \dfrac{-a_1}{V_r} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ K_1 \\ \dfrac{-a_1}{C_z} \end{bmatrix} \beta + \begin{bmatrix} 0 \\ \dfrac{-A_6}{V_r} \\ \dfrac{-1}{V_r} \end{bmatrix} V_w \tag{10}$$

$$i = \theta + \frac{\dot{z}}{V_r} - \frac{V_w}{V_r} \tag{11}$$

where the state vector is $x(t)=\begin{bmatrix}\theta & \dot{\theta} & \dot{z}\end{bmatrix}^T$. The input vector is β(t), the bounded external disturbance is V_w, and the output vector is $y(t)=\theta(t)$.

The coefficients A_6, K_1, a_1 and a_2 are the system variables that make the launcher's model non stationary and, therefore, difficult to control. Typical curves describing the variation of these parameters are shown in Figure 2 [8]. Operating points are chosen.

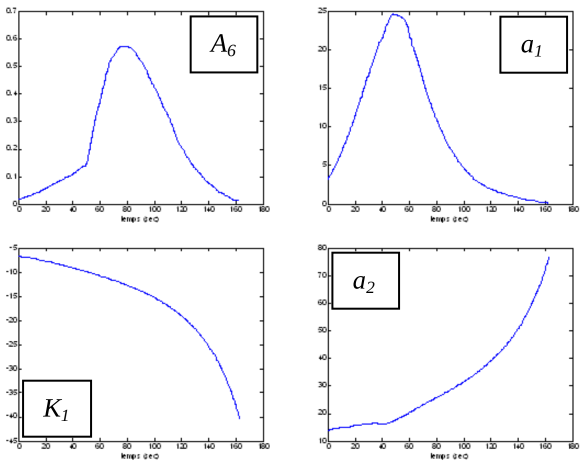

Figure 2. Evolution of the coefficients A_6, K_1, a_1 and a_2

III. Strategy Of Control And Fault Detection

The uncertain system (10) affected by actuator fault $f_a(t)$, can have the following form

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)+Ff_\alpha(t)+M\zeta(t,u,y)\\ y(t)=Cx(t)\end{cases} \quad (12)$$

where $x(t)\ \epsilon\,\Re^n$ is the state vector, $u(t)\ \epsilon\,\Re^m$ is the input vector, $y(t)\ \epsilon\,\Re^p$ is the output vector, $\zeta(t,u,y)\,\epsilon\,\Re^k$ includes uncertainties or perturbations affecting the system like the wind effect and $f_a(t)\ \epsilon\,\Re^q$ is the actuator faults vector. The system matrices A, B and M are defined in the previous Section . F is the repartition matrix of faults. We assume that $\|f_a(t)\|\le\alpha(t)$ and $\|\zeta(t,u,y)\|\le\beta$, where $\alpha:\Re_+ x\Re^m\rightarrow\Re_+$ a known function is and β is a known positive scalar.

In order to eliminate the effect of the actuator fault, a new control law is added to the nominal one. Therefore, the control applied to the system is given by

$$u(t)=\bar{u}(t)+u_0(t)\,.$$

Then (12) can be rewritten as

$$\dot{x}(t)=Ax(t)+B\bar{u}(t)+M\zeta(t,y,u)+Ff_\alpha(t)+Bu_0(t) \quad (13)$$

where $\bar{u}(t)=-KX(t)=-K_x x(t)-K_z z(t)$ is the control component that minimizes a quadratic functional:

$$J=\int_0^\infty x^TQx+\bar{u}^TR\bar{u}\ dt \quad (14)$$

where Q and R are diagonal matrices, weighting each state and control variables respectively in the common performance index (14), and the gain matrix K is determined from the expression:

$$K=-R^{-1}B^TP \quad (15)$$

where P is a positive matrix, solution of the well known Riccati equation.

The additional control law u_0, compensating the effect of faults, can be implemented such that the faulty system (13) is as close as possible to the nominal system, therefore:

$$Ff_a+Bu_0=0 \quad (16)$$

and, if the matrix B is of full row rank, then:

$$u_0=-B^+Ff_a \quad (17)$$

where $B^+=(B^TB)^{-1}B^T$ is the pseudo inverse of matrix B. In cases where the matrix B is not of full rank, the SVD theorem can be applied [9].

The control scheme is described in Figure 3.

