Inventory-Based Empty Container Repositioning in a Multi-Port System

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Abstract— **The purpose of this study is to develop an inventory-based control policy to reposition empty container in a multi-port system with uncertain customer demand. A single-level policy with repositioning rule in terms of minimizing the repositioning cost is proposed to manage the empty container with periodical review. The objective is to optimize the parameters of the policy to minimize the expected total cost per period incurred by repositioning empty container, holding unused empty container and leasing empty container. The problem is solved by applying non-linear programming and a gradient search approach with Infinitesimal Perturbing Analysis (IPA) estimator. Numerical examples are given to demonstrate the effectiveness of the proposed policy.**

Keywords- empty container repositioning; inventory control; simulation; Infinitesimal Perturbation Analysis (IPA).

I. INTRODUCTION

In the last few decades, the containerization of cargo transportation has been the fastest growing sector of the maritime industries. The growth of containerized shipping has presented challenges inevitably, in particular to the management of empty containers arising from the highly imbalanced trade between countries. It is reported that empty container movements constitute approximately 20% of the world ports handing activity ever since 1998 [1]. Song [2] reports that the cost of repositioning empty container is just under \$15 billions, which is 27% of the total world fleet running cost based on the data for 2002. If the cost of repositioning empty container can be reduced, the shipping company could increase profit and improve competitiveness. Therefore, how to effectively and efficiently manage ECs is a very important issue for shipping company and it is known as *empty container repositioning* (ECR) problem.

Much attention about ECR problem has been focused on utilizing mathematic models to solve this issue [3-7]. Mathematic models can often capture the nature of the problem, while give rise to concerns, such as requirement of a pre-specified planning horizon, sensitivity of the decisions to data accuracy and variability and implementation of the decisions in the stochastic systems [8, 9]. Recently, several authors turn to explore the inventory-based mechanism in addressing the ECR problem in the stochastic systems [2, 10, 11]. These studies demonstrate that the optimal repositioning policies are of the threshold control type, characterized by some parameters and rules, in some situations such as one-port and two-port systems. Researchers extend the above works to more general systems and focus on the implementation of threshold-type control policies [9, 12~14].

In this paper, we consider the ECR problem in a multi-port system which comprises a set of ports connected to each other and a fleet of owned containers are used to meet the stochastic customer demands. A single-level threshold policy with repositioning rule in terms of minimizing the repositioning cost is proposed to manage the EC with periodical review. The objective of the paper is to minimize the expected total cost per period, including transportation cost and holding and leasing cost by optimizing the parameters of the single-level policy. The paper is organized as follows. Section II presents the formulation for our problem. Section III describes the Infinitesimal Perturbation Analysis (IPA)-based gradient technique to solve the problem. Section IV illustrates the numerical studies. Conclusions are provided in the last section.

II. PROBLEM FORMULATION

We consider a multi-port system, consisting of ports connected with each other. A fleet of owned ECs meets exogenous customer demands, which are defined as the requirements for transforming ECs to laden containers and then transporting these laden containers from original ports to destination ports. A single-level threshold policy with periodical review is employed to manage the ECs. At the beginning of a period, the ECR decisions are made for each port, involving whether to reposition ECs, to/from which ports, and in what quantity. Then, when the customer demands occur in the period, we can use those containers that are currently stored at the port and those ECs that are repositioned to the port in the period to satisfy. If it is not enough, we need to lease additional ECs immediately from vendors. We make the following assumptions:

- The owned container fleet is fixed.
- Short-term leasing is considered and the quantity of the leased ECs is always available in the port at any time.
- The leased ECs are not distinguished from owned container.
- Twenty-foot equivalent unit (TEU) is used to represent a container.
- The travel time for each O-D pair (p, m) (from port p to port m) is less than one period length.
- When the repositioned ECs arrive at the destination ports, they will become available immediately.

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- When the laden containers arrive at the destination ports, they will become empty and be available at the beginning of next period.

A. Notation

To formulate the problem, following notations are introduced firstly.

N The fleet of owned empty containers
P The set of ports
t The discrete time decision period
P_t^S The surplus port subset in period t
P_t^D The deficit port subset in period t
P_t^B The balanced port subset in period t
$x_{p,t}$ The beginning on-hand inventory of port p in period t
$y_{p,t}$ The inventory position of port p in period t after making the ECR decisions
$z_{p,m,t}$ The amount of ECs repositioned from surplus port p to the port m in period t
$\varepsilon_{p,m,t}$ The random customer demand for the O-D pair (p,m) in period t
$\mathbf{x}_t$ The vector of the beginning on-hand inventory in period t
$\mathbf{y}_t$ The vector of the inventory position in period t
$\mathbf{Z}_t$ The array of repositioned quantities for all ports
$a_{p,t}^S$ The amount of estimated EC supply for surplus port p in period t
$a_{p,t}^D$ The amount of estimated EC demand for deficit port p in period t
ω_t The stochastic customer demands in period t, which is the array of a realization of the customer demands
$C_{p,m}^R$ The cost of repositioning an EC from port p to port m
C_p^H The cost of holding an EC at port p per period
C_p^L The cost of leasing an EC at port p per period
γ_p The threshold of port p
$\boldsymbol{\gamma}$ Vector of the thresholds

To simplify the narrative, the following notations are introduced.

