A Periodogram-based CFO Estimation Scheme for OFDM Systems

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***Abstract*—In this paper, we propose a novel carrier frequency offset (CFO) estimation scheme for orthogonal frequency division multiplexing (OFDM) systems in non-Gaussian noise environments. The proposed scheme has much wider estimation range compared with that of the conventional scheme, improving the overall CFO estimation performance. Numerical results demonstrate that the proposed scheme has better estimation performance than the conventional scheme.**

Keywords-estimation; carrier frequency offset; orthogonal frequency division multiplexing (OFDM); non-Gaussian; periodogram.

I. Introduction

Tho orthogonal frequency division multiplexing (OFDM) signal has been widely used for wireless communication systems including wireless local area network (WLAN), wireless metropolitan area network (WMAN), and digital video broadcasting (DVB) systems [1]. However, the OFDM system is very sensitive to the carrier frequency offset (CFO) caused by Doppler shift or oscillator instabilities. Thus, the CFO estimation is one of the most important issues in OFDM systems.

Several schemes [2]-[4] have been proposed to estimate the CFO of OFDM signals. Schmidl and Cox proposed a CFO estimation scheme using a training symbol with two identical halves [2], whose estimation range is equal to the sub-carrier spacing. In [3], a new CFO estimation scheme that utilizes a training symbol with more than two identical parts was proposed, increasing the estimation range twice that of the scheme in [2]. With the maximum-likelihood (ML) criterion, in [4], an optimal scheme for CFO estimation was derived using the same training symbol as in [3]. Recently, in [5], a periodogram-based CFO estimation scheme was proposed, whose estimation range is as large as the bandwidth of the OFDM signal while maintaining the same level of the estimation performance as those of the schemes based on training symbols with identical parts. However, the conventional schemes are developed under the assumption of the Gaussian distributed noise. Since it has been observed that the noise often exhibits non-Gaussian nature in wireless channels [6], the conventional estimators could suffer from performance degradation in the non-Gaussian noise environments.

In this paper, we propose a novel periodogram-based CFO estimation scheme for OFDM systems in non-Gaussian noise environments. We first investigate the influence of the non-Gaussian noise on the integer part of CFO estimation scheme in [5], and then, propose a novel fractional frequency offset (FFO) estimation scheme with wider estimation range. The numerical results show that the proposed FFO estimation scheme has better estimation performance than the FFO estimation scheme in [5] under the influence of the non-Gaussian noise.

II. Signal Model

After the inverse fast Fourier transform (IFFT) operation, at the transmitter, the nth complex-valued OFDM sample $x(n)$ can be expressed as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad n = 0, 1, \cdots, N-1, \quad (1)$$

where N is the size of the IFFT and X_k is a phase shift keying (PSK) or a quadrature amplitude modulation (QAM) symbol in the kth sub-carrier. The data part of the OFDM symbol has a duration of T seconds, and the cyclic prefix (CP), whose length is generally designed to be longer than the channel impulse response, is inserted to avoid the intersymbol interference (ISI).

The nth received OFDM sample $r(n)$ is obtained by sampling the received OFDM signal every $T_s = T/N$ seconds and can be expressed as

$$r(n) = s(n)e^{j2\pi(\varepsilon_I + \varepsilon_F)n/N} + w(n), \quad (2)$$

where $s(n) = \sum_{k=0}^{L-1} h_k x(n-k)$ is the signal component with the kth channel filter tap coefficient h_k and the channel memory size L, ε_I and ε_F represent the integer FO (IFO) and FFO normalized to the sub-carrier spacing $1/T$, respectively, and $w(n)$ is the non-Gaussian noise sample. In this paper, we assume the static channel during one OFDM symbol duration and perfect timing synchronization.

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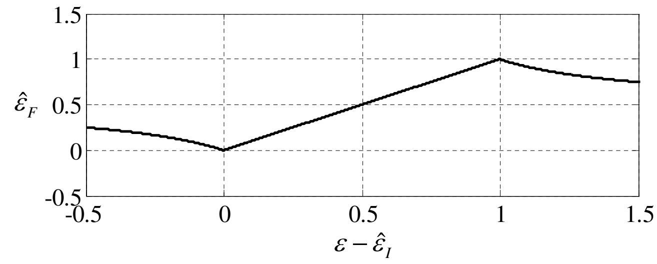

Figure 1. $\hat{\varepsilon}_F$ as a function of $\varepsilon - \hat{\varepsilon}_I$ in [5].