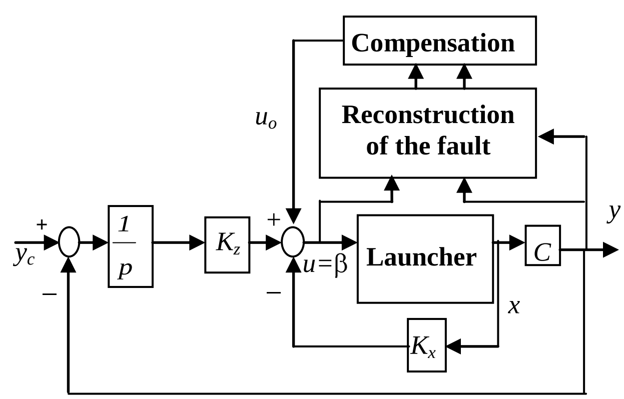

Figure 3. Tolerant control scheme

Using the reconstruction of the actuator fault $f_a(t)$ from the block "Reconstruction" determined in the next section, the component u_o is computed in the block "Compensation" as follows:

$$u_0=-B^+F\widehat{f_a}$$

and we obtain the control law

$$u(t) = -KX(t) - B^{+}F\ \hat{f}_a(t). \tag{18}$$

IV. SLIDING MODE OBSERVER

In the following work, a design method for a sliding mode observer for uncertain linear systems based methodology inspired from the work of Edwards and Spurgeon [10] is presented. The problem of a robust reconstruction of actuators faults can be implemented as shown on Figure 4.

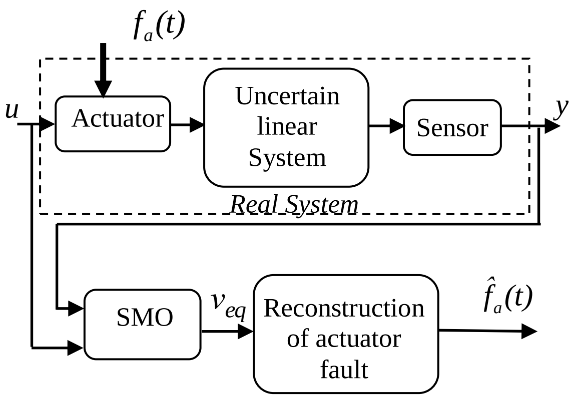

Figure 4. Reconstruction of actuator faults by a Sliding Mode Observer.

For the uncertain system (12), the structure of the SMO is defined by:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + G_l e_y(t) + G_n \upsilon \tag{19}$$

where $G_l \in \Re^{n\times p}$ is the linear gain and $G_n \in \Re^{n\times p}$ is the non-linear gain. The discontinuous vector υ is defined by

$$\upsilon = \begin{cases} -\rho(t,y,u)\dfrac{\bar{P}_0 e_y(t)}{\left\|\bar{P}_0 e_y(t)\right\|} & \text{if } e_y(t) \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

where $e_y = \hat{y} - y$ is the output estimation error, $\bar{P}_0 \in \Re^{pxp}$ is a symmetric positive definite (spd) matrix that will be determined later and the value of the function $\rho{:}\Re_+ \mathrm{x} \Re^p \mathrm{x} \Re^m \to \Re_+$ is a known positive scalar that acts as an upper bound on the uncertainties and the faults.

Edwards, Spurgeon, and Patton [11] have shown that a sliding motion exists if:

- rank$(CF) = q$ (21)
- invariant zeros of the system (A, F, C) are stable.

If these conditions are satisfied, then there exists a change of coordinates such that the triplet (A, F, C) will be as follows:

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \qquad \bar{F} = \begin{bmatrix} 0 \\ \bar{F}_1 \end{bmatrix} \qquad \bar{C} = \begin{bmatrix} 0 & T \end{bmatrix} \tag{22}$$

with $\bar{A}_{11} \in \Re^{(n-p)\times(n-p)}$, $\bar{A}_{12} \in \Re^{(n-p)\times p}$, $\bar{A}_{21} \in \Re^{p\times(n-p)}$, $\bar{A}_{22} \in \Re^{p\times p}$, $\bar{F}_1 \in \Re^{q\times q}$ is non singular and $T \in \Re^{p\times p}$ is orthogonal. Define $\bar{A}_{211}$ as the matrix obtained from the upper $(p-q)$ rows of $\bar{A}_{21}$. Tan and Edwards [5] proved that the pair $\bar{A}_{11}, \bar{A}_{211}$ is detectable since the unobservable modes of this pair are the invariant zeros of the system and they are stable. Define also $\bar{F}_2 \in \Re^{p\times q}$ to be the lower p rows of $\bar{F}$ such that $\bar{F}_1 \subset \bar{F}_2$. Then equations (12) are given by:

