Some Performance of Three-hop Wireless Relay Channels in the Presence of Rician Fading

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***Abstract*—Three-hop wireless relay channels in the presence of Rician fading will be examined in this article. This system model is generated by the product of three independent, but not necessarily identically distributed, Rician random variables (RVs). Some important performance of this system, such as cumulative distribution function (CDF), outage probability (Pout) and average fade duration (AFD) of wireless relay communication system working over Rician multipath fading environment will be calculated and graphically presented. The fading parameters' impact will be analyzed based on the obtained graphs.**

Keywords- average fade duration (AFD); outage probability (Pout); random variables; Rician fading; three-hop relaying system.

I. INTRODUCTION

In mobile channels in the presence of multipath fading, properties of communications systems are disturbed significantly due to the signal envelope fluctuations [1][2]. It is of vital importance to characterize those random variations in terms of the fading characteristics and derive both first and second order [3][4]. The first order performance we will calculate here is outage probability (Pout).

The closed-form expressions for Pout, channel capacity (CC), and average symbol error probability (ASEP) are derived in [5] for amplify-and-forward (AF) multi-hop relay network in the presence of Rayleigh fading. Based on approximation of multi-hop relay by dual-hop relay systems, the analytical expressions are obtained for some scenarios.

A three-hop communication system, as we analyze, is illustrated in Fig. 1 [6]. It consists of the source node, denoted by (S), sending the information signal to the destination (D) with the help of two consecutive relays, namely R1 and R2. The AF relay nodes are assumed to be untrusted and hence, they can overhear the transmitted information signal while relaying.

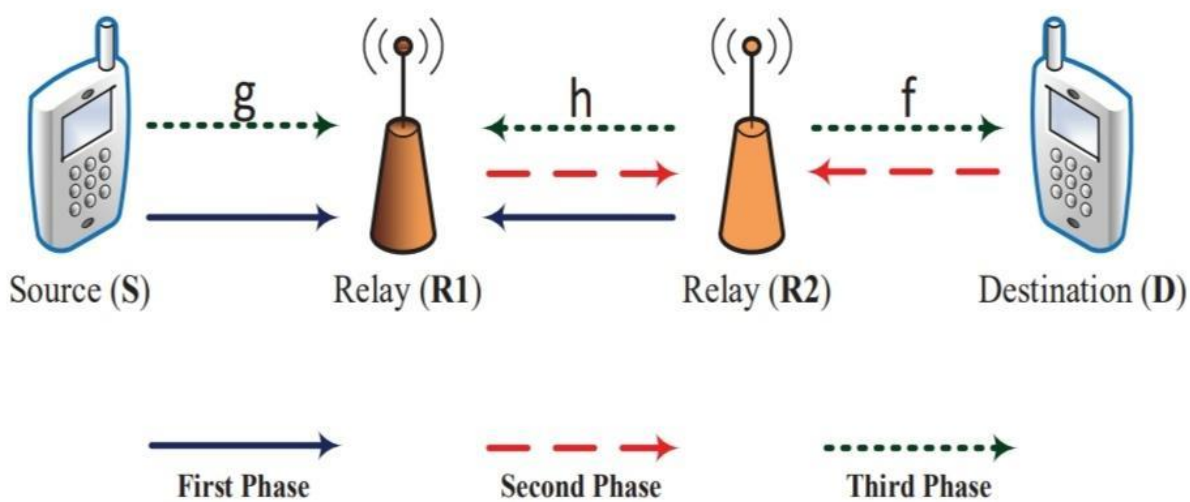

Figure 1. System model of a three-hop wireless relay [6].

All nodes are equipped with a single antenna operating in half-duplex mode. The consecutive relays are necessary helpers to deliver the information signal to the destination. This assumption is valid when the network nodes experience a heavy shadowing, or when the distance between terminals is large, or when the nodes suffer from limited power resources [6].

Three-hop system is analyzed in [7]. For three-hop relay system we will obtain the second order characteristics. The knowledge of second-order statistics of multipath fading channels (level crossing rate (LCR) and average fade duration (AFD)) can help us better understand and mitigate the effects of fading. For example, the AFD determines the average length of error bursts in fading channels [8]. So, in fading channels with relatively large AFD, long data blocks will be significantly affected by the channel fades than short blocks [9].