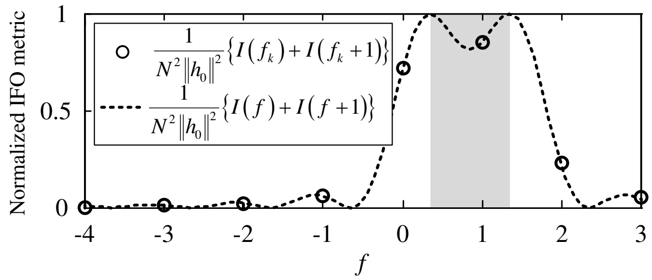

Figure 2. IFO metric $\{I(f) + I(f + 1)\}$ normalized to $N^2\|h_0\|^2$ as a function of the frequency $f \in [-N/2, N/2)$ for $\varepsilon_F = 0.3$ when $\varepsilon_I = 1$, $N = 8$, and the noise is absent.

III. Proposed Scheme

A. Influence of the non-Gaussian noise on the IFO estimation

In [5], the estimates $\hat{\varepsilon}_I$ and $\hat{\varepsilon}_F$ of the IFO and FFO are obtained as

$$\hat{\varepsilon}_I = \arg\max_{f_k} \{I(f_k) + I(f_k + 1)\} \tag{3}$$

and

$$\hat{\varepsilon}_F = \frac{\sqrt{I(\hat{\varepsilon}_I + 1)}}{\sqrt{I(\hat{\varepsilon}_I)} + \sqrt{I(\hat{\varepsilon}_I + 1)}}, \tag{4}$$

respectively, where 'arg' is the argument operation and $I(f_k)$ is the signal periodogram defined as

$$I(f_k) = \left| \sum_{n=0}^{N-1} r(n)e(n)e^{-j2\pi f_k n/N} \right|^2, \tag{5}$$

where $f_k \in \{-\frac{N}{2}, -\frac{N}{2} + 1, \cdots, \frac{N}{2} - 1\}$ is the kth IFO candidate, $|\cdot|$ is the absolute operation, and $e(n)$ is the envelope equalized processing factor, which removes the influence of the data modulation, defined by $\frac{x(n)^*}{\|x(n)\|^2}$ with the complex conjugation '$*$' and Euclidean norm $\|\cdot\|$.

In the absence of the noise, $\hat{\varepsilon}_F$ is given by $\frac{Z(\hat{\varepsilon}_I)}{Z(\hat{\varepsilon}_I)+Z(\hat{\varepsilon}_I+1)}$, where $Z(\alpha) = |\sin(\pi(\varepsilon-\alpha)/N)|$, and is drawn as a function of $\varepsilon - \hat{\varepsilon}_I$ as shown in Fig. 1, where $\varepsilon = \varepsilon_I + \varepsilon_F$. It is seen from the figure that the FFO can be correctly estimated only when $0 \leq \varepsilon - \hat{\varepsilon}_I < 1$ (i.e., $\hat{\varepsilon}_I \in (\varepsilon - 1, \varepsilon]$) which is referred to as the correct estimation range.

Fig. 2 shows the IFO metric $\{I(f) + I(f + 1)\}$ normalized to $N^2\|h_0\|^2$ as a function of the frequency $f \in [-N/2, N/2)$ for $\varepsilon_F = 0.3$ when $\varepsilon_I = 1$, $N = 8$, and the noise is absent, where '$\circ$' represents the IFO metric value corresponding to each f_k and the shaded region represents the correct estimation range. In this paper, the correct estimation probability of the IFO is defined as the probability that the maximum IFO metric corresponds to f_k stays within the correct estimation range.

From the figure, we can see that the IFO metric value (outside the correct estimation range) nearest to the correct estimation range is as large as that in the correct estimation range. Thus, under the influence of non-Gaussian noise with impulsive nature, the IFO estimation scheme (3) often outputs an incorrect estimate.