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\,\bar{x}(t) + \bar{B}\,u(t) + \bar{F}\ f_\alpha(t) + \bar{M}\ \zeta(t,u,y) \\ \bar{y}(t) = \bar{C}\,\bar{x}(t) \end{cases} \tag{23}$$

Firstly, assume that G_n, in the new coordinates, is given by:

$$\bar{G}_n = \begin{bmatrix} -LT^T \\ T^T \end{bmatrix} \tag{24}$$

where $L = \begin{bmatrix} L_o & 0 \end{bmatrix} \in \Re^{(n-p)\times p}$ with $L_o \in \Re^{(n-p)\times(p-q)}$ and T is defined in (22). For the case, when

$\xi(t,y,u) = 0$ and $\rho = \|CF\|\alpha(t) + \eta_o$ with η_o is a positive scalar, the following results are proven in [5]:

<u>Proposition 1</u>. There exists a Lyapunov symmetric positive definite matrix $\bar{P}$ satisfying:

$$\bar{P}(\bar{A} - \bar{G}_l\bar{C}) + (\bar{A} - \bar{G}_l\bar{C})^T\,\bar{P} < 0 \tag{25}$$

with

$$\bar{P} = \begin{bmatrix} \bar{P}_1 & \bar{P}_1 L \\ L^T\bar{P}_1 & \bar{P}_2 + L^T\bar{P}_1 L \end{bmatrix} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{11}^{\ T} & \bar{P}_{22} \end{bmatrix} > 0 \tag{26}$$

where $\bar{G}_l = TG_l$, $\bar{P}_1 \in \Re^{(n-p)\times(n-p)}$, $\bar{P}_2 \in \Re^{p\times p}$ and the matrix $\bar{P}_0$ in (20) is given by $\bar{P}_0 = T\bar{P}_2 T^T$.

The state estimation error $\bar{e}(t) = T(x(t) - \hat{x}(t))$ is then quadratically stable. Furthermore, a sliding motion occurs in

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finite time on $S_g = \{\bar{e} : C\bar{e} = 0\}$, governed by the stable matrix $(\bar{A}_{11} + L_o\bar{A}_{211})$. Then $\bar{e}(t) \to 0$ as $t \to \infty$.

In the following, these results to the case where $\xi(t, y, u) \neq 0$ are generalized. In this context, the state estimation error dynamical system is given by:

$$\dot{\bar{e}}(t) = (\bar{A} - \bar{G}_l\bar{C})\bar{e}(t) + \bar{G}_n\nu - \bar{F}f_a(t) - \bar{M}\zeta(t, y, u). \qquad (27)$$

Suppose there exists a symmetric definite positive matrix $\bar{P}$ witch satisfies proposition 1. Define the positive scalars

$$\mu_o = -\lambda_{\max}(\bar{P}(\bar{A} - \bar{G}_l\bar{C}) + (\bar{A} - \bar{G}_l\bar{C})^T\bar{P}) \qquad (28)$$

$$\mu_1 = \sqrt{\lambda_{\max}(\bar{M}^T\bar{P}^2\bar{M})}$$

where $\lambda_{\max}$ is the maximum eigenvalue. Suppose that

$$\rho(t, y, u) \geq \|\bar{C}\bar{F}\|\alpha(t) + \eta_o \qquad (29)$$

where η_o is a positive scalar.

In terms of (27), (28) and (29) we have the following result in lemma 1 [4]:

<u>*Lemma 1*</u>. The norm of the state estimation error $\bar{e}(t)$ belongs to the set:

$$\Omega_\varepsilon = \left\{\bar{e} : \|\bar{e}\| < \frac{2}{\mu_o}\mu_1\beta + \varepsilon\right\} \qquad (30)$$

where ε is an arbitrary small positive scalar.

Lemma 1 implies that the choice of $\rho(t, y, u)$ guarantees the sliding mode on S_g and provides an explication for the structures of the matrices defined by (22) after the coordinates change.