$u_{p,t}^O = \sum_{m\in P(p)} z_{p,m,t}$ The sum of ECs repositioned out from port p in period t
$u_{p,t}^I = \sum_{l\in P(p)} z_{l,p,t}$ The sum of ECs repositioned into port p in period t
$\eta_{p,t}^O = \sum_{m\in P(p)} \varepsilon_{p,m,t}$ The sum of exported laden containers of port p in period t
$\eta_{p,t}^I = \sum_{l\in P(p)} \varepsilon_{l,p,t}$ The sum of imported laden containers of port p in period t
$F_p(.)$ the cumulative distribution function for $\eta_{p,t}^O$
$\varphi_{p,t} = \eta_{p,t}^I - \eta_{p,t}^O$ The amount of the difference between the laden container inbound and outbound of port p in period t

It should be pointed out that $\mathbf{x}_1$ is a given state variable, i.e., the given initial on-handing inventory; while $\mathbf{x}_t$ is a decision variable for $t > 1$. The ECR decisions are made at the beginning of period t firstly. Then, the inventory position can be obtained by

$$y_{p,t} = x_{p,t} - u_{p,t}^O + u_{p,t}^I \quad \forall p \in P \tag{1}$$

After customer demands are realized and the laden containers become available, the beginning on-hand inventory for the next period can be updated by

$$x_{p,t+1} = y_{p,t} + \varphi_{p,t} \forall p \in P \tag{2}$$

Next, we present the single-level threshold policy to determine the repositioned quantities $\mathbf{Z}_t$ in period t.

B. A Single-Level Threshold Policy

To make the ECR decisions, a single-level threshold policy is developed, which tries to maintain the inventory position at a target threshold value $\boldsymbol{\gamma}$. More specifically, port p has a target threshold, namely γ_p; in each period, such as in period t, if the beginning on-handing inventory of port p, namely $x_{p,t}$ is greater than its threshold value, i.e., γ_p, then it is a surplus port and the quantity excess of γ_p can be repositioned out to other ports that may need it to try to bring the inventory position down to γ_p; if $x_{p,t}$ is less than γ_p, then it is a deficit port and ECs should be repositioned into this port from surplus ports to try to bring the inventory position up to γ_p; if $x_{p,t}$ is equal to γ_p, then it is a balanced port and nothing is done.

Without loss of generality, we consider the ECR decisions in period t. According to the threshold policy, three subsets, i.e., surplus port subset, deficit port subset and balanced port subset can be obtained as follows: $P_t^S = \{i: x_{i,t} > \gamma_i\}$;$P_t^D = \{j: x_{j,t} < \gamma_j\}$; $P_t^B = \{b: x_{b,t} = \gamma_b\}$. When either the surplus port subset or the deficit port subset is empty, we do nothing. That is, no ECs are repositioned and we can have $\mathbf{Z}_t = 0$. However, when P_t^S and P_t^D are nonempty, we can compute the amounts of EC supplies of surplus ports and EC demands of deficit ports by:

$$a_{i,t}^S = x_{i,t} - \gamma_i \quad \forall i \in P_t^S \tag{3}$$

$$a_{j,t}^D = \gamma_j - x_{j,t} \quad \forall j \in P_t^D \tag{4}$$

Then, the problem is about moving ECs from surplus ports to deficit ports in the right quantity at the least movement cost. A transportation model is formulated to solve this problem as follows:

$$\min \sum_{i\in P_t^S} \sum_{j\in P_t^D} C_{i,j}^R z_{i,j,t} \tag{5}$$

$$\text{s.t} \qquad \sum_{j\in P_t^D} z_{i,j,t} \le a_{i,t}^S \quad \forall i \in P_t^S \tag{6}$$

$$\sum_{i\in P_t^S} z_{i,j,t} \le a_{j,t}^D \quad \forall j \in P_t^D \tag{7}$$

$$\sum_{i\in P_t^S} \sum_{j\in P_t^D} z_{i,j,t} = \min\left(\sum_{i\in P_t^S} a_{i,t}^S, \sum_{j\in P_t^D} a_{j,t}^D\right) \tag{8}$$

$$z_{i,j,t} \ge 0 \quad \forall i \in P_t^S, j \in P_t^D \tag{9}$$

Constraints (6) and (7) are resource constraints. Constraint (8) implies that the amount of total exported ECs from the surplus ports is capacitated by the amount of total demands of all deficit ports; thus, we can try to bring the inventory position of each port back to its threshold level. Constraints (9) are the non-negative repositioned EC quantity constraints.

