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Economics of Intelligent Selection of Wireless Access Networks in a Market-Based Framework: A Game-Theoretic Approach

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***Abstract*—The Digital Marketplace is a market-based framework where network operators offer communications services with competition at the call level. It strives to address a tussle between the actors involved in a heterogeneous wireless access network. However, as with any market-like institution, it is vital to analyze the Digital Marketplace from the strategic perspective to ensure that all shortcomings are removed prior to implementation. In this paper, we analyze the selling mechanism proposed in the Digital Marketplace. The mechanism is based on a procurement first-price sealed-bid auction where the network operators represent the sellers/bidders, and the end-user of a wireless service is the buyer. However, this auction format is somewhat unusual as the winning bid is a composition of both the network operator's monetary bid and their reputation rating. We create a simple economic model of the auction, and we show that it is mathematically intractable to derive the equilibrium bidding behavior when there are N network operators, and we make only generic assumptions about the structure of the bidding strategies. We then move on to consider a scenario with only two network operators, and assume that network operators use bidding strategies which are linear functions of their costs. This results in the derivation of the equilibrium bidding behavior in that scenario.**

Keywords-Wireless access networks; network selection; Digital Marketplace; economics; auction theory

I. Introduction

This paper is an extension of the conference paper [1], and aims at providing a greater insight into the economics of intelligent network selection in the Digital Marketplace.

With the advent of 4th Generation wireless systems, such as WiMAX and 3GPP Long Term Evolution (LTE), the world of wireless and mobile communications is becoming increasingly diverse in terms of different wireless access technologies available [2], [3]; each of these technologies has its own distinct characteristics. Mirroring this diversity, multimode terminals (GSM/UMTS/Wi-Fi) currently dominate the market permitting the possibility of selecting the most appropriate access network to match the Quality of Service (QoS) requirements of a particular session/call. A number of approaches have examined this issue utilizing techniques as disparate as neural networks [4] and multiple attribute decision making [5]. The applicability of these techniques can be extended to fixed networks that employ multihoming where the problem becomes one of path selection [6], [7].

This work complements previous studies of intelligent network selection by considering economic aspects. From this perspective the exclusive one-to-one relationship between network operators and their subscribers no longer holds; subscribers are free to choose which operator and which access technology they would like to utilize at call set-up time. From the end-users' perspective, different coverage and QoS characteristics of each access network will lead to the ability to seamlessly connect at any time, at any place, and to the technology, which offers the best quality available for the best price. This is referred to as the *Always Best Connected* networking paradigm [8]. From the network operators' perspective, the integration of wireless access technologies will allow for more efficient usage of the network resources (by utilizing a wireless technology the most suitable to a particular service request), and may be the most economic way of providing both universal coverage and broadband access [2]. For example, a cellular network operator who also owns a set of Wi-Fi hot-spots will be able to offload the bandwidth intensive services from cellular base stations to Wi-Fi hot-spots. This should, in principle, reduce the potential cost to the network operator since instead of investing in additional cellular capacity, they can achieve the same (or better) results by investing into potentially cheaper Wi-Fi.

On the other hand, since many different actors with opposing interests are involved, it may also lead to a 'tussle' [9]. For example, the end-users seek to obtain the best quality for the best price, while the network operators are concerned with maximizing their profit and/or performing efficient load balancing. The conflict will become even more aggravated should the service provision be separated from the network operators [10]. Hence more sophisticated management techniques may be required to manage such a complex system.

In this paper, we analyze the network selection mechanism proposed in the Digital Marketplace (DMP) [11]. The DMP is a framework where network operators offer communications services with competition at the call level, and it strives to address the tussle between the actors involved in a heterogeneous wireless access network. Within this framework, the network selection mechanism constitutes a sealed-bid auction. We create a simple economic model of the auction, and show that it is mathematically intractable to derive the equilibrium bidding behavior when there are N network operators competing in the DMP, and we make only generic assumptions about the structure of the bidding strategies. We then move on to consider a scenario with only two network operators, and assume that network operators use bidding strategies which are linear functions of their costs. This results in the derivation of the equilibrium bidding behavior in that scenario. The main goal of this research is to demonstrate and analyze the boundary conditions for such a market to function in the future. In this context, the participants could be cellular network operators or, alternatively, localized Wi-Fi hotspot operators competing for business.

The rest of this paper is organized as follows. In Section II, a brief summary of related work by other authors is given, while in Section III, an overview of the DMP is provided. Section IV presents the results of the analysis. Section V discusses future work, while Section VI draws conclusions.

II. Related Work

Over the last decade, several different approaches have been proposed as possible solutions to the problem where economic competition is considered. Antoniou *et al.*, and Charilas *et al.* model the problem as a noncooperative game between wireless access networks, which aims at obtaining the best possible tradeoff between networks' efficiency and available capacity, while, at the same time, satisfying the end-users' QoS [12], [13]. Ormond *et al.* propose an algorithm for intelligent cost-oriented and performance-aware network selection, which maximizes consumer surplus [14], [15]. Niyato *et al.* propose two game-theoretic algorithms for intelligent network selection mechanism, which performs intelligent load balancing to avoid network congestion and performance degradation [16]. Khan *et al.* model the problem as a procurement second-price sealed-bid auction where network operators are the bidders and the end-user is the buyer [17], [18]. Lastly, Irvine *et al.* propose a market-based framework called the DMP, where network operators offer communications services with competition at the call level [11], [19], [20].

Although each proposed solution is technically valid, only the DMP strives to address tussle between the actors involved. Not only does the DMP consider the technical challenges but also the economic issues. However, as with any market-like institution, it is vital to analyze the DMP from the strategic perspective (using game theory, or otherwise) to ensure that all shortcomings are removed prior to implementation. This paper presents results of such an analysis.

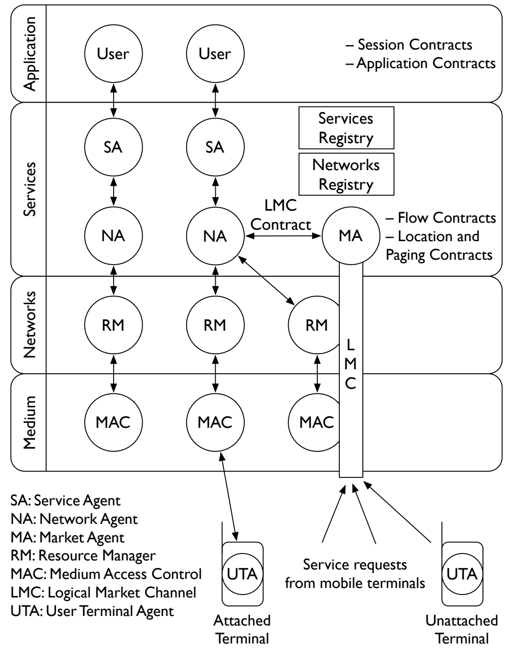

Figure 1. The Digital Marketplace (adapted from [11])

III. The Digital Marketplace

The DMP was developed with the heterogeneous mobile and wireless communications environment in mind, where the end-users have the ability to select a network operator that reflects their preferences best on a per-call basis. In other words, the end-users have the freedom of choice, while the network operators manage service requests appropriately.

The conceptual framework of the DMP is shown in Figure 1. The DMP is defined using a four-layer communications stack: *application layer*, *services layer*, *networks layer*, and *medium layer*. The end-users who effectively reside in the application layer are able to negotiate network access on a per-call basis. To this end, they have two ways of accomplishing it: they can either go into a business relationship with a service provider (service agent, SA, in Figure 1) who will act on their behalf, or they can personally participate in the negotiation process with a network operator (network agent, NA). In both cases, the process is supervised by a market provider (market agent, MA), and takes place in the services layer. Before the negotiation occurs, the end-user is required to forward their service requirements to either the SA or the NA. This is done using a common communications channel referred to as a logical market channel (LMC). The LMC itself is negotiated between the MA and the registered NAs at the marketplace initialization stage.

The network selection mechanism in the DMP is based on a procurement first-price sealed-bid (FPA) auction. The network operators represent the sellers/bidders who compete for the right to sell their product (bearer service) to the end-user. However, unlike in a standard procurement FPA auction, here, network operators do not bid only on prices, but also on reputation; i.e., when selecting the winner, the end-user takes into consideration both the offered price of the product and the network operator's reputation. The reputation is directly proportional to the number of calls that have been decommitted in the past by the respective network operator. Since the network selection is intended to be performed on a per call basis, in a wireless environment, an important factor to consider while selecting an access network is the link/connection quality. There exists an extensive research base in the literature discussing technical constraints of the network selection problem (for example, see [21] for a survey of approaches); however, a few consider economic aspects. In this research, we suggest that poor link/connection quality strongly implies poor reputation.

Out of sealed-bid and sequential-bid auctions (such as English or Dutch auctions), an FPA auction was chosen as a selection mechanism due to the following reasons. Firstly, given the timing constraints in the DMP (e.g., the waiting time of the end-user for the call to be admitted), and the difficulty in predicting the number of bids placed until the winner is selected in a sequential-bid auction, sealed-bid auctions are deemed as the most appropriate [11]. Secondly, the rules governing a second-price sealed-bid auction may appear as counter-intuitive to the end-user; that is, the lowest bid secures the auction but the price paid equals the second-lowest bid [22]. Lastly, since the end-users not only base their network selection strategy on the offered price, but also on reputation, an FPA auction is the best fit to such a requirement.