The application of a second change of coordinates defined in [5] by

$$\tilde{T} : \bar{e} \mapsto \tilde{e} : \quad \tilde{T} = \begin{bmatrix} I_{n-p} & L \\ 0 & T \end{bmatrix} \qquad (31)$$

where L is given by (24), transforms $(\bar{A}, \bar{F}, \bar{C})$ into the following matrices**:**

$$\tilde{A} = \tilde{T}\bar{A}\tilde{T}^{-1} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{21} \\ \tilde{A}_{12} & \tilde{A}_{22} \end{bmatrix}$$

$$\tilde{M} = \tilde{T}\bar{M} = \begin{bmatrix} \tilde{M}_1 \\ \tilde{M}_2 \end{bmatrix} = \begin{bmatrix} \bar{M}_1 + L\bar{M}_2 \\ T\bar{M}_2 \end{bmatrix} \qquad (32)$$

$$\tilde{C} = \bar{C}\tilde{T}^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix}$$

$$\tilde{F} = \tilde{T}\bar{F} = \begin{bmatrix} 0 \\ \tilde{F}_2 \end{bmatrix}$$

where $\tilde{A}_{11} = \bar{A}_{11} + L_o\bar{A}_{211}$ and $\tilde{F}_2 = T\bar{F}_2$.

Thus, the nonlinear gain and the Lyapunov matrix become:

$$\tilde{G}_n = \tilde{T}\bar{G}_n = \begin{bmatrix} 0 \\ I_p \end{bmatrix}. \qquad (33)$$

The new Lyapunov matrix is given by

$$\tilde{P} = (\tilde{T}^{-1})^T\bar{P}(\tilde{T}^{-1}) = \begin{bmatrix} \bar{P}_1 & 0 \\ 0 & \bar{P}_o \end{bmatrix}. \qquad (34)$$

The new estimation error system is:

$$\dot{\tilde{e}}(t) = (\tilde{A} - \tilde{G}_l\tilde{C})\tilde{e}(t) + \tilde{G}_n - \tilde{F}f_a(t) - \tilde{T}\bar{M}\zeta(t, y, u). \qquad (35)$$

Partitioning this error according to the dimensions of (35), we get

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) + (\tilde{A}_{12} - \tilde{G}_{l_1})\tilde{e}_y - (\bar{M}_1 + L\bar{M}_2)\zeta(t, y, u) \qquad (36)$$

$$\begin{aligned} \dot{\tilde{e}}_y(t) = \tilde{A}_{21}\tilde{e}_1(t) + (\tilde{A}_{22} - \tilde{G}_{l_2})\tilde{e}_y(t) \\ + \nu - \tilde{F}_2 f_a(t) - T\bar{M}_2\zeta(t, y, u) \end{aligned} \qquad (37)$$

where $\tilde{G}_{l_1}$ and $\tilde{G}_{l_2}$ are appropriate partitions of the matrix $\tilde{G}_l = \tilde{T}G_l$.

<u>*Proposition2*</u>: If the gain function $\rho(t, y, u)$ from (20) satisfies the inequality:

$$\rho(t, y, u) \geq 2\|\tilde{A}_{21}\|\mu_1\beta/\mu_o + \|\bar{M}_2\|\beta + \|\tilde{F}_2\|\alpha(t) + \eta_o \qquad (38)$$

where η_o is a positive scalar, then a sliding mode occurs on S_g in finite time, with the presence of faults and matched uncertainties.

V. ROBUST RECONSTRUCTION OF ACTUATOR FAULT

In this part, assume that the SMO (19) is designed and can give a robust reconstruction of the faults $f_a(t)$ with minimization of the effect of $\zeta(t, y, u)$.