Solving the transportation model, we can obtain the repositioned quantities from the surplus ports to the deficit

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ports. To further complete the value of $\mathbf{Z}_t$, which involves the repositioned quantities for all ports, we set $z_{l,b,t} = z_{b,m,t} = 0\ \forall l \in P(b), m \in P(b)$ for a balanced port $b \in P_t^B$, $z_{l,j,t} = 0\ \forall l \in P(j)$ for a surplus port $j \in P_t^S$ and $z_{i,m,t} = 0\ \forall m \in P(i)$ for a deficit port $i \in P_t^D$, which reflect the facts that a balanced port does not reposition in or out ECs, a surplus port does not reposition in ECs and a deficit port does not reposition out ECs, respectively.

C. The Optimization Problem

Let $J(N,\boldsymbol{\gamma})$ be the expected total cost per period with the fleet size N and policy parameter $\boldsymbol{\gamma}$. The problem, which is to find the optimal parameters of the given policy, namely $\boldsymbol{\gamma}^*$ that minimizes the expected total cost per period can be formulated as

$$\min_{\gamma} J(N,\boldsymbol{\gamma}) \tag{10}$$

subject to the single-level threshold policy, the given fleet size N and the inventory dynamics equations (1) and (2). With a slight misuse of the notation, we drop the subscript t in the notations of $\mathbf{x}_t, \mathbf{y}_t, \omega_t, \mathbf{Z}_t$ and $\eta_{p,t}^O$ for ease of description. More specifically, $J(N,\boldsymbol{\gamma})$ can be formulated as:

$$J(N,\boldsymbol{\gamma}) = EJ(\mathbf{x},\boldsymbol{\gamma},\omega) = E\left(H(\mathbf{x},\boldsymbol{\gamma}) + G(\mathbf{y},\omega)\right) \tag{11}$$

where $J(\mathbf{x},\boldsymbol{\gamma},\omega)$ is the total cost in one period; $H(\mathbf{x},\boldsymbol{\gamma})$ and $G(\mathbf{y},\omega)$ are the EC repositioning cost and the total EC holding and leasing cost in one period, respectively. We have

$$H(\mathbf{x},\boldsymbol{\gamma}) = H(\mathbf{Z}) = \sum_{p\in P}\sum_{m\in P(p)} C_{p,m}^R z_{p,m} \tag{12}$$

$$\begin{aligned} G(\mathbf{y},\omega) &= \sum_{p\in P} g(y_p,\eta_p^O) \\ &= \sum_{p\in P}\left(C_p^H (y_p-\eta_p^O)^+ + C_p^L(\eta_p^O - y_p)^+\right) \end{aligned} \tag{13}$$

where $g(y_p,\eta_p^O)$ represents the EC holding and leasing cost of port p in one period; $x^+ = max\,(0,x)$.

Next, we consider the problem under balanced scenario, followed by that under unbalanced scenario. Here, balanced (unbalanced) scenario is the scenario in which the total amount of estimated EC supply is equal (not equal) to the total amount of estimated EC demand in each period. From (3) and (4), it is observed that $\sum_{i\in P_t^S} a_{i,t}^S - \sum_{j\in P_t^D} a_{j,t}^D = N - \sum_{p\in P}\gamma_p$. Hence, a scenario with $\sum_{p\in P}\gamma_p = N$ is a balanced scenario and a scenario with $\sum_{p\in P}\gamma_p \neq N$ is an unbalanced scenario.

1) Balanced Scenario: Consider the problem in the balanced scenario. We can obtain the optimal solution of (10) analytically, since it only depends on the holding and leasing cost function. The explanations are as follows.

From the transportation models, we know that the repositioning out (in) requirement of each surplus (deficit) port can be fully satisfied in each period in a balanced scenario. Thus, after making ECR decisions, the inventory position level of each port can be always kept at its target threshold level. It implies that the estimated EC supply (demand) of the surplus (deficit) port in a period, except the initial period, will be independent from the parameters of the fleet size and the thresholds, and just depend on its customer demands in the previous period. Consequently, the EC transportation cost and the total EC holding and leasing cost in one period will be independent; and the expected EC transportation cost per period will be independent from parameters of N and $\boldsymbol{\gamma}$. Further speaking, the optimal solution only depends on the expected total EC holding and leasing cost function. The problem (10) in the balanced scenario can be simplified to an non-linear programming as:

$$\min_{\gamma}\ E\left(\sum_{p\in P}\left(C_p^H(\gamma_p-\eta_p^O)^+ + C_p^L(\eta_p^O-\gamma_p)^+\right)\right)$$

$$\text{s.t. } \sum_{p\in P}\gamma_p = N$$

where the value of fleet size N is given.