It is necessary to know this fact for choosing the frame length for coded packetized systems, designing interleaved or non-interleaved concatenated coding methods [10], optimizing the interleaver size, choosing the buffer depth for adaptive modulation schemes [11] [12], throughput (efficiency) estimation of communication protocols, ... Empirically-verified formulas for the LCR and AFD of common multipath fading models are necessary for all observed applications.

The output signal from multi-hop relay system is product of random variables (RVs) at hops outputs. In [13], multi-hop system in the presence of Nakagami fading is analyzed through N*Nakagami distribution as suitable for modeling of realistic wireless fading channels. Statistical analysis of cascaded Rician fading channels is given in [14]. Different performance is derived for both fading channels in terms of the Meijer G-function.

We observed different products of RVs and derived a number of system performance for dual and three-hop relay systems in closed forms in [15]-[18]. Wireless relay system with two sections (dual-hop) in κ-μ short term fading channel is observed in [15]. In [16], an analytical approach for evaluating performance of dual-hop cooperative link over shadowed Ricean fading channels is presented. All performance of product of three Rayleigh RVs are presented in [17] and the statistics of product of three Nakagami-*m* RVs is given in [18].

Here, we consider a three-hop relay channels as a special case of multi-hop relay network in the presence of Rician fading. This case is important for the channels where an optical line of sight is present. We opine that there are not enough reported works in this area.

This paper is organized through four sections. In introduction, the basic points are given. In the second section, the first order characteristics of the product of three Rician RVs are introduced. In the third section, the second order characteristics of the product of three Rician RVs are presented and graphs for all performance are plotted. The acknowledgement and conclusions close the article.

II. The First Order Performance of Product of Three Rician Random Variables

For description of three-hop wireless relay system it is necessary to derive the first-order characteristics of the product of three Rician RVs.

A. PDF of Product of Three Rician RVs

Rician fading is a stochastic model for radio propagation where the signal arrives at the receiver by several different paths when one of the paths, typically a line of sight signal or some strong reflection signals, is much stronger than the others. In Rician fading, the amplitude gain is characterized by a Rician distribution. It was named after Stephen O. Rice [19].

Rician RVs x_i have Rician distribution [19]:

$$p_{x_i}\left(x_i\right)=\frac{2\left(\kappa_i+1\right)}{\Omega_{\mathrm{i}}\mathrm{e}^{\kappa_i}}\sum_{j_i=0}^{\infty}\left(\frac{\left(\kappa_i+1\right)\kappa_i}{\Omega_i}\right)^{j_i}\frac{1}{\left(j_i!\right)^2}\cdot$$

$$\cdot\, x_i^{2j_i+1}e^{-\frac{\kappa_i+1}{\Omega_{\mathrm{i}}}x_i^2},\; x_i\geq 0, \quad (1)$$

where Ω_i are mean powers of RVs x_i, and κ_i are Rician factors. Rician factor is defined as a ratio of signal power of dominant component and power of scattered components. It can have values from [0, ∞].

A random variable x is product of three Rician RVs [17 eq. (2)]:

$$x=\prod_{i=1}^{3}x_i\;, \quad (2)$$

which implies: $x_1=x/\,x_2\,x_3$.

Probability density function of product of three Rician RVs x is [20, eq. (7)]:

$$p_x\left(x\right)=\frac{2\left(\kappa_1+1\right)}{\Omega_1\mathrm{e}^{\kappa_1}}\sum_{j_1=0}^{\infty}\left(\frac{\left(\kappa_1+1\right)\kappa_1}{\Omega_1}\right)^{j_1}\frac{1}{\left(j_1!\right)^2}\cdot$$

$$\cdot\frac{2\left(\kappa_2+1\right)}{\Omega_2\mathrm{e}^{\kappa_2}}\sum_{j_2=0}^{\infty}\left(\frac{\left(\kappa_2+1\right)\kappa_2}{\Omega_2}\right)^{j_2}\frac{1}{\left(j_2!\right)^2}\cdot$$