An FPA auction, in an economic terminology, is an example of an allocation mechanism; that is, a system where economic transactions take place and goods are allocated [23]. As briefly mentioned in the Introduction, it is vital to analyze it from the strategic perspective, and establish what the most probable outcome will be; how the network operators will most likely bid; etc. In this way, all the shortcomings and inefficiencies can be addressed prior to implementation.

IV. Modeling and Analysis

The following notation and concepts are assumed throughout the rest of this paper.

1) Probability Theory and Statistics: Let X denote a random variable (r.v.) with the support $[a, b]$, where $a < b$ and $a, b \in \mathbb{R}$. By F_X we mean a cumulative distribution function of the X r.v.; therefore, for any $x \in \mathbb{R}$, $F_X(x) = P\{X \leq x\}$, where $P\{X \leq x\}$ denotes the probability of the event such that $X \leq x$. If F_X admits a density function, it shall be denoted by $f_X \equiv F'_X$.

The expected value of X, denoted by $E[X]$, is defined as $E[X] = \int_{-\infty}^{\infty} x dF_X(x)$. Similarly, if u is a function of X, then the expected value of $u(X)$ is defined as $E[u(X)] = \int_{-\infty}^{\infty} u(x) dF_X(x)$.

Let $X_1, \ldots, X_n$ be independent continuous r.v.s with distribution function F and density function $f \equiv F'$. If we let $X_{i:n}$ denote the ith smallest of these r.v.s, then $X_{1:n}, \ldots, X_{n:n}$ are called the *order statistics* [24], [25]. In the event that the r.v.s are independently and identically distributed (i.i.d.), the distribution of $X_{i:n}$ is

$$F_{X_{i:n}}(x) = \sum_{k=i}^{n} \binom{n}{k} (F(x))^k (1 - F(x))^{n-k}, \quad (1)$$

while the density of $X_{i:n}$ can be obtained by differentiating Equation (1) with respect to x [26]. Hence,

$$f_{X_{i:n}}(x) = \frac{n!}{(n-i)!(i-1)!} f(x)(F(x))^{i-1}(1 - F(x))^{n-i}.$$

2) Game Theory: Let $\Gamma^B = [N, \{S_i\}, \{u_i\}, \Theta, F]$ be a *Bayesian game with incomplete information*. Formally, in this type of games, each player $i \in N$ has a utility function $u_i(s_i, s_{-i}, \theta_i)$, where $s_i \in S_i$ denotes player i's action, $s_{-i} \in S_{-i} = \times_{j \neq i} S_j$ denotes actions of all other players different from i, and $\theta_i \in \Theta_i$ represents the type of player i. Letting $\Theta = \times_{i \in N} \Theta_i$, the joint probability distribution of the $\theta \in \Theta$ is given by $F(\theta)$, which is assumed to be common knowledge among the players [27]–[29].

In game Γ^B, a *pure strategy* for player i is a function $\psi_i : \Theta_i \to S_i$, where for each type $\theta_i \in \Theta_i$, $\psi_i(\theta_i)$ specifies the action from the feasible set S_i that type θ_i would choose. Therefore, player i's pure strategy set Ψ_i is the set of all such functions.

Player i's *expected utility* given a profile of pure strategies $(\psi_1, \ldots, \psi_N)$ is given by

$$\tilde{u}_i(\psi_1, \ldots, \psi_N) = E[u_i(\psi_1(\theta_1), \ldots, \psi_N(\theta_N), \theta_i)], \quad (2)$$

where the expectation is taken over the realizations of the players' types, $\theta \in \Theta$. Now, in game Γ^B, a strategy profile $(\psi_1^*, \ldots, \psi_N^*)$ is a *pure-strategy Bayesian Nash equilibrium* if it constitutes a Nash equilibrium of game $\Gamma^N = [N, \{\Psi_i\}, \{\tilde{u}_i\}]$; that is, if for each player $i \in N$,

$$\tilde{u}_i(\psi_i^*, \psi_{-i}^*) \geq \tilde{u}_i(\psi_i, \psi_{-i}^*) \quad (3)$$

for all $\psi_i \in \Psi_i$, where $\tilde{u}_i(\psi_i, \psi_{-i})$ is defined as in Equation (2).

3) Incentive Compatibility, Individual Rationality and the Revelation Principle: Let $(\mathbf{Q}, \mathbf{M})$ be a direct mechanism where $\mathbf{Q} = (Q_1, Q_2, \ldots, Q_{|N|})$ is an allocation rule, and $\mathbf{M} = (M_1, M_2, \ldots, M_{|N|})$ a payment rule. Let, as before, Θ_i be the set of all types of player i. The allocation rule Q_i for each player $i \in N$ is then defined as $Q_i : \Theta_i \to \Delta_i$ where Δ_i is the set of all probability distributions over Θ_i. Similarly, the payment rule M_i for each player $i \in N$ is defined as $M_i : \Theta_i \to \mathbb{R}$ [22], [30].

A direct mechanism $(\mathbf{Q}, \mathbf{M})$ is said to satisfy *incentive compatibility* (IC) constraint if for all $i \in N$, for all $\theta_i \in \Theta_i$, and for all $\hat{\theta}_i \in \Theta_i$,

$$\tilde{\tilde{u}}_i(\theta_i) \equiv q_i(\theta_i)\theta_i - m_i(\theta_i) \geq q_i(\hat{\theta}_i)\theta_i - m_i(\hat{\theta}_i),$$

where

$$q_i(\hat{\theta}_i) = E[Q_i(\hat{\theta}_i, \theta_{-i})],$$

and

$$m_i(\hat{\theta}_i) = E[M_i(\hat{\theta}_i, \theta_{-i})].$$

In both cases, the expectation is taken over the realizations of all but player i types, $\theta_{-i} \in \Theta_{-i}$.

A direct mechanism $(\mathbf{Q}, \mathbf{M})$ is said to satisfy *individual rationality* (IR) constraint if for all $i \in N$, and for all $\theta_i \in \Theta_i$,

$$\tilde{\tilde{u}}_i(\theta_i) \geq 0.$$

In the paper, we will also make use of the very powerful Revelation Principle theorem [22], [31], [32]:

Theorem 1 (Revelation Principle)**.** *Given a mechanism and an equilibrium for that mechanism, there exists a direct mechanism in which (1) it is an equilibrium for each buyer to report his or her value truthfully and (2) the outcomes are the same as in the given equilibrium of the original mechanism.*

A. Problem Definition and Assumptions

The formal description of the network selection mechanism employed in the DMP is as follows. The model is a modified version of procurement FPA auction. Thus, formally, it represents a Bayesian game of incomplete information, Γ^B, as defined in Section IV-2. There are N network operators who bid for the right to sell their product to the end-user. With some abuse of notation, we will write N to denote the cardinality of the set N unless it becomes ambiguous where we will succumb to the standard notation of $|N|$.

Let $\beta : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$, defined by

$$\beta(b_i, r_i) = w_{price} \cdot b_i + w_{penalty} \cdot r_i \quad \text{for all } i \in N, \tag{4}$$

denote the *compound bid*. Each network operator i is characterized by the utility function u_i such that

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \text{if } \beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j), \\ 0 & \text{if } \beta(b_i, r_i) > \min_{j \neq i} \beta(b_j, r_j), \end{cases} \tag{5}$$

where $b = (b_i, b_{-i})$ represents the monetary bid (or offered price) vector, $c = (c_i, c_{-i})$ the type vector, and $r = (r_i, r_{-i})$ the reputation rating vector. The type of each network operator is assumed to represent the cost of (or the minimum price for) the service under consideration. The winner of the auction is determined as the network operator whose compound bid is the lowest one; i.e., network operator i is the winner if

$$\beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j).$$

In the event that there is a tie

$$\beta(b_i, r_i) = \min_{j \neq i} \beta(b_j, r_j),$$

the winner is randomly selected with equal probability.

It is, moreover, assumed that the price and reputation weights $(w_{price}, w_{penalty})$ are announced by the end-user to all network operators before the auction. Thus, there is no uncertainty in knowing how much the end-user values the offered price of the service over the reputation of the network operator (or vice versa). Furthermore,

$$w_{price} + w_{penalty} = 1, \quad 0 \leq w_{price}, w_{penalty} \leq 1.$$

In order to simplify the notation, it is assumed throughout the rest of this paper that $w = w_{price}$. This reduces the definition of the compound bid in Equation (4) to

$$\beta(b_i, r_i) = wb_i + (1-w)r_i \quad \text{for all } i \in N.$$

The set of network operators, N, is finite and the network operators are risk neutral. Furthermore, the end-user is risk neutral and does not have any budget constraints; that is, the end-user is prepared to accept any offer from the network operator.