Considering the convexity of the above cost function and taking use of the K.K.T. conditions, we can obtain the optimal solution of the NLP by solving (14) and (15).

$$\left(C_p^H + C_p^L\right)\bullet F_p(\gamma_p) - C_p^L + \lambda_N = 0\ \ \forall p\in P \tag{14}$$

$$\sum_{p\in P}\gamma_p - N = 0 \tag{15}$$

where λ_N is the Lagrange Multiplier of the balance constraint.

<u>Remark:</u> Let $(\boldsymbol{\gamma}^*)^B$ be the optimal thresholds in the balanced scenario with given fleet size. Given customer demands, we know that $(\boldsymbol{\gamma}^*)^B$ can achieve the minimum holding and leasing cost for the problem (10). However, it may not achieve the minimum expected total cost per period for the problem, because we can find other thresholds achieving less transportation costs than that achieved by $(\boldsymbol{\gamma}^*)^B$. For example, when we set all the thresholds going to infinite so that no ECs will be repositioned, the transportation cost in this scenario will be zero and less than that in the scenario with $(\boldsymbol{\gamma}^*)^B$. Thus, we next consider the unbalanced scenario.

2) Unbalanced Scenario: Consider the problem in the unbalanced scenario. By taking advantage of the structure of the problem, we find an interesting property about the transportation cost as follows:

Property I: *In an unbalanced scenario, the transportation cost in a period, except the initial period, could be less than or equal to that in a balanced scenario with same customer demands.*

Intuitively, for example, if a exported-dominated port has the probability to become a surplus port, i.e., it needs to reposition out ECs to other ports, repositioning in less ECs than its threshold in this port in advance when it becomes a surplus port will reduce its EC repositioning out quantity. Hence, less transportation cost in a period in an unbalanced scenario could be occurred.

Since there is no closed-form formulation for the computation of expected total cost per period in the unbalanced scenario involving the repositioned EC quantities from the transportation models, we adopt the simulation to estimate $J(N,\boldsymbol{\gamma})$ given values of N and $\boldsymbol{\gamma}$ as shown in (16).

$$J(N,\boldsymbol{\gamma}) \approx \frac{1}{T}\sum_{t=1}^{T} J(\mathbf{x}_t,\boldsymbol{\gamma},\omega_t) = \frac{1}{T}\sum_{t=1}^{T}\left(H(\mathbf{x}_t,\boldsymbol{\gamma}) + G(\mathbf{y}_t,\omega_t)\right) \tag{16}$$

where $J(\mathbf{x}_t,\boldsymbol{\gamma},\omega_t)$ is the total cost in period t; $H(\mathbf{x}_t,\boldsymbol{\gamma})$ and $G(\mathbf{y}_t,\omega_t)$ are the EC repositioning cost and the total EC

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holding and leasing cost in period t and can be obtained from (12) and (13), respectively; T is the amount of the simulation periods. It is significant to highlight that solving (10) in the unbalanced scenario is difficult. In order to find an optimal solution to the problem, we need to use a search-based method. In next section, we develop an optimization technique namely IPA-based gradient technique.

Summarizing above discussions, we can get that given fleet size and customer demands, the minimum expected total cost per period could be achieved in either the balanced scenario or unbalanced scenario. The optimal solution under balanced scenario can be obtained analytically by solving (14) and (15), and under unbalanced scenario by applying the proposed IPA-based gradient technique.

III. IPA-BASED GRADIENT TECHNIQUE

IPA is able to estimate the gradient of the objective function from one single simulation run, thus reducing the computational time. Moreover, it has been shown that variance of IPA estimator is lower, compared with many other gradient estimators [15]. Thus, we propose an IPA-based gradient technique to search the optimal solution in the unbalanced scenario. The overall IPA-based gradient technique is briefly described in Fig. 1.

As shown in Fig. 1, given the parameters of the fleet size N and the policy $\boldsymbol{\gamma}$ with $\sum_{p\in P}\gamma_p \neq N$, we first calculate the total cost and estimate the gradient of total cost with respect to the thresholds in all periods. We estimate the gradient in a period using the concept of perturbation propagation from IPA [16], the dual information of the LP model and the chain rule. Then, we can obtain the expected total cost per period and the gradient of $J(N,\boldsymbol{\gamma})$. This gradient can provide a direction for finding new parameters of the policy that may have a lower expected total cost per period and hence the hill climbing algorithm is used to update the parameters of the policy. Finally, when the termination criteria are satisfied, the simulation is stopped.