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During the sliding motion, $e_y = \dot{e}_y = 0$, equations (36) and (37) become

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) - (\bar{M}_1 + L\bar{M}_2)\zeta(t,y,u) \quad (39)$$

$$0 = \tilde{A}_{21}\tilde{e}_1(t) + \nu_{eq} - \tilde{F}_2 f_a(t) - T\bar{M}_2\zeta(t,y,u) \quad (40)$$

where ν_{eq} is the equivalent output error injection. ν_{eq} can be approximated to any degree of accuracy by replacing υ in (20) with:

$$\nu_{eq} = \begin{cases} -\rho(t,y,u)\dfrac{\bar{P}_0 e_y(t)}{\|\bar{P}_0 e_y(t)\| + \delta} & \text{if } e_y(t) \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (41)$$

where δ is a small positive constant representing the smoothing term. Since ν_{eq} is required for maintaining the sliding motion in presence of faults and uncertainties, the analysis of this term allows us to find the estimated actuator faults $\hat{f}_a(t)$. Now, define an estimate as

$$\hat{f}_a(t) = WT^T\nu_{eq} = G(s)\zeta(t,y,u) + f_a(t). \quad (42)$$

The transfer matrix *G(s)* is defined by

$$G(s) = W\bar{A}_{21}(sI - \tilde{A}_{11})^{-1}(\bar{M}_1 + L\bar{M}_2) + W\bar{M}_2 \quad (43)$$

where $\bar{A}_{21} = T^T\tilde{A}_{21}$ and $WT^TT\bar{M}_2 = W\bar{M}_2$. However, in this case, the transfer matrix *G(s)* links the exogenous input signal $\zeta(t,y,u)$ and the reconstructed faults signal $\hat{f}_a(t)$; thus, obtaining $\hat{f}_a(t) \approx f_a(t)$ (i.e., zero uncertainty case) is equivalent to minimizing the H_∞ norm of *G(s)*, with an appropriately chosen W. To formulate and solve this problem with LMI techniques, the *Bounded Real Lemma* [12] and a numerical development in [4] are used. Then, an optimization problem is address, in which $\|G_{\xi\hat{f}}\|_\infty < \gamma$, where γ is a positive scalar to be minimized with respect to the variable matrices $\bar{P}$, L, and W subject to the following matrix inequalities:

$$\begin{bmatrix} \bar{P}_{11}\tilde{A}_{11} + \tilde{A}_{11}^T\bar{P}_{11} & -\bar{P}_{11}\tilde{M}_1 & -(W\bar{A}_{21})^T \\ -\tilde{M}_1^T\bar{P}_{11} & -\gamma I & (W\bar{M}_2)^T \\ -W\bar{A}_{21} & W\bar{M}_2 & -\gamma I \end{bmatrix} < 0 \quad (44)$$

and

$$\begin{bmatrix} \bar{P}\bar{A} + \bar{A}^T\bar{P} - \gamma_o\bar{C}^T(D_dD_d^T)^{-1}\bar{C} & -\bar{P}B_d & E^T \\ -B_d^T\bar{P} & -\gamma_0 I & H^T \\ E & H & -\gamma_0 I \end{bmatrix} < 0 \quad (45)$$

where $D_d = [D_1 \;\; 0]$, $H = [0 \;\; H_2]$, $E = [E_1 \;\; E_2]$, $\bar{P}_{11}\tilde{A}_{11} = \bar{P}_{11}\bar{A}_{11} + \bar{P}_{12}\bar{A}_{21}$ and $\bar{P}_{11}\tilde{M}_{11} = \bar{P}_{11}\bar{M}_1 + \bar{P}_{12}\bar{M}_2$.

Note that inequalities (44) and (45) are affine with respect of the variables $\bar{P}_{11}$, $\bar{P}_{12}$, W and γ. Thus, the resulting observer is robust enough for the reconstruction of the faults, which affect the linear uncertain system, assuming that the linear gain $\bar{G}_l$ satisfies

$$\bar{G}_l = \gamma_o\bar{P}^{-1}\bar{C}^T(D_dD_d^T)^{-1}. \quad (46)$$

Inequality (44) is a necessary condition for the feasibility of inequality (45) and imposes the following equations $E_1 = -W\bar{A}_{21}$ and $H_2 = W\bar{M}_2$.

Consequently, this method consists of minimizing γ, with respect to the variables $\bar{P}$ and W subject to (44) and (45) where $\gamma_o \in \Re_+$ and $D_1 \in \Re^{p\times p}$ are arbitrary parameters which adjust the observer's gain. It's clear that when γ_o increases, the value of γ decreases, which results in $\bar{G}_l$ having a larger gain. Decreasing the gain of D_1 has the same effect. Let $\gamma_{\min}$ be the minimum value of γ satisfying (44). Then, equation (44) is a sub-block of (45), so, it is logical to always have $\gamma_{\min} \leq \gamma_o$. Moreover, to solve this convex optimization problem, a software like *MATLAB's LMI Control Toolbox* [13] is available to find γ, $\bar{P}$ and *W*.