The costs c_i for each network operator i are private knowledge. Thus, they are particular realizations of the r.v.s C_i for each i. Furthermore, it is assumed that each C_i is i.i.d. over the interval $[0,1]$, and admits a continuous distribution function F_C and its associated density function f_C.

The reputation ratings r_i for each network operator $i \in N$ are common knowledge. It is assumed that each $r_i \in [0,1]$ such that the higher the reputation, the lower the rating r_i. In earlier work [1], it was assumed that ratings are private knowledge. However, after analysis, it was concluded that this would contradict its purpose. The reputation of each network operator, in order to be meaningful, must be freely available to everyone, including the competitors of the network operators. For example, in the Amazon.com Marketplace, the buyers have the right to rate the seller they buy from on a scale from one to five (with five being the best), and these ratings are publicly available [33]. Similarly, on eBay, the buyers can leave sellers feedback (negative, neutral, or positive), which over time is viewed as reputation, and is also publicly available [34].

The bidding strategy functions $b_i : [0,1] \to \mathbb{R}_+$ are nonnegative in value for all $i \in N$. The aim is to solve the game for pure-strategy Bayesian Nash equilibrium(-a) as defined in Equation (3), Section IV-2.

The problem will be divided into two cases: generic and restricted case. In the former, no additional assumptions about the game than those already stated in the previous section will be made, and we will concentrate on finding a symmetric equilibrium. In the latter, on the other hand, the problem will be simplified by considering only two network operators, letting the costs be drawn from the uniform distribution, and focusing on bidding strategies, which are linear functions of cost.

B. Generic Case

Suppose that all network operators use the same strictly increasing in c_i bidding strategy function; i.e., $b_i = b_i(c_i) = b(c_i)$ for all $i \in N$. In this case, the equilibrium profile $(b^*, \dots, b^*)$ is called *symmetric*. In its generic form, the problem proves complicated enough for the analytical solution not to be achievable using the existing methods of solving auctions. It would seem that since the problem is a modified version of the standard FPA, the standard analytical approach, found for example in [22], [35]–[37], should apply. However, this is not the case. To see why, note that each network operator i faces an optimization problem

$$\max_{b_i} E\left[b_i - c_i \,\middle|\, wb_i + (1-w)r_i < \min_{j\neq i}(wb(C_j) + (1-w)r_j) \right].$$

Noting that

$$\min_{j\neq i}(wb(C_j) + (1-w)r_j) \geq w \min_{j\neq i} b(C_j) + (1-w)\min_{j\neq i} r_j,$$

and assuming that $w \neq 0$, yields

$$\max_{b_i} E\left[b_i - c_i \,\middle|\, b^{-1}\left(b_i + \frac{1-w}{w}(r_i - \min_{j\neq i} r_j)\right) < \min_{j\neq i} C_j \right] \quad (6)$$

where we have used the fact that b is strictly increasing, and hence, it is invertible and $\min_x b(x) = b(\min_x x)$ for all x.

Let $C_{1:N-1} = \min_{j\neq i} C_j$ be the lowest order statistic of an i.i.d. random sample C_j for all $j \neq i$ with the distribution function $F_{C_{1:N-1}}$. Hence, the identity (6) becomes

$$\max_{b_i}\left(b_i - c_i\right)\left(1 - F_C\left(b^{-1}\left(b_i + \frac{1-w}{w}(r_i - \min_{j\neq i} r_j)\right)\right)\right)^{N-1} \quad (7)$$

where we have used the fact that the distribution function of an i^{th} order statistic of an i.i.d. random sample is defined as in Equation (1).

Finally, recalling that at a symmetric equilibrium $b_i = b(c_i)$ and letting $k = \frac{(1-w)}{w}(r_i - \min_{j\neq i} r_j)$, the identity (7) becomes

$$\begin{aligned} &\frac{d}{dc_i} b\left(b^{-1}(b(c_i)+k)\right) \cdot \left[1 - F_C(b^{-1}(b(c_i)+k))\right]^{N-1} \\ &= (N-1)(b(c_i) - c_i)\left[1 - F_C(b^{-1}(b(c_i)+k))\right]^{N-2} \\ &\cdot f_C(b^{-1}(b(c_i)+k)). \end{aligned} \quad (8)$$

It is rather difficult (if even possible) to solve the resulting ordinary differential equation in (8). Therefore, it can be concluded that even serious simplification of the problem is not enough to heuristically derive an optimal bidding strategy function for each network operator i.

However, it is possible to gain some insight into the problem by analyzing a handful of boundary (or special) cases; that is, $w = 0$, $w = 1$, and $r_i = r_j$ for all $i \neq j$. In all three cases, the problem simplifies enough for the analytical analysis to be tractable, as presented below.

1) Special Case $w = 0$: When $w = 0$, the utility function simplifies to

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \text{if } r_i < \min_{j\neq i} r_j, \\ 0 & \text{if } r_i > \min_{j\neq i} r_j. \end{cases} \quad (9)$$

Since the reputation ratings, r_i, are common knowledge, the probability of winning, i.e., the probability of the event such that $r_i < \min_{j\neq i} r_j$ for all i, is either 0 or 1, and does not depend on the value of the bid, b_i. In other words, each network operator knows in advance whether they won, tied, or lost based on their own and their opponents reputation ratings since these are deterministic in nature. Hence, it is clear that the network operator with the lowest reputation rating will have an incentive to bid abnormally high since they are guaranteed a win regardless of the value of their bid. The remaining network operators, on the other hand, will be indifferent to the value of the submitted bids as it is impossible for them to win regardless of the values of their bids. In case of a tie, i.e., in case there is more than one network operator with the lowest reputation rating, each has an equal probability of winning the auction, and this probability is independent of the values of their bids. Hence, in this case, the network operators also have an incentive to bid abnormally high. Formally,

Proposition 1. *Suppose c_i is i.i.d. over the interval $[0,1]$ for all $i \in N$ and $r_i \in [0,1]$ for all $i \in N$ is common knowledge. Let $N_0 \subseteq N$ be the set of all those network operators with the lowest reputation rating. If $w = 0$, then every network operator $j \in N_0$ will have an incentive to bid abnormally high, i.e., $b_j \to \infty$, while every remaining network operator $k \in N \setminus N_0$ will be indifferent to the value of their bid.*

The formal proof of Proposition 1 is given in Appendix A.

In real life, the end-user will be constrained by a fixed budget. Therefore, when $w = 0$, the real value of the bid will not tend to infinity; rather it is expected to oscillate in the region of the highest price the end-user is willing to pay for the service. In this way, the network operator will extract the entire consumer surplus from the end-user who is looking for a premium service of the best possible quality.

2) Special Case $w = 1$: When $w = 1$, on the other hand, the problem reduces to that of standard FPA auction. The utility of each network operator i becomes

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \text{if } b_i < \min_{j\neq i} b_j, \\ 0 & \text{if } b_i > \min_{j\neq i} b_j. \end{cases} \quad (10)$$

Network operator i, conjecturing that other network operators follow the symmetric equilibrium bidding strategy b and submit their costs truthfully, solves

$$\begin{aligned}
&\max_{b_i} E\left[b_i - c_i \,\middle|\, b_i < \min_{j\neq i} b(C_j)\right] \\
&= \max_{b_i} E\left[b_i - c_i \,\middle|\, b^{-1}(b_i) < \min_{j\neq i} C_j\right] \\
&= \max_{b_i} E\left[b_i - c_i \mid b^{-1}(b_i) < C_{1:N-1}\right] \\
&= \max_{b_i} \int_{b^{-1}(b_i)}^{1} (b_i - c_i) dF_{C_{1:N-1}}(t) \\
&= \max_{b_i} (b_i - c_i)(1 - F_{C_{1:N-1}}(b^{-1}(b_i))), \qquad (11)
\end{aligned}$$

where, as before, $C_{1:N-1} = \min_{j\neq i} C_j$ is the lowest order statistic of an i.i.d. random sample C_j for all $j \neq i$ with the distribution function $F_{C_{1:N-1}}$, and its associated density $f_{C_{1:N-1}}$. The first-order condition yields

$$1 - F_{C_{1:N-1}}(b^{-1}(b_i)) - (b_i - c_i)\frac{f_{C_{1:N-1}}(b^{-1}(b_i))}{\frac{d}{db_i} b(b^{-1}(b_i))} = 0. \qquad (12)$$

Recalling that at a symmetric equilibrium $b_i = b(c_i)$, the identity (12) becomes

$$\frac{d}{dc_i} b(c_i) - b(c_i)\frac{f_{C_{1:N-1}}(c_i)}{1 - F_{C_{1:N-1}}(c_i)} = -c_i \frac{f_{C_{1:N-1}}(c_i)}{1 - F_{C_{1:N-1}}(c_i)}.$$

Since $b(1) = 1$, we have

$$\begin{aligned}
b(c_i) &= \frac{1}{1 - F_{C_{1:N-1}}(c_i)} \int_{c_i}^{1} t dF_{C_{1:N-1}}(t) \\
&= \frac{N-1}{(1 - F_C(c_i))^{N-1}} \int_{c_i}^{1} t(1 - F_C(t))^{N-2} f_C(t) dt. \qquad (13)
\end{aligned}$$

The symmetric bidding strategy in Equation (13) constitutes a symmetric pure-strategy Bayesian Nash equilibrium of the standard FPA auction when $w = 1$. Formally,

Proposition 2. *Suppose c_i is i.i.d. over the interval $[0,1]$ for all $i \in N$ and $r_i \in [0,1]$ for all $i \in N$ is common knowledge. If $w = 1$, then the symmetric equilibrium bidding strategy function of the standard procurement first-price sealed-bid auction,*

$$b^*_{FPA}(c_i) = \frac{1}{1 - F_{C_{1:N-1}}(c_i)} \int_{c_i}^{1} t dF_{C_{1:N-1}}(t), \qquad (14)$$

constitutes a symmetric pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction.