To estimate the gradient of expected total cost per period, we take a partial derivation of (16) with respect to the threshold of port i. With the help of (13), we can obtain

$$\begin{aligned}\frac{\partial J(N,\boldsymbol{\gamma})}{\partial\gamma_i} &\approx \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\partial J(\mathbf{x}_t,\boldsymbol{\gamma},\omega_t)}{\partial\gamma_i}\right)\\ &=\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\partial H(\mathbf{Z}_t)}{\partial\gamma_i}+\sum_{p\in P}\frac{\partial E\left(g(y_{p,t},\eta_{p,t}^{O})\right)}{\partial y_{p,t}}\bullet\frac{\partial y_{p,t}}{\partial\gamma_i}\right)\end{aligned} \tag{17}$$

where for the inventory holding and leasing cost function, we use the expected holding and leasing cost function to estimate the gradient instead of using the sample path since we are able to get the explicit function to evaluate the average gradient; $\partial H(\mathbf{Z}_t)/\partial\gamma_i$ measures the impact of the transportation cost in period t when the threshold is changed; $\partial g(y_{p,t},\eta_{p,t}^{O})/\partial y_{p,t}$ measures the impact of the holding and leasing cost function of port p in period t when the inventory position level is changed; $\partial y_{p,t}/\partial\gamma_i$ measures the impact of the inventory position level of port p in period t when the threshold is changed.

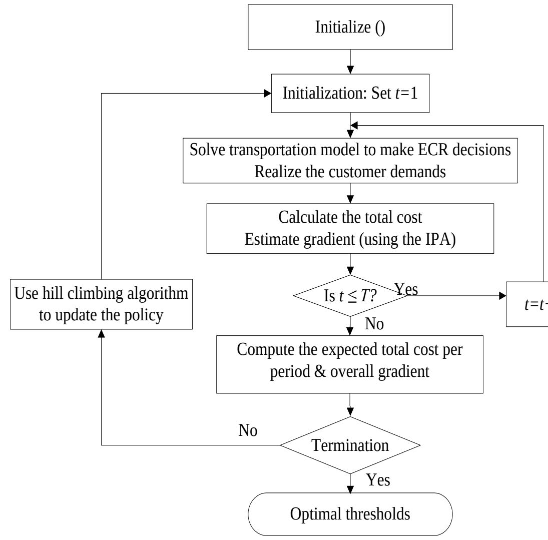

Figure 1. The flow of the IPA-based gradient technique

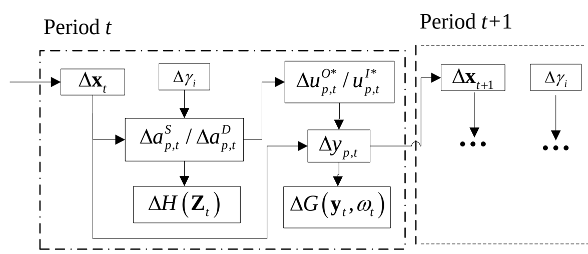

Figure 2. The Perturbation Flow

We define the nominal path as the sample path generated by the simulation model with parameter $\boldsymbol{\gamma}$ and the perturbed path as the sample path generated using the same model and same random seeds, but with parameter $(\boldsymbol{\gamma})'$, where $(\boldsymbol{\gamma})' = \boldsymbol{\gamma}+\Delta\boldsymbol{\gamma}$. Without loss of generality, we only perturb the threshold of port i and keep the thresholds of the other ports unchanged, i.e., $(\gamma_i)' = \gamma_i + \Delta\gamma_i$ and $(\gamma_j)' = \gamma_j$ for other port j, where the value of $\Delta\gamma_i$ is infinitesimally small. By "sufficiently" small, we mean such that the surplus port subset and deficit port subset are same in the both nominal and perturbed paths in every period. Oftentimes, we will present the changes in various quantities by displaying with argument Δ. We perturb γ_i in all periods and the representative perturbation flow in period t is shown in the Fig. 2.

In our problem with real variables, the probability of having balanced port is close to 0. In other word, port i should be either surplus port or deficit port in period t. From (3) and (4), we can derive that $\Delta a_{p,t}^{S} = \Delta x_{p,t} - \Delta\gamma_p\ \forall p \in P_t^S$ and $\Delta a_{p,t}^{D} = \Delta\gamma_p - \Delta x_{p,t}\ \forall p\in P_t^D$. Hence, in Fig. 2, the perturbation of $\Delta\mathbf{x}_t$ will work together with the perturbation of $\Delta\gamma_i$ to affect the estimated EC supply/demand, namely $\Delta a_{p,t}^{S}/\Delta a_{p,t}^{D}$ for some ports. We know that the estimated EC supply/demand of a port is the RHS of the corresponding port's constraint in the transportation model. It implies that the perturbations of

