

# Maximal Ratio Combining SC-FDMA Performance over Correlated Ricean Channels

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**Abstract**— Long Term Evolution (LTE) system has selected Single Carrier Frequency Division Multiple Access (SC-FDMA) for uplink transmission. Multiple Input Multiple Output (MIMO) can be used in order to improve throughput, robustness, coverage and capacity. Although it is well known that the performance of SC-FDMA is worse than that of Orthogonal Frequency Division Multiplexing (OFDM) for Rayleigh channels, the existence of a Line Of Sight (LOS) is able to increase SC-FDMA resistance to fading further than that of OFDM. In this work, the effect of antenna correlation over SC-FDMA is investigated for Rice and Rayleigh fading channels. Performance of MRC SC-FDMA is compared to that of ZF and MMSE equalizers for several LOS power and antenna correlation values. Results show that SC-FDMA coherent combination performance is better than that of OFDM for both Rayleigh and Rice channels. Influence of fading frequency correlation function on SC-FDMA performance is kept under MRC.

**Keywords**-SC-FDMA, MRC, Rice

## I. INTRODUCTION

Single Carrier Frequency Division Multiple Access (SC-FDMA) is used for the E-UTRA Long Term Evolution (LTE) mobile communication system. SC-FDMA, also referred to as Discrete Fourier Transform (DFT) spread Orthogonal Frequency Division Multiple Access (OFDM), has been selected for uplink transmission for LTE [1]. The main important advantage of SC-FDMA compared to standard OFDM is its low Peak to Average Power Ratio (PAPR) [2], which enables low complexity implementation of mobile terminal.

In general, it is accepted that SC-FDMA link level performance is worse than that of OFDM [3]. However, under certain conditions, SC-FDMA behavior improves that of OFDM. Specifically, the existence of a Line Of Sight (LOS) increases SC-FDMA resistance to fading over OFDM [4]. Roughly speaking, SC-FDMA BER is obtained from the harmonic average of the channel response at the allocated subcarriers. On the other hand, BER in OFDM is evaluated as the average of the BER for each subcarrier. Under high probability of deeply faded subcarriers (as in Rayleigh channels), SC-FDMA basically behaves as the worst subcarrier. However, the existence of a LOS greatly reduces the probability of deep fading. Without that burden, SC-FDMA is able to add frequency diversity to an OFDM system, thus reducing BER.

In the receiver at base station, frequency domain Zero Forcing (ZF) or Minimum Mean-Squared Error (MMSE) linear equalization [5], [6] might be applied. For OFDM, both techniques obtain similar results [7]. However, MMSE

SC-FDMA performance outperforms MMSE OFDM [8][9] due to a similar reason to that of the existence of a LOS: MMSE modifies the harmonic average previously described by including a constant for each subcarrier.

Multiple Input Multiple Output (MIMO) techniques [10] take advantage of the spatial separation between antenna elements to create uncorrelated spatial channels and to exploit higher levels of the spatial diversity. This improves spectral and power efficiency. MIMO techniques are very attractive at base station, where large antenna spacing is easily accommodated. On the mobile unit, however, single antenna is more feasible, thus Single Input Multiple Output (SIMO) techniques are advisable for uplink.

Diversity combining is well known to mitigate the performance degradation in multipath fading. Specifically, Maximal Ratio Combining (MRC) represents an optimal combiner over fading channels: multiple copies of the same information signal are blended so as to maximize the instantaneous SNR at the output [11]. Exact closed form expressions of average symbol error rate (SER) can be found for uncorrelated [12] and correlated [13] received Rayleigh channels, and certain efforts for analysis under LOS reception can also be found [14]. In general, it is well known that antenna correlation degrades the system performance as less diversity is present at the receiver.

MRC for received uplink SC-FDMA signals is implicitly assumed in many works with multiple antennas at the base station [15][6]. However, works regarding evaluation of MRC SC-FDMA link level performance for Rice channels and studies on the effect of antenna correlation are difficult to find.

In this paper, BER performance of MRC SC-FDMA system is evaluated. The effects of antenna correlation and Rice factor on SC-FDMA link level performance are studied and compared to those over Rayleigh channel and for OFDM. Two distinct realistic channel models [16] are used for simulation, as power delay profile greatly influences SC-FDMA performance.

The rest of the paper is organized as follows. In Section II, we summarize the maximal ratio combining receiver diversity scheme. Our system model is described in Section III. In Section IV, we evaluate BER of MRC SC-FDMA system by simulating a BPSK signaling scheme. Finally, some concluding remarks are given.

## II. MAXIMAL RATIO COMBINING

MRC is a SIMO technique allowing coherent combination of signals received over a set of antennas. In particular, signals from antenna elements are weighted and combined to maximize the output Signal to Noise Ratio (SNR).