The formal proof of Proposition 2 can be found in [1].

The next natural question to ask is whether b^*_{FPA} constitutes an equilibrium for $w \neq 1$. The following conjecture summarizes this point,

Conjecture 3. *Suppose c_i are i.i.d. over the interval $[0,1]$ for all $i \in N$ and $r_i \in [0,1]$ for all $i \in N$ are common knowledge. If the symmetric equilibrium bidding strategy function of the standard procurement first-price sealed-bid auction, b^*_{FPA}, constitutes a symmetric pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction, then $w = 1$.*

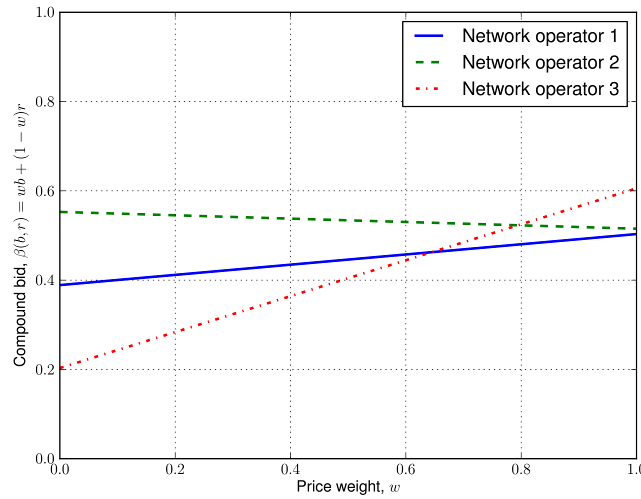

Figure 2. The performance of standard FPA bidding strategy, b^*, for costs, reputation ratings and bids aggregated in Table I

The conjecture can be rephrased as "If $w \neq 1$, then b^*_{FPA} does not constitute a symmetric pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction." The formal proof of this statement is rather difficult. However, the following argument shows why it might hold.

Suppose for the time being that $b^*(c_i) = b^*_{FPA}(c_i)$ for every value of the price weight $w \in [0,1]$. It is possible to estimate numerically how well such a bidding strategy performs for all values of w. To this end, a simple Monte Carlo simulation scenario was constructed where the network operators' costs and reputation ratings were pseudo-randomly generated and drawn from a uniform distribution $\mathcal{U}[0,1]$.

Table I and Figure 2 depict the output from a single simulation for $N = 3$ network operators. In this particular example, for $w \in (0.65, 1]$, network operator 1 who is characterized by the lowest cost of all three network operators, wins the auction; that is, his compound bid is the lowest. At $w = 0.65$, an intersection occurs of network operator 1's and 3's compound bids, and after that, for $w \in [0, 0.65)$, network operator 3 becomes the winner. If the simulation was repeated n times, and the intersection would fall within a close neighborhood of $w = 0.65$ in the vast majority of cases, then b^* is quite likely to be an equilibrium bidding strategy in the interval $w \in (0.65, 1]$. This is predicated on the fact that, as $w \to 1$, the offered price dominates the value of the compound bid; that is, the offered price is weighted more than the reputation rating (see Equation (4)).

The methodology is as follows:

1) Generate cost/reputation rating/bid triplet using the Monte Carlo methods.
2) Find the winner for $w = 1$, network operator i, say (in Figure 2 that would be network operator 1).
3) Decrease the value of w until network operator i no

Table I
THE OUTPUT FROM A SINGLE RUN OF THE MONTE CARLO SIMULATION FOR $N = 3$ NETWORK OPERATORS

	Cost, c_i	**Reputation rating**, r_i	**Bid**, $b^*(c_i)$
Network operator 1	0.2548	0.3889	0.5032
Network operator 2	0.2728	0.5528	0.5152
Network operator 3	0.4084	0.2031	0.6056

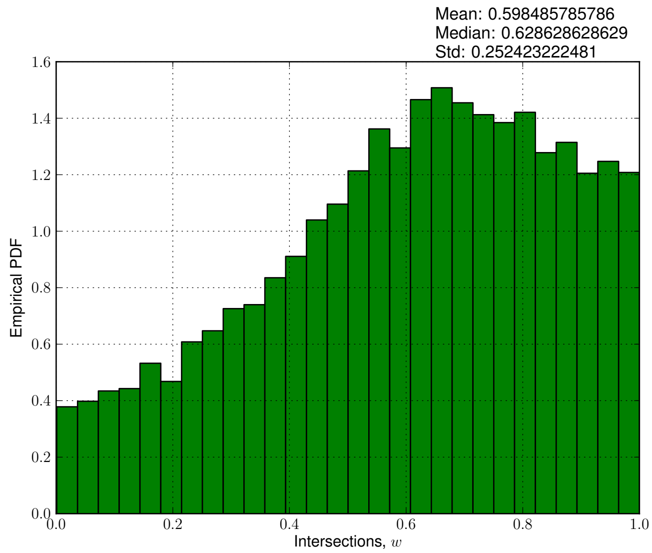

Figure 3. The histogram of intersections, simulated for $n = 10,000$ runs and $N = 3$ network operators

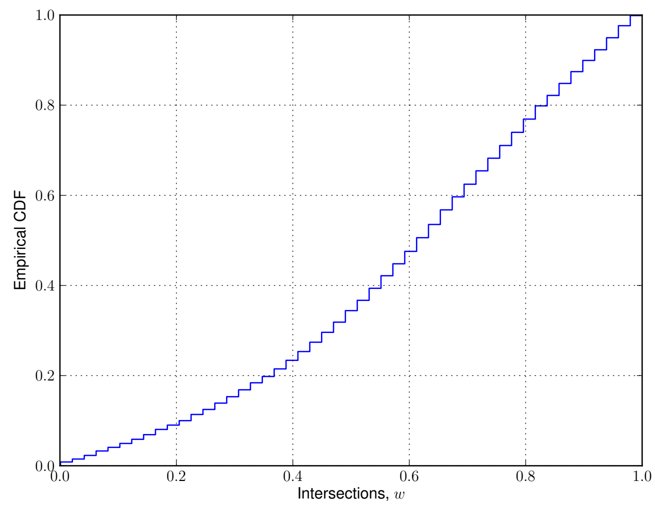

Figure 4. The empirical probability distribution associated with the histogram in Figure 3

longer wins, and save the value of w for which that happens. Henceforth, such an event shall be denoted by I, and called *the event when an intersection has occurred.*

4) If the intersection did not occur, $I = 0$, increase the counter that counts the frequency of such an event, and then discard that run.
5) Repeat n number of times.

By way of example, Figure 3 depicts the empirical density function of the intersections simulated for $n = 10,000$ runs and $N = 3$ network operators, while Figure 4 shows the associated empirical distribution function. The probability of an intersection occurring equals $P\{I = 1\} = 0.67$. It can be concluded from the figures that, on average, the intersections occur at $\bar{w} \approx 0.6$, which represents the mean of the distribution. However, the peak observed in a close neighborhood of $\bar{w}$ is not significant enough to conclude that bidding according to b^* is the best strategy one can take for $w \in (\bar{w}, 1]$.

A more formal argument goes as follows. Figure 4 depicts the probability that an intersection has occurred within an interval $(-\infty, w]$ given that an intersection has occurred, $I = 1$; that is, if the former event is denoted by W, then the figure describes $P\{W \in (-\infty, w] \mid I = 1\}$. From this, the probability of winning for network operator i (as defined in the list above) given any w is

$$\begin{aligned} &P\{\text{winning} \mid w\} = \\ &= 1 - P\{W \in [w, \infty) \cap I = 1\} \\ &= 1 - P\{W \in [w, \infty) \mid I = 1\}P\{I = 1\} \\ &= 1 - (1 - P\{W \in (-\infty, w] \mid I = 1\})\, P\{I = 1\}. \end{aligned} \tag{15}$$

In order to verify Equation (15), set $w \in \{0.25, 0.75\}$ and run a Monte Carlo simulation, which counts the number of times when the network operator with the lowest cost is the winner; i.e., the winner of the auction for $w = 1$. When $w = 0.25$,

$$P\{\text{winning} \mid w = 0.25\} = 1 - (1 - 0.13)0.67 = 0.4171$$

according to Equation (15), while the numerically obtained result equals

$$P\{\text{winning} \mid w = 0.25\} = 0.4136.$$

When $w = 0.75$,

$$P\{\text{winning} \mid w = 0.75\} = 1 - (1 - 0.68)0.67 = 0.7856$$

according to Equation (15), while the numerically obtained result equals

$$P\{\text{winning} \mid w = 0.75\} = 0.7866.$$

Clearly, the prediction based on Equation (15) converges to the numerically obtained result. Moreover, it is worth noting that for $w = 0.25$, bidding according to b^* guarantees the probability of winning for the network operator with the lowest cost of only 0.4171, which is below 50%. Thus, the network operators will definitely deviate from b^* for low values of w. On the other hand, for $w = 0.75$, b^* seems to achieve a relatively high probability of winning for the network operator with the lowest cost; i.e., the probability of 0.7856. However, the argument is incomplete in the sense that it only considers the probability of winning rather than the expected utility.