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$\Delta a_{p,t}^{S}/\Delta a_{p,t}^{D}$ of some ports could affect the optimal repositioning quantities of some ports, which, of course, will affect the total optimal repositioned out/in quantities of some ports, namely $\Delta u_{p,t}^{O*}/\Delta u_{p,t}^{I*}$. From (1), we know that $\Delta y_{p,t} = \Delta x_{p,t} - u_{p,t}^{O} + u_{p,t}^{i}\ \forall p \in P$. The perturbation of $\Delta \mathbf{x}_t$ will work together with the perturbation of $\Delta u_{p,t}^{O*}/\Delta u_{p,t}^{I*}$ of some ports to affect the perturbation on EC inventory positions, namely $\Delta y_{p,t}$ for some ports. Furthermore, the perturbation of $\Delta a_{p,t}^{S}/\Delta a_{p,t}^{D}$ will affect the transportation cost and the perturbation of $\Delta y_{p,t}$ will affect the total holding and leasing cost. From (2), we have $\Delta \mathbf{x}_t = \Delta \mathbf{y}_t$, which implies that the perturbation on the inventory position will be fully propagated to the beginning on-hand inventory of next period.

The flowing notations are introduced.

- $Q_{p,t}$ the set of ports whose beginning on-hand inventory in period t are affected by perturbing threshold of port p, $Q_{p,t} \subset P$
- $E_{p,t}$ the set of ports whose total optimal repositioned quantities are changed by perturbing the estimated EC supply/demand of port p in period t, $E_{p,t} \subset P$
- $\pi_{p,t}$ the corresponding dual variable for port p constraint in the transportation model in period t
- $I\{\text{condition}\}$ a indicator function, which takes 1 if the condition is true and otherwise 0

Tracing the perturbations by following the flow in Fig. 2, we can obtain that in period t, $E_{i,t}$ will be either empty or consist of a pair of ports, i.e., $E_{i,t} = \emptyset$ or $E_{i,t} = (i,\ e_{i,t})$. Similarly, $Q_{i,t}$ will be either empty or consist of a pair of ports, i.e., $Q_{i,t} = \emptyset$ or $Q_{i,t} = (i,\ q_{i,t})$. We can obtain the gradient of expected total cost per period with respect to γ_i in (17) can be approximated by (18). In (18), the first term of the RHS presents the perturbation on the transportation cost when $Q_{i,t} = \emptyset$; the second term of the RHS presents the perturbation on the transportation cost when $Q_{i,t} \neq \emptyset$; the third term of the RHS presents the perturbation on the holding and leasing cost when $Q_{i,t} = \emptyset$ and $E_{i,t} \neq \emptyset$; the forth term of the RHS presents the perturbation on the holding and leasing cost when $Q_{i,t} \neq \emptyset$ and $E_{i,t} = \emptyset$; the fifth term of the RHS presents the perturbation on the holding and leasing cost when $Q_{i,t} \neq \emptyset$, $E_{i,t} \neq \emptyset$ and $e_{q_{i,t},t} \neq i$; $E_{i,t}$ can be obtained by applying a proposed modified stepping stone approach with perturbing the estimated supply/demand of port i in period t. We know that $Q_{i,1} = \emptyset$ in the initial period; and for $t > 1$, $Q_{i,t}$ can be obtained as follows: (a) $Q_{i,t} = E_{i,t}$, when $Q_{i,t-1} = \emptyset$; (b) $Q_{i,t} = Q_{i,t-1}$, when $Q_{i,t-1} \neq \emptyset$ and $E_{q_{i,t-1},t} = \emptyset$; (c) $Q_{i,t} = \emptyset$, when $Q_{i,t-1} \neq \emptyset$, $E_{q_{i,t-1},t} \neq \emptyset$ and $e_{q_{i,t-1},t} = i$; (d) $Q_{i,t} = \left(i, e_{q_{i,t-1},t}\right)$, when $Q_{i,t-1} \neq \emptyset$, $E_{q_{i,t-1},t} \neq \emptyset$ and $e_{q_{i,t-1},t} \neq i$.