$$\cdot\frac{2\left(\kappa_3+1\right)}{\Omega_3\mathrm{e}^{\kappa_3}}\sum_{j_1=0}^{\infty}\left(\frac{\left(\kappa_3+1\right)\kappa_3}{\Omega_3}\right)^{j_1}\frac{1}{\left(j_3!\right)^2}\cdot$$

$$\int_0^{\infty}dx_2\int_0^{\infty}dx_3\,x_2^{-1-2j_1+2j_2}\,x_3^{-1-2j_1+2j_3}$$

$$\cdot\, x^{2j_1+1}e^{-\frac{\kappa_1+1}{\Omega_1}\left(\frac{x}{x_2x_3}\right)^2-\frac{\kappa_2+1}{\Omega_2}x_2^2-\frac{\kappa_3+1}{\Omega_3}x_3^2}. \quad (3)$$

B. CDF of Product of Three Rician RVs

Cumulative distribution function (CDF) of product of three Rician RVs is:

$$F_x(x)=\int_0^\infty dt p_x(t)=$$

$$=\frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}}\sum_{j_1=0}^{\infty}\left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1}\right)^{j_1}\frac{1}{(j_1!)^2}\cdot$$

$$\cdot\frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}}\sum_{j_2=0}^{\infty}\left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2}\right)^{j_2}\frac{1}{(j_2!)^2}\cdot$$

$$\cdot\frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}}\sum_{j_1=0}^{\infty}\left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3}\right)^{j_1}\frac{1}{(j_3!)^2}\cdot$$

$$\int_0^\infty dx_2\int_0^\infty dx_3 x_2^{-1-2j_1+2j_2}x_3^{-1-2j_1+2j_3}e^{-\frac{\kappa_2+1}{\Omega_2}x_2^2-\frac{\kappa_3+1}{\Omega_3}x_3^2}$$

$$\int_0^x dt t^{2j_1+1}e^{-\frac{\kappa_1+1}{\Omega_1}\frac{t^2}{x_2^2x_3^2}}=$$

$$=\frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}}\sum_{j_1=0}^{\infty}\left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1}\right)^{j_1}\frac{1}{(j_1!)^2}\cdot$$

$$\cdot\frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}}\sum_{j_2=0}^{\infty}\left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2}\right)^{j_2}\frac{1}{(j_2!)^2}\cdot$$

$$\cdot\frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}}\sum_{j_1=0}^{\infty}\left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3}\right)^{j_1}\frac{1}{(j_3!)^2}\cdot$$

$$\int_0^\infty dx_2\int_0^\infty dx_3 x_2^{-1-2j_1+2j_2+2j_1+2}x_3^{-1-2j_1+2j_3+2j_1+2}$$

$$\cdot e^{-\frac{\kappa_2+1}{\Omega_2}x_2^2-\frac{\kappa_3+1}{\Omega_3}x_3^2}$$

$$\cdot\frac{1}{2}\left(\frac{\Omega_1}{\kappa_1+1}\right)^{j_1+1}\gamma\left(j_1+1,\frac{\kappa_1+1}{\Omega_1}\frac{x^2}{x_2^2x_3^2}\right). \tag{4}$$

Rayleigh fading is a model for stochastic fading when there is no line of sight signal. Because of that it is considered as a special case of the more generalized concept of Rician fading. Rayleigh fading is obtained for Rician factor κ=0. For this reason, derived expressions for CDF of product of three Rician RVs can be used for evaluation a CDF of product of three Rayleigh RVs, also for CDF of product of two Rayleigh RVs and Rician RV, and CDF of product of two Rician RVs and Rayleigh RV. The obtained results can be used in performance analysis of wireless relay communication radio system with three sections in the presence of multipath fading. This means that derived CDFs are used for the next cases: 1) when Rician fading is present in all three sections ($\kappa_i \neq 0$, i=1,2,3), then 2) when Rayleigh fading is present in all three sections ($\kappa_1=\kappa_2=\kappa_3=0$), the next 3) when Rayleigh fading is present in two sections and Rician in one ($\kappa_1=\kappa_2=0$, $\kappa_3 \neq 0$), and 4) when Rayleigh fading is present in one and Rician fading in two sections ($\kappa_1=0$, $\kappa_2 \neq 0$, $\kappa_3 \neq 0$). A case with κ →∞ present the scenario without fading.