3) Special Case $r_i = r_j$: In the last extreme case, when all the network operators are characterized by the same reputation rating, i.e., when $r_i = r_j$ for all $i \neq j$ and $w \neq 0$, it can easily be verified that the problem simplifies to the special case $w = 1$. To see why, let $r = r_i$ for all $i \in N$. Then, for all $i \in N$ and $w \neq 0$

$$\begin{aligned}
&\beta(b_i, r) < \min_{j\neq i} \beta(b_j, r) \\
\iff &\frac{1}{w}\left(b_i + \frac{1-w}{w} r\right) < \frac{1}{w} \min_{j\neq i} \left(b_j + \frac{1-w}{w} r\right) \\
\iff &b_i + \frac{1-w}{w} r < \min_{j\neq i} b_j + \frac{1-w}{w} r \\
\iff &b_i < \min_{j\neq i} b_j.
\end{aligned}$$

Hence, the utility of each network operator i simplifies to

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \text{if } b_i < \min_{j\neq i} b_j, \\ 0 & \text{if } b_i > \min_{j\neq i} b_j. \end{cases}$$

Formally,

Corollary 4. *Suppose c_i is i.i.d. over the interval $[0, 1]$ for all $i \in N$ and $r_i \in [0, 1]$ for all $i \in N$ is common knowledge. Suppose $r_i = r_j$ for all $i \neq j$, and $w \neq 0$. Then, the problem simplifies to the special case $w = 1$, and hence, b^*_{FPA} is the symmetric equilibrium bidding strategy (Proposition 2).*

C. Restricted Case $N = 2$

In this section, we will restrict our attention to only two network operators. Since the problem in its generic form proved too complex to be solved analytically, this section will explore whether in a much simplified scenario it is possible to find a closed-form solution. From the mathematical standpoint, restricting the number of network operators to two considerably simplifies the optimization problem that each network operator faces, since it is no longer necessary to consider the minimum of β in the specification of network operators' utility function (Equation (5)).

To this end, let $N = 2$. The utility function for each network operator i thus becomes

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \text{if } \beta(b_i, r_i) < \beta(b_j, r_j), \\ \frac{1}{2}(b_i - c_i) & \text{if } \beta(b_i, r_i) = \beta(b_j, r_j), \\ 0 & \text{otherwise}. \end{cases} \tag{16}$$

Furthermore, the assumption about the symmetric equilibrium profile is relaxed; that is, network operators are permitted to use differing bidding strategies.

The analysis is conducted in two steps. Firstly, it is assumed that information is complete; that is, that each network operator not only knows their own cost and reputation rating, but also those of their opponent's. Secondly, the standard case is considered; that is, that the reputation ratings of the network operators are assumed to be known, while the costs are private knowledge.

1) Complete Information: Here, we assume that information is complete; i.e., that each network operator knows their own and their opponent's cost and reputation rating. In total, there are 7 different bidding scenarios to consider.

Figure 5 shows the first 4 cases for which $r_i < r_j$. (Notice that exactly the same reasoning applies to the situation when $r_i > r_j$.) If $c_i < c_j$, network operator i is guaranteed a victory and a positive profit as long as they bid within the highlighted part of the $\beta(b, r)$ curve depicted in Figure 5a. Thus, their optimal bidding strategy would be to bid slightly less than their opponent's compound bid evaluated at their opponent's cost, $\beta(c_j, r_j)$; that is, $b_i = c_j + \frac{1-w}{w}(r_j - r_i) - \epsilon$ where $\epsilon > 0$ is very small. Network operator j, on the other hand, should find it optimal to bid $b_j = c_j$. To see why, suppose network operator j bids $\hat{b}_j > c_j$. Since network operator i's reputation and cost are strictly lower than those of network operator j's, they can undercut the network operator j's bid by a small amount so that $\hat{b}_i < \hat{b}_j$ and still make positive profit. But, in response, network operator j will find it optimal to lower their bid so that it undercuts that of network operator i's; that is, $\hat{b}_j < \hat{b}_i$. This process will continue until one of the network operators is forced to bid their cost. Since network operator i's reputation and cost are strictly lower than those of network operator j's, we conclude that $b_j = c_j$ and $b_i = c_j + \frac{1-w}{w}(r_j - r_i) - \epsilon$ where $\epsilon > 0$ is very small.

If $c_i = c_j$, arguing in the similar manner as previously, network operator i's optimal bidding strategy would be to bid $b_i = c_j + \frac{1-w}{w}(r_j - r_i) - \epsilon$ where $\epsilon > 0$ is very small; while network operator j should bid $b_j = c_j$ (Figure 5b).

If $c_i > c_j$, there are two cases to consider. If $\beta(c_i, r_i) < \beta(c_j, r_j)$, then network operator i still has some room for maneuver, and should find it optimal to bid $b_i = c_j + \frac{1-w}{w}(r_j - r_i) - \epsilon$ where $\epsilon > 0$ is very small; while network operator j to bid $b_j = c_j$ (Figure 5c). If $\beta(c_i, r_i) \geq \beta(c_j, r_j)$, on the other hand, the roles are reversed, and network operator j should find it optimal to bid $b_j = c_i + \frac{1-w}{w}(r_i - r_j) - \epsilon$ where $\epsilon > 0$ is very small; while network operator i to bid $b_i = c_i$ (Figure 5d).

Figure 6 depicts the remaining 3 cases for which $r_i = r_j$. If $c_i < c_j$, network operator i's optimal bidding strategy would be to bid $b_i = c_j - \epsilon$ where $\epsilon > 0$ is very small; while network operator j should bid $b_j = c_j$ (Figure 6a).

If $c_i = c_j$, both network operators should bid their costs;

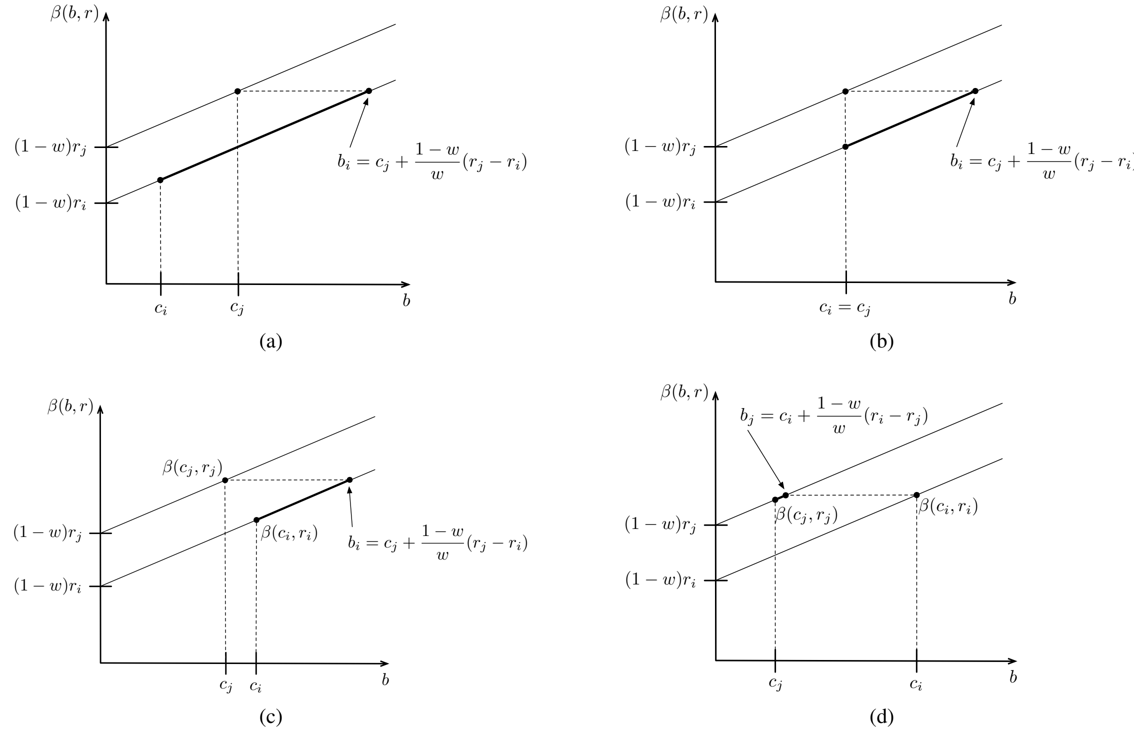

Figure 5. Different bidding scenarios for $r_i < r_j$: (a) $c_i < c_j$, (b) $c_i = c_j$, (c) $c_i > c_j$ with $\beta(c_i, r_i) < \beta(c_j, r_j)$, and (d) $c_i > c_j$ with $\beta(c_i, r_i) \geq \beta(c_j, r_j)$