$$\frac{\partial J(N,\boldsymbol{\gamma})}{\partial \gamma_i} \approx \frac{1}{T}\sum_{t=1}^{T}\frac{\partial J(\mathbf{x}_t,\boldsymbol{\gamma},\omega_t)}{\partial \gamma_i}$$
$$= \frac{1}{T}\sum_{t=1}^{T}\left(\begin{array}{l} I(Q_{i,t}=\emptyset)\bullet(-1)^{I(i\in P_t^S)}\bullet\pi_{i,t} + I(Q_{i,t}\neq\emptyset)\bullet(-1)^{I(q_{i,t}\in P_t^S)}\bullet\pi_{q_{i,t},t} \\ +I\begin{pmatrix} Q_{i,t}=\emptyset, \\ E_{i,t}\neq\emptyset \end{pmatrix}\bullet\left(\dfrac{\partial E\left(g\left(y_{i,t},\eta_{i,t}^{O}\right)\right)}{\partial y_{i,t}} - \dfrac{\partial E\left(g\left(y_{e_{i,t},t},\eta_{e_{i,t},t}^{O}\right)\right)}{\partial y_{e_{i,t},t}}\right) \\ +I\begin{pmatrix} Q_{i,t}\neq\emptyset, \\ E_{q_{i,t},t}=\emptyset \end{pmatrix}\bullet\left(\dfrac{\partial E\left(g\left(y_{i,t},\eta_{i,t}^{O}\right)\right)}{\partial y_{i,t}} - \dfrac{\partial E\left(g\left(y_{q_{i,t},t},\eta_{q_{i,t},t}^{O}\right)\right)}{\partial y_{q_{i,t},t}}\right) \\ +I\begin{pmatrix} Q_{i,t}\neq\emptyset, \\ E_{q_{i,t},t}\neq\emptyset, \\ e_{q_{i,t},t}\neq i \end{pmatrix}\bullet\left(\dfrac{\partial E\left(g\left(y_{i,t},\eta_{i,t}^{O}\right)\right)}{\partial y_{i,t}} - \dfrac{\partial E\left(g\left(y_{e_{q_{i,t},t},t},\eta_{q_{i,t},t}^{O}\right)\right)}{\partial y_{e_{q_{i,t},t},t}}\right) \end{array}\right) \tag{18}$$

where the value of $\partial E(.)/\partial(.)$ in (18) is calculated by

$$\frac{\partial E\left(g(y_{p,t},\eta_{p,t}^{O})\right)}{\partial y_{p,t}} = \begin{cases} -C_p^L & \text{if } y_{p,t} < 0 \\ \left(C_p^H + C_p^L\right)\bullet F_p\left(y_{p,t}\right) - C_p^L & \text{if } 0 \le y_{p,t} \end{cases} \tag{19}$$

IV. NUMERICAL RESULTS

In this section we aim to evaluate the performance of the proposed single-level threshold policy (STP). For comparison, a match back policy (MBP) is introduced. Such policy is widely accepted and applied in practice and its basic principle is to match the containers back to the original port. Mathematically,

$$z_{p,m,t+1} = \left(\varepsilon_{m,p,t} - \varepsilon_{p,m,t}\right)^{+} \tag{20}$$

The NLP in the balanced scenario is solved by Matlab (version 7.0.1). The IPA-gradient based algorithm is coded in Visual C++ 5.0. All the numerical studies are tested on and Intel Duo Processor E6750 2.67GHz CPU with 4.00 GB RAM under the Microsoft Vista Operation System. We set the simulation period $T = 10{,}100$ with warm-up period T_0 =100. For the STP, the termination criteria are that the maximum iteration for finding the optimal thresholds, namely r_{max} is achieve, or the expected total cost in the iteration r is larger than that in the previous iteration. We set $r_{max} = 1{,}000$. For the MBP, since the transportation cost is independent from the parameter of fleet size, we set the inventory position in the initial period be equal to the optimal inventory position which minimizing the expected holding and leasing cost.

For a three-port system, we compare the performance of both policies based on the expected total cost per period. we give the fleet size from 483 TEUs to 1128 TEUs to investigate the effect of the fleet size on the expected total cost. Fig. 3 shows the results. It is observed that STP outperforms MBP for all cases. The expected total cost per period savings achieved by STP over MBP are of the order of 12.75%~37.18%. One possible explanation is that STP makes the ECR decisions in terms of minimizing the transportation cost. Hence, it is important for operators to use intelligent method in repositioning ECs, instead of resorting to simple way such as the MBP.

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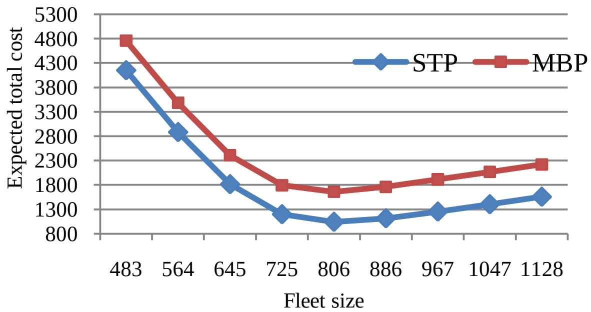

Figure 3. Expected total cost per period comparison for three-port system

From Fig. 3, it reveals that the optimal average total cost appears to be convex with respect to the fleet size for each system. It reflects the intuition that the optimal fleet size is the trade-off between the transportation cost and the holding and leasing cost.