B. Proposed FFO Estimation Scheme

First, we define a new function similar to the periodogram, which can be expressed as

$$I_p(f_k) = \sum_{n=0}^{N-1} r(n)e(n)e^{-j2\pi f_k n/N}. \tag{6}$$

In the absence of the noise and interferences, $I_p(f_k)$ can be re-written as

$$I_p(f_k) = \sum_{n=0}^{N-1} h_0 e^{j2\pi(\varepsilon - f_k)n/N}. \tag{7}$$

Next, using the ratio $I_p(\hat{\varepsilon}_I)$ to $I_p(\hat{\varepsilon}_I + 1)$, we can remove the channel component h_0 as follows

$$\begin{aligned} \frac{I_p(\hat{\varepsilon}_I)}{I_p(\hat{\varepsilon}_I + 1)} &= \frac{h_0 \cdot \frac{1-e^{j2\pi(\varepsilon-\hat{\varepsilon}_I)}}{1-e^{j2\pi(\varepsilon-\hat{\varepsilon}_I)/N}}}{h_0 \cdot \frac{1-e^{j2\pi(\varepsilon-\hat{\varepsilon}_I)}e^{-j2\pi}}{1-e^{j2\pi(\varepsilon-\hat{\varepsilon}_I)/N}e^{-j2\pi/N}}} \\ &= \frac{1 - e^{j2\pi(\varepsilon-\hat{\varepsilon}_I)/N}e^{-j2\pi/N}}{1 - e^{j2\pi(\varepsilon-\hat{\varepsilon}_I)/N}}. \end{aligned} \tag{8}$$

From (8), the estimate of the FFO $\hat{\varepsilon}_F$ ($\approxeq \varepsilon - \hat{\varepsilon}_I$) can be obtained as

$$\hat{\varepsilon}_F = \frac{N}{2\pi}\angle\left(\frac{1 - M(\hat{\varepsilon}_I)}{e^{j2\pi/N} - M(\hat{\varepsilon}_I)}\right), \tag{9}$$

where $M(\hat{\varepsilon}_I) = I_p(\hat{\varepsilon}_I)/I_p(\hat{\varepsilon}_I + 1)$ and $\angle(y)$ denotes an angle of y. From (9), we can see that the estimation range of the proposed scheme is $-\frac{N}{2} \leq \hat{\varepsilon}_F < \frac{N}{2}$. Thus, the proposed scheme (9) can estimate the FFO in both correct IFO estimate and incorrect IFO estimate cases.

IV. Numerical Results

In this section, we compare the performance of the proposed and conventional [5] FFO estimation schemes in terms of the correct estimation probability. In the simulation, we assume the following parameters: quadrature PSK (QPSK) data modulation, the FFT size of $N = 64$, a CP with a length of 8 samples, and the maximum Doppler shift of 125 Hz (corresponding to a mobile speed of 54 km/h with a carrier

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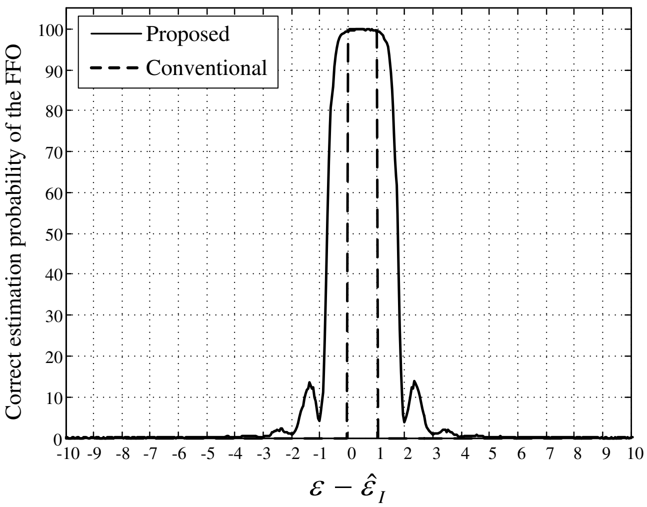

Figure 3. Correct estimation probabilities of the FFO as a function of $\varepsilon - \hat{\varepsilon}_I$ for the proposed and conventional schemes in the Rayleigh fading channel model when G-SNR is 5 dB.