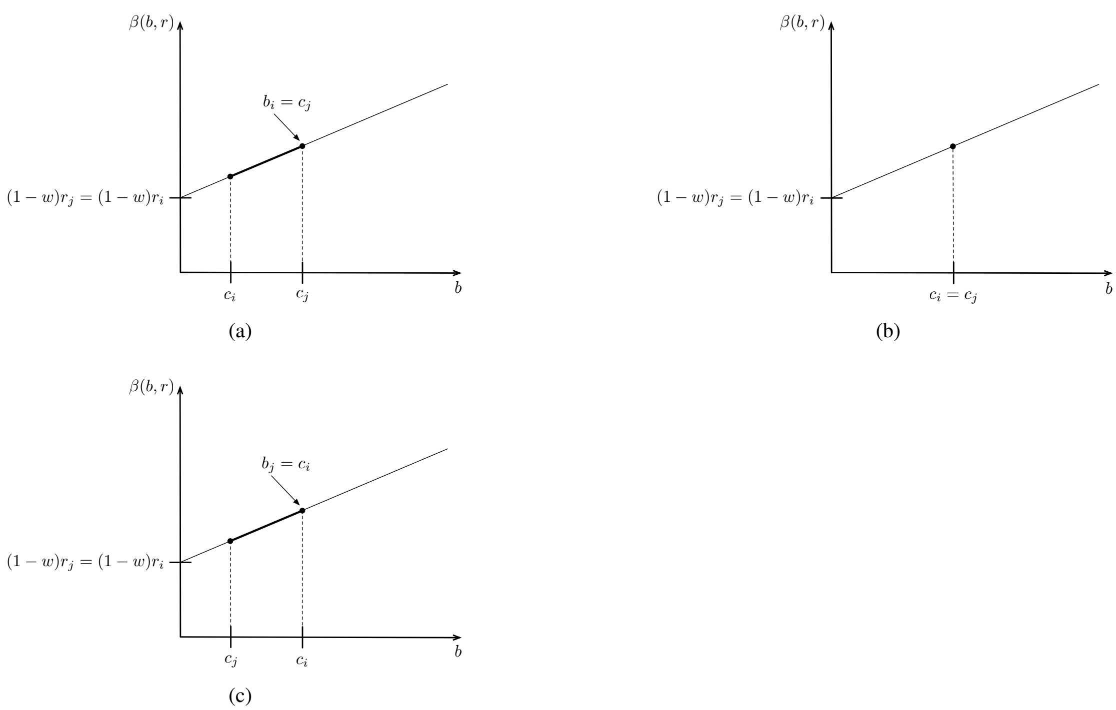

Figure 6. Different bidding scenarios for $r_i = r_j$: (a) $c_i < c_j$, (b) $c_i = c_j$, and (c) $c_i > c_j$

that is, $b_i = c_i$ and $b_j = c_j$ (Figure 6b).

If $c_i > c_j$, network operator j's optimal bidding strategy would be to bid $b_j = c_i - \epsilon$ where $\epsilon > 0$ is very small; while network operator i should bid $b_i = c_i$ (Figure 6c).

It can be concluded that the bidding strategies depend only on costs if $r_i = r_j$. In the remaining cases, they are asymmetric in the sense that the winning network operator is characterized by

$$b_i = c_j + \frac{1-w}{w}(r_j - r_i) - \epsilon \quad \text{with } \epsilon > 0 \text{ being very small,}$$

while the losing network operator by bidding their own cost

$$b_j = c_j.$$

Hence, when dealing with incomplete information, we will exploit these results by concentrating on equilibrium bidding strategies, which are linear functions of cost.

2) Incomplete Information: Here, we assume the standard case; that is, that reputation ratings for both network operators are known at the time of bidding; however, their costs are private knowledge. Suppose that the network operators use a strategy function $b_i : [0,1] \to \mathbb{R}$ defined by the rule

$$b_i(c_i) = m_i + n_i c_i, \quad \text{for all } m_i \in \mathbb{R}, n_i > 0, \tag{17}$$

and the costs are independently drawn from the uniform distribution over the interval $[0,1]$. In other words, (although somewhat counter-intuitive) we allow for negative bids from the network operators. The motivation for such an assumption will become clear later on in this section. Notice, moreover, that the strategy function is assumed to be linear in cost. Each network operator i faces an optimization problem

$$\max_{b_i} E\left[b_i - c_i \mid wb_i + (1-w)r_i < w(m_j + n_j C_j) + (1-w)r_j\right]. \tag{18}$$

If $w = 0$, then the result described in Proposition 1, Section IV-B1, holds. Otherwise, for $0 < w \leq 1$, each network operator i solves

$$\begin{aligned}
&\max_{b_i} E\left[b_i - c_i \,\middle|\, \frac{1}{n_j}\left(b_i + \frac{1-w}{w}(r_i - r_j) - m_j\right) < C_j\right] \\
&= \max_{b_i} \int_{\frac{1}{n_j}\left(b_i + \frac{1-w}{w}(r_i - r_j) - m_j\right)}^{1} (b_i - c_i) dF_C(t) \\
&= \max_{b_i} \left(b_i - c_i\right)\left(1 - \frac{1}{n_j}b_i - \frac{1}{n_j}\left(\frac{1-w}{w}(r_i - r_j) - m_j\right)\right).
\end{aligned} \tag{19}$$

The first-order condition yields

$$\begin{aligned}
&1 - \frac{2}{n_j}b_i + \frac{1}{n_j}c_i - \frac{1}{n_j}\left(\frac{1-w}{w}(r_i - r_j) - m_j\right) = 0 \\
&\iff b_i = \frac{n_j}{2} - \frac{1}{2}\left(\frac{1-w}{w}(r_i - r_j) - m_j\right) + \frac{1}{2}c_i.
\end{aligned} \tag{20}$$

(Notice that the second-order condition is satisfied; i.e., $\frac{d^2}{db_i^2}E[\cdot|\cdot] = -\frac{2}{n_j} < 0$ since $n_j > 0$.) Similar argument for network operator j yields

$$b_j = \frac{n_i}{2} - \frac{1}{2}\left(\frac{1-w}{w}(r_j - r_i) - m_i\right) + \frac{1}{2}c_j. \tag{21}$$

Table II
AN EXEMPLARY SET OF COST-REPUTATION PAIRS FOR TWO NETWORK OPERATORS

	Cost, c_i	**Reputation rating**, r_i
Network operator 1	0.75	0.25
Network operator 2	0.25	0.75

Thus, it follows

$$\begin{cases}
n_i &= n_j = \frac{1}{2}, \\
m_i &= \frac{n_j}{2} - \frac{1}{2}\left(\frac{1-w}{w}(r_i - r_j) - m_j\right), \\
m_j &= \frac{n_i}{2} - \frac{1}{2}\left(\frac{1-w}{w}(r_j - r_i) - m_i\right).
\end{cases}$$

Solving the above equations simultaneously yields the equilibrium bidding strategy, for all i

$$b_i'(c_i) = \frac{1}{2} - \frac{1-w}{3w}(r_i - r_j) + \frac{1}{2}c_i.$$

Formally,

Proposition 5. *Let there be $N = 2$ network operators. Suppose c_i is independently drawn from uniform distribution over the interval $[0,1]$ for all $i \in N$, and $r_i \in [0,1]$ for all $i \in N$ is common knowledge. Then the equilibrium bidding strategy for all $w \in (0,1]$ is given by*

$$b_i'(c_i) = \frac{1}{2} - \frac{1-w}{3w}(r_i - r_j) + \frac{1}{2}c_i. \tag{22}$$

The formal proof of Proposition 5 is given in Appendix A. Observe that the pair of strategies (b_i', b_j') does not constitute a symmetric equilibrium.