C. Outage probability of Product of Three Rician RVs

The outage probability is an important performance measure of communication links operating over fading channels. Outage probability is defined as the probability that information rate is less than the required threshold information rate Γ_{th}. Pout is the probability that an outage will occur within a specified time period:

$$P_{out}=\int_0^{\Gamma_{th}} p_x(t)dt, \tag{5}$$

where $p_x(x)$ is the PDF of the signal and Γ_{th} is the system protection ratio depending on the type of modulation employed and the receiver characteristics [21].

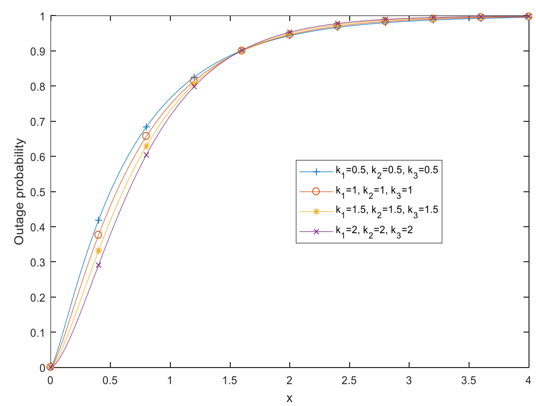

Figure 2. Outage probability of product of three Rician RVs versus signal envelope x for different values of Rician factor κ_I and signal power Ω=1.

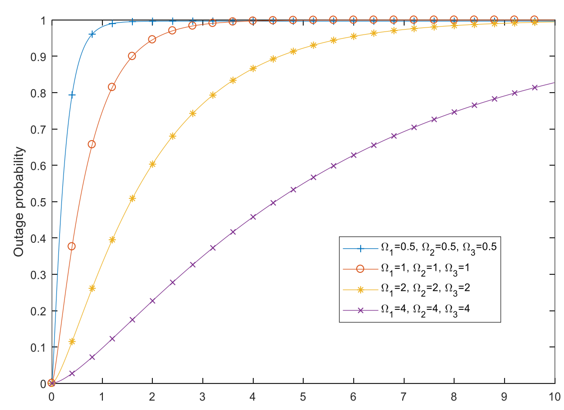

Figure 3. Outage probability of product of three Rician RVs depending on signal envelope for different values of signal power Ω_i and Rician factor κ=1.

Using (4), Pout can be expressed as:

$$P_{out} = F_x\left(\Gamma_{th}\right). \tag{6}$$

Plots of the outage probability, for different values of parameters, are shown in Figs. 2 and 3. The choice of parameters is intended to illustrate the broad range of shapes that the curves of the resulting distribution can exhibit. It is evident that performance is improved with an increase in Rician factors κ_i. Also, higher values of fading powers Ω_i tend to reduce the outage probability and improve system performance, as it is expected.

III. The Second Order Performance of the Product of Three Rician Random Variables

Level crossing rate (LCR) and average fade duration (AFD) of the signal envelope are two important second-order statistics of wireless channel. They give useful information about the dynamic temporal behavior of multipath wireless fading channels.

A. LCR of Product of Three Rician RVs

Level crossing rate is one of the most important second-order performance measures of wireless communication system, which has already found application in modelling and design of communication system but also in the design of error correcting codes, optimization of interleave size and throughput analysis.

The envelope LCR is defined as the expected rate (in crossings per second) at which a fading signal envelope crosses the given level in the downward direction [4]. The LCR of RV tells how often the envelope crosses a certain threshold x [22]. We should determine the joint probability density function (JPDF) between x and $\dot{x}$, $p_{x\dot{x}}\left(x\dot{x}\right)$ first, then apply the Rice's formula [19, Eq. (2.106)] to finally calculate the LCR. LCR is defined as [2]:

$$N_x = \int_0^\infty d\dot{x}\,\dot{x}\,p_{x\dot{x}}\left(x\dot{x}\right). \tag{7}$$