The gain matrices can be obtained from [3] as

$$L = \bar{P}_{11}^{-1}\bar{P}_{12}, \quad \bar{G}_l = \gamma_o\bar{P}^{-1}C^T(D_dD_d^T)^{-1}, \quad \bar{G}_n = \begin{bmatrix} -LT^T \\ T^T \end{bmatrix},$$

$$\bar{P}_o = T(\bar{P}_{22} - \bar{P}_{12}^T\bar{P}_{11}^{-1}\bar{P}_{12})T^T.$$

The SMO is then completely determined.

VI. SIMULATION RESULTS

The simulation is carried out with Matlab software. The system parameters of the unstable launcher are given as

$$A_6 = 0.57,\ K_1 = -12.2,\ a_1 = 11.3,\ a_2 = 26.5$$

$$V_r = 544.53,\ F_o = 93.81,\ M = 3169.18$$

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In Figure 5, the first two curves show the shape of the fault acting on the actuator at $t = 7$ seconds and its reconstruction. The third curve shows the fault reconstruction error. It can be seen that the fault $f_a(t)$ is faithfully reconstructed.

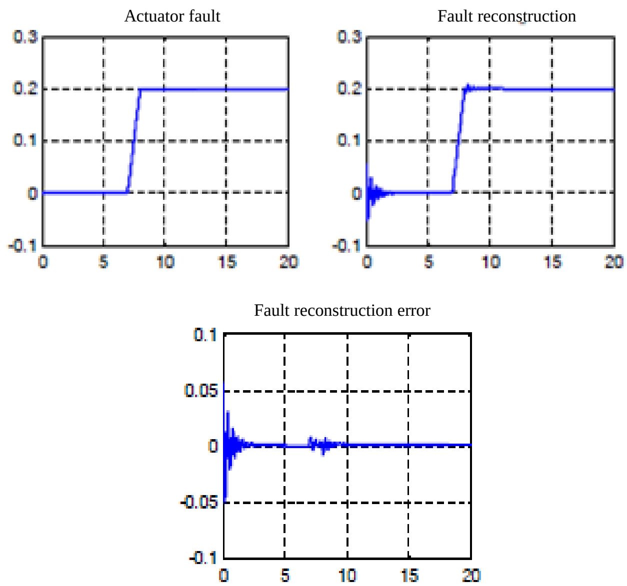

Figure 5. Actuator fault, fault reconstruction and fault reconstruction error

The fault reconstruction is then used to determine the additional term $u_o(t)$ in the control law $u(t)$ according to equation (18).

Figure 6 shows the evolution of the launcher attitude in the normal case (blue curve) and the estimation of its attitude obtained by the SMO observer when a fault appears on the actuator (green curve). To avoid bending forces that can destabilize the launcher, it is important to keep its attitude around zero. It is clear that the control law rejects the fault effect and stabilizes the launcher attitude.

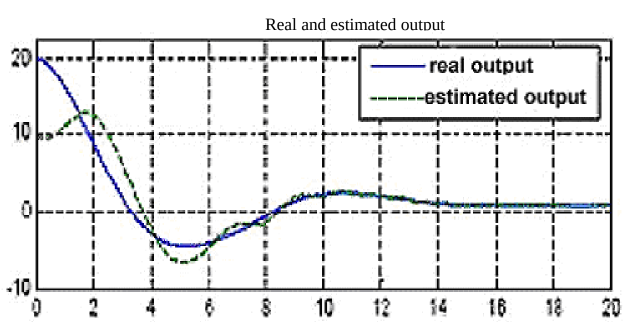

Figure 6. Evolution of the launcher attitude and its estimate

VII. CONCLUSIONS

In this paper, an approach for a robust control system based on fault estimates obtained by reconstruction techniques is proposed for an aerospace launcher system. An SMO was used to reconstruct actuator faults. This approach is based on the minimization of the effect of uncertainty on the faults reconstructed signal by the minimization of the H_∞ norm of the transfer matrix between the unknown inputs and the estimated actuator faults. A signal, built from the fault reconstruction, is then added to the control law and permitted the compensation of the fault effect. A numerical simulation example was provided to verify and validate the developed theoretical results.

Further work will extend the approach to non linear models and will specifically consider the launcher parameters' variations.

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