By way of example, Table II depicts a particular set of cost-reputation pairs of two network operators. Figure 7 shows the value of the compound bid, β, for different values of w for both network operators, while Figure 8 depicts the value of the bid (or offered price), b_i', for different values of w for both network operators. The numerical data in Table II suggests that network operator 2 should be the winner for the values of $w \to 1$ since network operator 2's cost is strictly lower than that of their opponent's. On the other hand, network operator 1 should be the winner for the values of $w \to 0$ since network operator 1's reputation rating is strictly lower that that of their opponent's (which implies that network operator 1's reputation is strictly higher than that of their opponent's). This prediction agrees with the numerical output shown in Figures 7 and 8. Let w_c denote the value of w for which an intersection between the compound bids of both network operators occurs (if it exists). In Figure 7, $w_c = 0.4$. Hence, network operator 2 wins the auction for the values of $w \in (w_c, 1]$, while network operator 1 for the values of $w \in [0, w_c)$. Notice, moreover, that since the range of the strategy function, b_i, was modified to span the entire real line, that is,

$$b_i : [0,1] \to \mathbb{R},$$

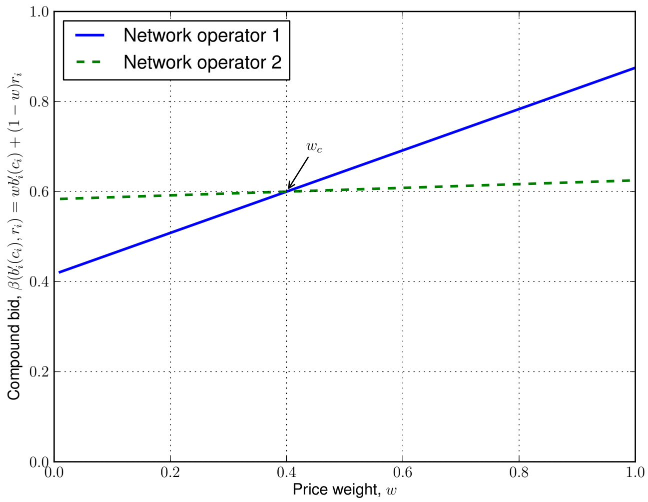

Figure 7. Compound bid plotted against the price weight

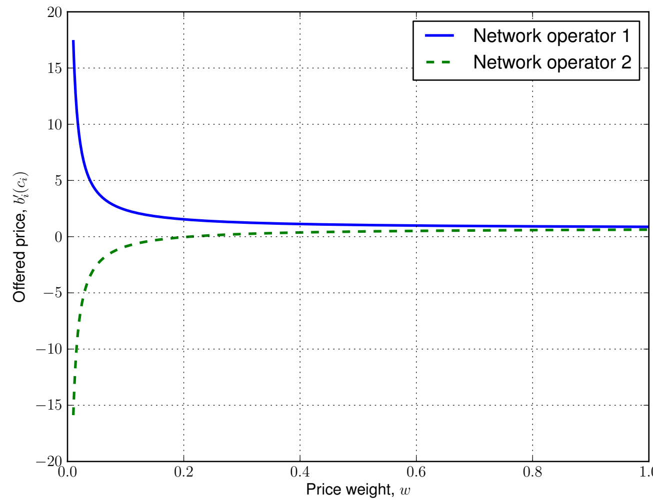

Figure 8. Offered prices (bids) plotted against the price weight

network operator 2, as a result, bids below their cost for values of $w < w_c$ (Figure 8). However, this does not automatically disqualify the equilibrium bidding strategies given by Equation (22). The following observations show why.

Firstly,

Proposition 6. *Suppose both network operators bid according to b_i' bidding strategies. Then they are guaranteed nonnegative profit in case of winning (or a tie).*

The formal proof of Proposition 6 is given in Appendix A.

The proposition implies that even though the equilibrium bidding strategies suggest that one of the network operators may bid negatively, they will not win the auction, and hence, are guaranteed profit at worst equal to zero. Therefore, the possibility of one of the network operators bidding below their cost or negatively will not matter to any of the network operators, and will not lead to an outcome in which the service is sold for a negative price.

Secondly, let $(\mathbf{Q}, \mathbf{M})$ be the direct mechanism induced by the equilibrium bidding strategies, b_i', in Equation (22) where $\mathbf{Q} = (Q_i, Q_j)$ and $\mathbf{M} = (M_i, M_j)$. Here, Q_i represents the allocation rule defined by

$$Q_i(c_i, c_j) = \begin{cases} 1 & \text{if } \beta(b_i'(c_i), r_i) < \beta(b_j'(c_j), r_j), \\ \frac{1}{2} & \text{if } \beta(b_i'(c_i), r_i) = \beta(b_j'(c_j), r_j), \\ 0 & \text{otherwise,} \end{cases} \tag{23}$$

while M_i is the payment rule defined by

$$M_i(c_i, c_j) = Q_i(c_i, c_j) b_i'(c_i). \tag{24}$$

Suppose network operator j reveals their cost truthfully. The equilibrium payoff function for network operator i characterized by cost c_i but revealing $\hat{c}_i$ is

$$\begin{aligned} \tilde{u}_i(\hat{c}_i) &= E\left[M_i(\hat{c}_i, C_j) - c_i Q_i(\hat{c}_i, C_j)\right] \\ &= E\left[(b_i'(\hat{c}_i) - c_i) Q_i(\hat{c}_i, C_j)\right] \\ &= E\left[b_i'(\hat{c}_i) - c_i \mid \beta(b_i'(\hat{c}_i), r_i) < \beta(b_j'(C_j), r_j)\right]. \end{aligned} \tag{25}$$

It turns out that it is in network operator i's best interest to reveal their cost truthfully as well; i.e., $\hat{c}_i = c_i$. Moreover, both network operators cannot be better off by not participating in the auction; i.e., their equilibrium payoff function is nonnegative, $\tilde{u}_i(c_i) \geq 0$. Formally,

Proposition 7. *The direct mechanism $(\mathbf{Q}, \mathbf{M})$ where $\mathbf{Q} = (Q_i, Q_j)$ and $\mathbf{M} = (M_i, M_j)$ (with Q_i and M_i defined in Equations (23) and (24) respectively) satisfies both the IC and IR constraints.*

Thirdly, suppose that economic agents are computers who bid on behalf of the network operators. This assumption is reasonable since there currently are estimated 6.1 billion mobile subscribers around the world [38]. In other words, bidding on a per-call basis would have to be automated by the network operators in order to make the process manageable. One way of achieving such an automation would be to utilize the concept of a direct mechanism. In a direct mechanism, economic agents submit their costs (which need not be truthful) directly to the mechanism, which then computes the bids and chooses the winner on their behalf. By the Revelation Principle (which is stated in Section IV-3), we know that for every mechanism and an equilibrium for that mechanism, there exists an incentive compatible direct mechanism, which yields the same outcomes as in the given equilibrium of the original mechanism. In our case, the direct mechanism $(\mathbf{Q}, \mathbf{M})$ is the direct representation of the DMP variant of an FPA. Since it is incentive compatible, it is in best interest of the economic agents to reveal their costs truthfully. Furthermore, because it is individually rational, it is also in their best interest to participate in the mechanism [22]. Therefore, the possibility of one of the network operators bidding below their cost or negatively will not matter to any of the network operators and will not lead to an outcome in which the service is sold for a negative price.

V. Future Work

In the restricted case, the possibility of one of the network operators bidding below their cost or negatively might seem counter-intuitive and irrational. Therefore, one of the future directions will include an in-depth analysis of this problem. The most straightforward solution is to constrain the optimization problem in Equation (18); that is, each network operator i tries to solve

$$\max_{b_i} E\left[b_i - c_i \mid wb_i + (1-w)r_i < w(m_j + n_j C_j) + (1-w)r_j\right]$$
$$\text{subject to} \quad c_i - b_i \leq 0.$$

The constraint $c_i - b_i \leq 0$ ensures that each network operator bids above or equal to their cost. However, this problem is much more complicated than its unconstrained version in Equation (18). Not only is it necessary to solve the nonlinear constrained optimization problem for each network operator i, but also it needs to be done simultaneously [39]. The preliminary analysis of the problem, which employs the application of the Karush-Kuhn-Tucker Conditions theorem seems to suggest that the most likely candidate for the solution would be

$$b_i^*(c_i) = \max\left\{c_i, b_i'(c_i)\right\} \quad \text{for all } i \in N.$$

However, this has yet to be verified.

VI. Conclusions

This paper has presented the results of the game-theoretic analysis of network selection mechanism proposed in the Digital Marketplace. All things considered, it can be concluded that the analytical analysis of the Digital Marketplace variant of procurement first-price sealed-bid auction is mathematically intractable for all but special cases considered in this paper. It is, however, vital to have at least partially accurate predictions of the behavior of the network operators prior to implementation.

In the generic case, where there are N network operators and costs are drawn from an arbitrary continuous distribution, derivation of the equilibrium bidding behavior is complicated. Nevertheless, some light was shed on the problem in a handful of special cases: $w = 0$, $w = 1$, and $r_i = r_j$. In the first case, we showed that network operators will find it beneficial to submit abnormally high bids, since their bid is independent of the probability of winning the auction. In the remaining two cases, when $w = 1$ and $r_i = r_j$, we showed that the problem reduces to a standard procurement first-price sealed-bid auction, and therefore, the symmetric equilibrium bidding behavior of the standard procurement first-price auction constitutes an equilibrium of the Digital Marketplace auction.

In the restricted case, where there are two network operators and costs are uniformly distributed, we successfully derived the equilibrium bidding strategies that are linear functions of cost. However, we showed that the derived bidding strategy functions constitute an asymmetric equilibrium; that is, their closed-form expression is not identical for both network operators. This implies that the analysis of the case with more than two network operators might not be analytically possible, and hence, indirectly explains the reason for unsuccessful analysis of the generic case. Furthermore, we showed that although the derived equilibrium bidding behavior allows for negative bids, it does not lead to negative profit in case of winning (or a tie) of either network operator. In fact, we established that the direct mechanism representation of the Digital Marketplace auction satisfies both individual rationality and incentive compatibility constraints. Therefore, if the auction were to be automated through the use of a direct mechanism, the network operators would find it in their best interest to participate in the auction, and they would reveal their costs truthfully.