V. Conclusion And Future Work

In this paper, the EC repositioning problem in a multi-port system is considered. A single-level inventory-based policy with the repositioning rule in terms of minimizing transportation cost is developed to reposition ECs periodically by taking into account demand uncertainty and dynamic operations. Two approaches, non-linear programming and IPA-based gradient technique are developed to solve the problem optimizing thresholds of policy under balanced and unbalanced scenarios, respectively. The numerical results provide insights that by repositioning the ECs intelligently, we can significantly reduce the total operation cost.

The main contributions of the study are as follows: (a) a single-level threshold policy with a repositioning rule in terms of minimizing transportation cost is developed for repositioning ECs in a multi-port system. To the best of our knowledge, few works consider the repositioning rule which is related to the transportation costs; (b) by developing the method to solve the difficult ECR problem, i.e., using IPA to estimate the gradient, it is innovative and provides a potential methodology contribution in this field

We strongly assumed that the ECs are dispatched between each pair of ports in one period. It may not be the right one period in some general cases. Further research is needed to relax the one-period assumption and consider the problem with different time dimension for the repositioning time. The main challenge is to track the perturbations along the sample path.

References

[1] U. Nations, Regional Shipping and Port Development (Container Traffic Forecast). 2007.

[2] D. P. Song, "Optimal threshold control of empty vehicle redistribution in two depot service systems," IEEE Transactions on Automatic Control, vol. 50(1), Jan. 2005, pp. 87-90, doi: 10.1109/TAC.2004.841134(410) 50.

[3] P. Dejax and T. Crainic, "Survey Paper--A Review of Empty Flows and Fleet Management Models in Freight Transportation," Transportation Science, vol. 21(4), Nov. 1987, pp. 227-248, doi: 10.1287/trsc.21.4.227.

[4] T. Crainic, M. Gendreau, and P. Dejax, "Dynamic and stochastic models for the allocation of empty containers," Operations Research, vol. 41(1), Jan. - Feb. 1993, pp. 102-126.

[5] W. Shen and C. Khoong, "A DSS for empty container distribution planning," Decision Support Systems, vol. 15(1), Sep. 1995, pp. 75-82, doi: 10.1016/0167-9236(94)00037-S.

[6] R. Cheung and C. Chen, "A two-stage stochastic network model and solution methods for the dynamic empty container allocation problem," Transportation Science, vol. 32(2), May 1998, pp. 142-162, doi: 10.1287/trsc.32.2.142.

[7] S. W. Lam, L. H. Lee, and L. C. Tang, "An approximate dynamic programming approach for the empty container allocation problem," Transportation Research Part C, vol. 15(4), Aug. 2007, pp. 265-277, doi: 10.1016/j.trc.2007.04.005.

[8] S. Choong, M. Cole, and E. Kutanoglu, "Empty container management for intermodal transportation networks," Transportation Research Part E, vol. 38(6), Nov. 2002, pp. 423-438, doi: 10.1016/S1366-5545(02)00018-2.

[9] J. X. Dong and D. P. Song, "Container fleet sizing and empty repositioning in liner shipping systems," Transportation Research Part E, vol. 45(6), Nov. 2009, pp. 860-877, doi: 10.1016/j.tre.2009.05.001.

[10] D. P. Song, "Characterizing optimal empty container reposition policy in periodic-review shuttle service systems," Journal of the Operational Research Society, vol. 58(1), 2007, pp. 122-133, doi: 10.1057/palgrave.jors.2602150.

[11] J. A. Li, K. Liu, S. C. H. Leung, and K. K. Lai "Empty container management in a port with long-run average criterion," Mathematical and Computer Modeling, vol. 40(1-2), July 2004, pp. 85-100, doi: 10.1016/j.mcm.2003.12.005.

[12] D. P. Song and J. Carter, "Optimal empty vehicle redistribution for hub-and-spoke transportation systems," Naval Research Logistics, vol. 55(2), Mar. 2008, pp. 156-171, doi: 10.1002/nav.20274.

[13] D. P. Song and J. X. Dong, "Empty Container Management in Cyclic Shipping Routes," Maritime Economics & Logistics, vol. 10(4), Dec. 2008, pp. 335-361, doi: 10.1057/mel.2008.11.

[14] J. A. Li, S. C. H. Leung, Y. Wu, and K. Liu, "Allocation of empty containers between multi-ports," European Journal of Operational Research, vol. 182(1), Oct. 2007, pp. 400-412, doi: 10.1016/j.ejor.2006.09.003.

[15] R. Suri, "Perturbation analysis: the state of the art and research issuesexplained via the GI/G/1 queue", Proceedings of the IEEE, vol. 77(1), Jan. 1989, pp. 114-137, doi: 10.1109/5.21075.

[16] Y.C.Ho and X. R. Cao, Discrete Event Dynamic Systems and Perturbation Analysis. Boston,UK, Kluwer Academic Publishers, 1991.

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