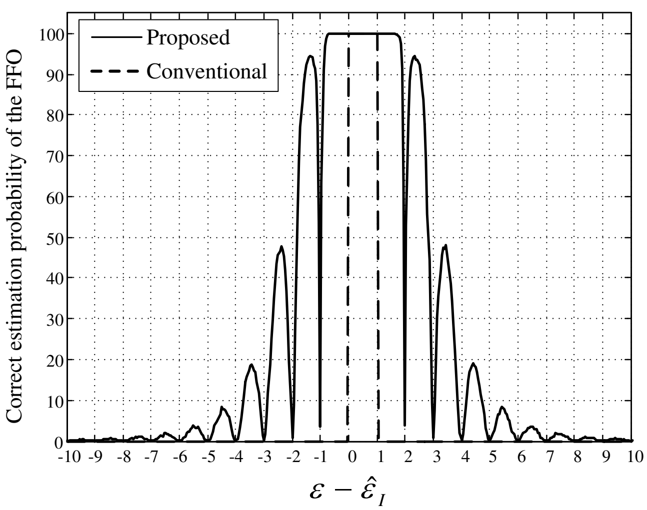

Figure 4. Correct estimation probabilities of the FFO as a function of $\varepsilon - \hat{\varepsilon}_I$ for the proposed and conventional schemes in the Rayleigh fading channel model when G-SNR is 25 dB.

frequency of 2.5 GHz). The non-Gaussian noise is modeled as Cauchy noise [7]. Since the variance is not defined in Cauchy noise, the standard signal-to-noise ratio (SNR) becomes meaningless. Thus, we employ the geometric SNR (G-SNR), which provides a mathematically and conceptually valid characterization of the relative strength between the signal and noise [8], is defined as

$$\text{G-SNR} = \frac{\sigma_s^2}{2C\gamma^2}, \tag{10}$$

where $\sigma_s^2 \triangleq \mathbf{E}\{|s(n)|^2\}$ with the expectation operator $\mathbf{E}\{\cdot\}$, $C = \exp\{\lim_{b\to\infty}(\sum_{a=1}^{b}\frac{1}{a} - \ln b)\} \approxeq 1.78$ is the exponential of the Euler constant, and γ is the dispersion parameter, which is set to be 1 in this paper. We consider four-path Rayleigh fading channel model with path delays of 0, 2, 4, and 6 samples and exponential power delay profile of $\mathbf{E}\{A_l^2\} = \exp(-0.768l)$ for $l = 0, 1, 2,$ and 3 (i.e., the power ratio of the first and last paths is set to 10 dB).

Figs. 3 and 4 show the correct estimation probabilities of the FFO as a function of $\varepsilon - \hat{\varepsilon}_I$ for the proposed and conventional schemes. In the figures, we can see that the proposed scheme has wider estimation range than the conventional scheme. Thus, the proposed scheme can estimate the FFO when $\varepsilon - \hat{\varepsilon}_I < 0$ and $\varepsilon - \hat{\varepsilon}_I > 1$ (outside of the correct estimation range of the IFO). However, we can observe that the estimation range of the proposed scheme is narrower than that in the ideal case of $-\frac{N}{2} \leq \hat{\varepsilon}_F < \frac{N}{2}$ in the absence of noise. This can be explained as follows. The proposed scheme is based on $I_p(\hat{\varepsilon}_I)$ and $I_p(\hat{\varepsilon}_I + 1)$. As shown in Fig. 2, when the chosen $\hat{\varepsilon}_I$ is far from the correct estimation range of the IFO, $I_p(\hat{\varepsilon}_I)$ and $I_p(\hat{\varepsilon}_I + 1)$ have small values, and thus, they would suffer from more severe influence of the non-Gaussian noise. Thus, the correct estimation probability of the proposed scheme becomes smaller as $|\varepsilon - \hat{\varepsilon}_I|$ increases and the estimation range of the proposed scheme is narrower than that in the ideal case. However, as mentioned in Section III-A and shown in Fig. 2, in the most cases, the proposed scheme can improve the overall CFO estimation performance.

In passing, we would like to stress that the proposed scheme is applicable to any kind of OFDM system employing training symbol. However, in the applications adopting the training symbol solely dedicated to the frequency offset estimation (e.g., IEEE 802.11 [9], long term evolution (LTE) [10]), the performance of the proposed scheme may not be better than the schemes dedicated to the application only.

V. Conclusion

In this paper, we have proposed a novel CFO estimation scheme for OFDM systems in the non-Gaussian noise environments. We have first investigated the influence of the non-Gaussian noise on the IFO estimation, and then, proposed a novel FFO estimation scheme with wider FFO estimation range. From numerical results, we have confirmed that the proposed scheme has better estimation performance than the conventional scheme.

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