Appendix
Proofs

Proof of Proposition 1: Let $w = 0$ and let $|N_0| = M$ be the number of network operators with the lowest reputation rating such that $M \in \mathbb{Z}_+$. Since N is finite and $N_0 \subset N$, then $M \leq |N|$. Now, each $j \in N_0$ is facing a maximization problem

$$\max_{b_j} \frac{1}{M}\left(b_j - c_j\right), \quad \text{for all } j \in N_0.$$

Since $1 \leq M \leq |N|$, and since $b_j \in \mathbb{R}_+$ and $\mathbb{R}_+$ is not bounded from above, this implies that the maximization problem is unbounded; that is, $b_j \to \infty$ for all $j \in N_0$.

The remaining network operators $k \in N - N_0$ will try to solve

$$\max_{b_k} 0, \quad \text{for all } k \in N - N_0,$$

since $r_k > r_j = \min_{i \in N} r_i$. Hence, each network operator $k \in N - N_0$ is indifferent to the value of their bid, which concludes the proof. ■

Proof of Proposition 5: Suppose there are two network operators: network operator 1 and network operator 2 with cost-reputation pairs (c_1, r_1) and (c_2, r_2) respectively. Suppose that network operator 2 follows b_2' equilibrium bidding strategy. We will argue that it is optimal for network operator 1 to follow b_1' equilibrium bidding strategy. First, notice that b_1' is strictly increasing and continuous function of cost (similarly is b_2'). Suppose that network operator 1 bids an amount b_1. Since b_1' is strictly increasing, there exists unique cost $\hat{c}_1$

such that $\hat{c}_1 = b_1'^{-1}(b_1)$. Network operator 1's expected utility when bidding $b_1'(\hat{c}_1)$ is

$$\begin{aligned}&\tilde{u}_1(b_1'(\hat{c}_1), c_1)\\&= E\left[b_1'(\hat{c}_1) - c_1 \,\middle|\, wb_1'(c_1) + (1-w)r_1 < wb_2'(c_2) + (1-w)r_2\right]\\&= \frac{1}{2}\left(1 - \frac{2}{3}\cdot\frac{1-w}{w}(r_1 - r_2) + \hat{c}_1 - 2c_1\right)\\&\cdot\left(1 - \hat{c}_1 - \frac{2}{3}\cdot\frac{1-w}{w}(r_1 - r_2)\right).\end{aligned}$$

We thus obtain that

$$\tilde{u}_1(b_1'(c_1), c_1) - \tilde{u}_1(b_1'(\hat{c}_1), c_1) = \frac{1}{2}(c_1 - \hat{c}_1)^2 \geq 0$$

regardless of whether $\hat{c}_1 \geq c_1$ or $\hat{c}_1 \leq c_1$. We have thus argued that if network operator 2 follows b_2', network operator 1 with a cost c_1 cannot benefit by bidding anything other than $b_1'(c_1)$. Similar argument can be used to show that it is optimal for network operator 2 to follow b_2' when network operator 1 is following b_1'. Hence, (b_1', b_2') constitutes a Bayesian-Nash equilibrium profile. Similar argument can be used to show that it is optimal for network operator 2 to reveal their cost truthfully, which concludes the proof. ■

Proof of Proposition 6: Let there be two network operators: network operator 1 and network operator 2 with cost-reputation pairs (c_1, r_1) and (c_2, r_2) respectively. Suppose that both network operators follow the equilibrium bidding strategy, b_i'. Without loss of generality, we need to show that network operator 1's bid is always at least as high as their cost whenever they win or draw with network operator 2; that is, $b_1'(c_1) \geq c_1$.

First of all, notice that if $r_1 \leq r_2$,

$$b_1'(c_1) = \frac{1}{2} - \frac{1-w}{3w}(r_1 - r_2) + \frac{1}{2}c_1 \geq \frac{1}{2}(1 + c_1) \geq c_1,$$

for all $c_1 \in [0,1]$. Thus, we need only to consider the case when $r_1 > r_2$.

Suppose $r_1 > r_2$. If $c_1 > c_2$, and since $b_1'(c_1)$ is strictly increasing in c_1, network operator 1 will lose for all values of $w \in (0,1]$. If $c_1 = c_2$, network operator 1 will lose for all values of $w \in (0,1)$, except at $w = 1$ when there will be a draw. But at $w = 1$, network operator 1's bid is at least as high as their cost; i.e.,

$$b_1'(c_1) = \frac{1}{2}(1 + c_1) \geq c_1, \quad \text{for all } c_1 \in [0,1].$$

If $c_1 < c_2$, it is sufficient to show that the intersection of $b_1'(c_1)$ and c_1 in terms of w can never occur before the intersection of $\beta(b_1'(c_1), r_1)$ and $\beta(b_2'(c_2), r_2)$. First of all, we need to check that both intersections do occur; that is,

$$b_1'(c_1) = c_1 \iff w = \frac{1}{1 + \frac{3}{2}\cdot\frac{1-c_1}{r_1-r_2}}.$$

Similarly,

$$\beta(b_1'(c_1), r_1) = \beta(b_2'(c_2), r_2) \iff w = \frac{1}{1 + \frac{3}{2}\cdot\frac{c_2-c_1}{r_1-r_2}}.$$

Since $r_1 > r_2$ and $c_1 < c_2$, we have $0 < r_1 - r_2 \leq 1$ and $0 < c_2 - c_1 \leq 1$. Therefore, this implies

$$0 < w = \frac{1}{1 + \frac{3}{2}\cdot\frac{1-c_1}{r_1-r_2}} \leq 1,$$

and

$$0 < w = \frac{1}{1 + \frac{3}{2}\cdot\frac{c_2-c_1}{r_1-r_2}} \leq 1.$$

Now, suppose that the intersection of $b_1'(c_1)$ and c_1 occurs before that of $\beta(b_1'(c_1), r_1)$ and $\beta(b_2'(c_2), r_2)$. We must thus have

$$\frac{1}{1 + \frac{3}{2}\cdot\frac{c_2-c_1}{r_1-r_2}} < \frac{1}{1 + \frac{3}{2}\cdot\frac{1-c_1}{r_1-r_2}} \iff \frac{1-c_2}{r_1-r_2} < 0.$$

But since $c_2 \in [0,1]$ and $r_1 > r_2$ by assumption,

$$0 < \frac{1-c_2}{r_1-r_2}$$

we reach a contradiction, and this concludes the proof. ■

Proof of Proposition 7: Let there be two network operators: network operator 1 and network operator 2 with cost-reputation pairs (c_1, r_1) and (c_2, r_2) respectively. Suppose that both network operators participate in the direct mechanism $(\mathbf{Q}, \mathbf{M})$. Firstly, we show that the mechanism is incentive compatible. Without loss of generality, suppose that network operator 2 truthfully submits their cost to the mechanism. We argue that it is optimal for network operator 1 to also submit their cost truthfully. Suppose to the contrary; that is, that network operator 1 has an incentive not to reveal their cost truthfully by submitting $\hat{c}_1$. Thus, their expected utility becomes

$$\begin{aligned}&\tilde{\tilde{u}}_1(\hat{c}_1)\\&= E\left[b_1'(\hat{c}_1) - c_1 \,\middle|\, 2b_1'(\hat{c}_1) - 1 + \frac{4}{3}\cdot\frac{1-w}{w}(r_1 - r_2) < C_2\right]\\&= \left(\frac{1}{2} - \frac{1}{3}\cdot\frac{1-w}{w}(r_1 - r_2) + \frac{1}{2}\hat{c}_1 - c_1\right)\\&\cdot\left(1 - \hat{c}_1 - \frac{2}{3}\cdot\frac{1-w}{w}(r_1 - r_2)\right).\end{aligned}$$

The first-order condition yields $\hat{c}_1 = c_1$ and the second-order condition is satisfied. Hence, this shows that $(\mathbf{Q}, \mathbf{M})$ is incentive compatible.

Secondly, we show that $(\mathbf{Q}, \mathbf{M})$ is individually rational. Since the mechanism is incentive compatible, each network operator reveals their cost truthfully. Hence, for all c_1

$$\begin{aligned}\tilde{\tilde{u}}_1(c_1) &= \left(\frac{1}{2} - \frac{1}{3}\cdot\frac{1-w}{w}(r_1 - r_2) - \frac{1}{2}c_1\right)\\&\cdot\left(1 - c_1 - \frac{2}{3}\cdot\frac{1-w}{w}(r_1 - r_2)\right)\\&= \frac{1}{2}\left(1 - c_1 - \frac{2}{3}\cdot\frac{1-w}{w}(r_1 - r_2)\right)^2 \geq 0.\end{aligned}$$

Therefore, $(\mathbf{Q}, \mathbf{M})$ is individually rational. ■

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