LCR of product of three Rician RVs is derived in [23, eq. (20)]:

$$N_x = \frac{1}{\sqrt{2\pi}}\pi f_m \frac{\Omega_1^{1/2}}{\left(\kappa_1+1\right)^{1/2}} \cdot \frac{2\left(\kappa_1+1\right)}{\Omega_1} \cdot \frac{2\left(\kappa_2+1\right)}{\Omega_2} \cdot \frac{2\left(\kappa_3+1\right)}{\Omega_3} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty} \left(\frac{\kappa_1\left(\kappa_1+1\right)}{\Omega_1}\right)^{i_1} \frac{1}{\left(i_1!\right)^2} \left(\frac{\kappa_2\left(\kappa_2+1\right)}{\Omega_2}\right)^{i_2} \frac{1}{\left(i_2!\right)^2}$$

$$\cdot \left(\frac{\kappa_3\left(\kappa_3+1\right)}{\Omega_3}\right)^{i_3} \frac{1}{\left(i_3!\right)^2} x^{2i_1+1}$$

$$\cdot \int_0^\infty dx_2 \int_0^\infty dx_3 \left(1 + \frac{x^2}{x_2^4 x_3^2}\frac{\Omega_2}{\kappa_2+1}\frac{\kappa_1+1}{\Omega_1} + \frac{x^2}{x_2^2 x_3^4}\frac{\Omega_3}{\kappa_3+1}\frac{\kappa_1+1}{\Omega_1}\right)^{1/2} \cdot$$

$$\cdot x_2^{-2i_1-1+2i_2+1} x_3^{-2i_1-1+2i_3+1} e^{-\frac{\kappa_1+1}{\Omega_1}\frac{x^2}{x_2^2 x_3^2} - \frac{\kappa_2+1}{\Omega_2}x_2^2 - \frac{\kappa_3+1}{\Omega_3}x_3^2} \tag{8}$$

Last integral can be solved by using Laplace approximation theorem for solution the two-fold integrals solved in [24] through equations (22)-(29):

$$\int_0^\infty dx_2 \int_0^\infty dx_3\, g\left(x_2, x_3\right) e^{\lambda f\left(x_2, x_3\right)} =$$

$$= \frac{\pi}{\lambda} g\left(x_{20}, x_{30}\right) e^{\lambda f\left(x_{20}, x_{30}\right)} \frac{1}{\left(B\left(x_{20}, x_{30}\right)\right)^{1/2}}. \tag{9}$$

We give in this subsection some new graphs for normalized LCR of product of three Rician RVs depending on this product x with Rician factor κ_i and average power Ω_i as parameters of curves in Figs. 4 and 5.

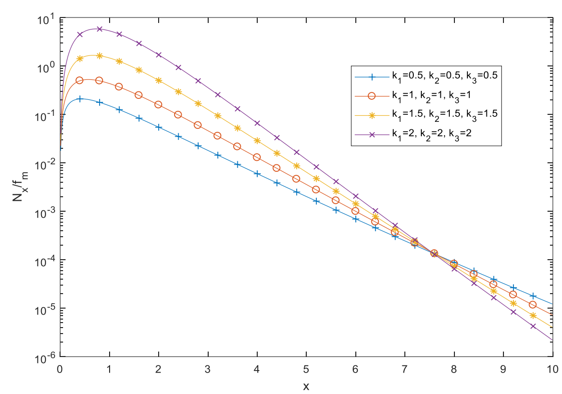

Figure 4. LCR normalized by f_m depending on signal envelope x for various values of Rician factor κ_i and signal power Ω=1.

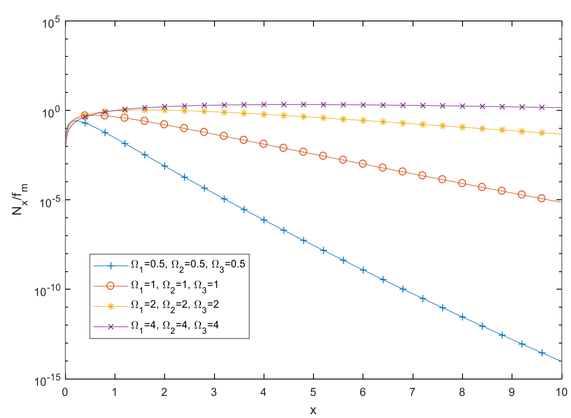

Figure 5. LCR normalized by f_m versus signal envelope x for various values of signal powers Ω_i.

LCR grows as Rician signal power increases. The impact of signal envelope power on the LCR is higher for bigger values of Rician factor κ_i. LCR increases with increasing of Ω_i for all values of signal envelope. The impact of signal envelope on the LCR is larger for higher values of the signal envelope when Ω_i changes. It is important bring to mind that system has better performance for lower values of the LCR.

B. AFD of Product of Three Rician RVs

Average fade duration measures how long a signal's envelope or power stays below a given target threshold derived from the LCR [4]. According to that, AFD is [25, eq. (9)]:

$$T_x(x) = \frac{P(x \le X)}{N_x(x)} = \frac{\int_0^X p_x(x)dx}{N_x(x)}. \qquad (10)$$

The numerator is the cumulative distribution function of x from (4), and $N_x(x)$ is LCR given in (8) [26].

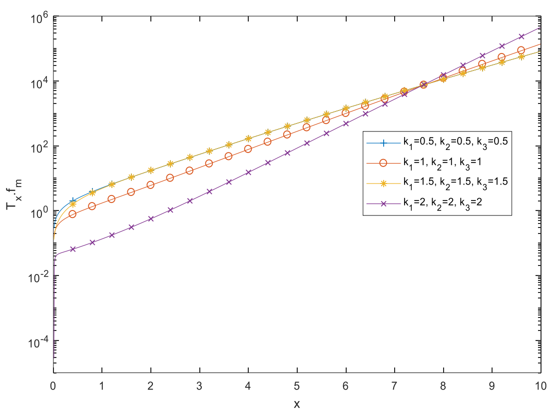

Figure 6. AFD normalized by f_m versus signal envelope x for different values of Rician factor κ_i and signal powers Ω_i=1.

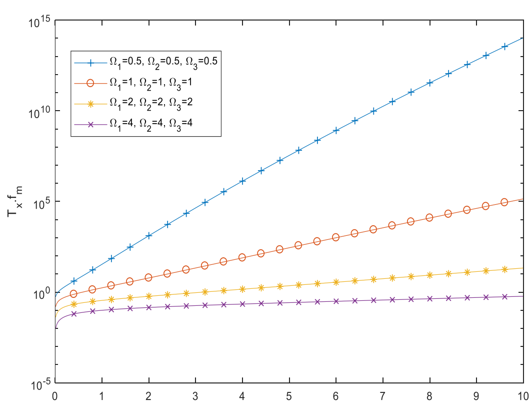

Figure 7. AFD normalized by f_m depending on signal envelope x for κ=1 and different values of signal powers Ω_i.

The normalized AFD ($T_x f_m$) of product of three Rician RVs is plotted in Figs. 6 and 7 versus signal envelope x. One can see that for higher values of κ_i and lower x, AFD has smaller values. Also, it is visible from Fig. 7 that AFD increases for all signal envelopes and lower Ω_i. The impact of Ω_i is bigger at higher envelopes.

IV. CONCLUSION

Due to transmit power limitations, the multi-hop communication in relay systems is introduced for improving the quality of transmission in cellular and ad hoc networks. These benefits of multi-hop relays are especially visible in rural areas with small population and low level of traffic density.

In this work, we presented previously determined formulas for the PDF and LCR and derived important expressions for CDF, Pout and AFD of the three-hop wireless relay system in the presence of Rician fading. This system output signal is the product of three Rician RVs.

Outage probability is defined as the point at which the receiver power value falls below the threshold (where the power value relates to the minimum signal or signal to noise ratio (SNR) within a cellular networks). It is said that the receiver is out of the range of Base Station in cellular communications. Average fade duration is used to determine how long a user is in continuous outage. This is important for coding design.

Based on the presented results it is possible to anticipate the behavior of the real wireless relay system in the presence of analyzed fading. Future works will introduce general fading distributions in consideration of three-hop relay systems' performance.

ACKNOWLEDGMENT

This article was realized under the projects: TR-33035 and III-44006 of the Ministry of Education, Science and Technological Development of the Republic of Serbia.

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