Consider a receiver diversity system with  $N_R$  antennas as shown in Fig. 1. The channel can be expressed as

$$\mathbf{h} = [h_1 \ h_2 \ h_3 \ \dots \ h_{N_R}]^T \quad (1)$$

The received set of signals  $\mathbf{y} = [y_1 \ y_2 \ y_3 \ \dots \ y_{N_R}]^T$  is then

$$\mathbf{y} = \sqrt{\frac{E_x}{N_0}} \mathbf{h}x + \mathbf{n} \quad (2)$$

being  $\mathbf{n}$  a vector of noise AWGN samples and  $x$  the transmitted symbol. Let  $\gamma_i$  be the instantaneous SNR for the  $i^{\text{th}}$  branch, which is given by

$$\gamma_i = \frac{E_x}{N_0} |h_i|^2 \quad (3)$$

In MRC, all  $N_R$  branches are combined by the following weighted sum:

$$y_{MRC} = [W_1 W_2 W_3 \dots W_{N_R}] \mathbf{y} = \mathbf{W}^T \mathbf{y} \quad (4)$$

Power of the instantaneous signal and noise part are respectively given as

$$P_{signal} = \frac{E_x}{N_0} |\mathbf{W}^T \mathbf{h}|^2 \quad (5)$$

and

$$P_{noise} = \|\mathbf{W}^T\|_2^2 \quad (6)$$

where  $\|\cdot\|_2$  represents the usual Euclidean norm. From equation (5) and (6) the average SNR for MRC is given as

$$SNR_{MRC} = \frac{P_{signal}}{P_{noise}} = \frac{E_x}{N_0} \frac{|\mathbf{W}^T \mathbf{h}|^2}{\|\mathbf{W}^T\|_2^2} \quad (7)$$

Invoking the Cauchy-Schwartz inequality, SNR is maximized at  $\mathbf{W} = \mathbf{h}^*$  which yields

$$SNR_{MRC} = \frac{E_x}{N_0} \|\mathbf{h}\|_2^2 \quad (8)$$

Roughly, MRC process corrects the channel phase and blends the two received signals in the correct direction. Further, signals are amplitude scaled in such a way that stronger signals are more influent in the final value. Later, the amplitude scaling step makes sure that the received sequence has similar amplitude as the transmitted sequence. These steps together remove the channel effect and replace the equalizer.

### III. SYSTEM MODEL

We consider Fig. 2 - Fig. 3 system model with one transmit and  $N_R$  receive antenna, i.e. a SIMO  $1 \times N_R$  system.

For a given user, a sequence of transmitted bit is mapped to a constellation of complex symbols (e.g. BPSK, QAM). The precoded complex symbol  $\mathbf{X}$  is obtained by performing N-DFT operation over the resulting complex sequence  $\mathbf{x}$ . Then,  $\mathbf{X}$  is mapped on a subset of different allocated subcarriers per user,

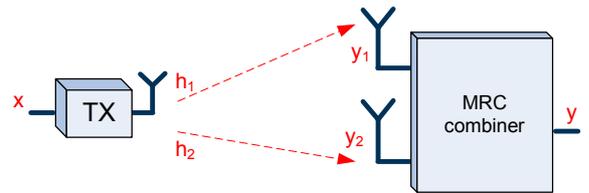


Figure 1. Transmitter and receiver configuration for MRC

i.e.,  $N$  out of  $M$  sub-carriers in which the total system bandwidth is divided. The subset may consist of a group of adjacent localized SC-FDMA (L. SC-FDMA) or of distributed interleaved SC-FDMA (I. SC-FDMA) subcarriers [2]. Non-allocated subcarriers are forced to zero. From this point onwards transmission is similar to that of OFDMA.

The channel system with  $N_R$  diversity branches at the receiver can be represented by the channel vector

$$\mathbf{h} = [h_j] \quad (9)$$

where  $h_j$  is the channel coefficient between the transmit antenna and the  $i^{\text{th}}$  receive antenna. Certain correlation, measured through the correlation factor  $\rho$ , can exist among paths. Moreover,  $h_j$  can be described by multiple paths, which arise from spreading. If there are  $N_j$  distinct paths from the transmitter to the receiver, the impulse response for this channel will be:

$$h_j(t, \tau) = \sum_i^{N_j} a_i \delta(t, \tau_i) \quad (10)$$

where  $t$  stands for time variability and  $\tau$  for delay. This is the well known tapped-delay line model. Path amplitudes are well described by Rayleigh distributed amplitudes varying according to a classical Doppler spread and with average power as given by the Power Delay Profile (PDP). Moreover, a Line of Sight (LOS) component can also exist between the transmitter and the receiver. The Rice factor  $K$  [10] measures the relative strength of the LOS compared to that of the whole varying amplitude. It is a measure of the severity of the fading, being  $K = 0$  the most severe fading case (Rayleigh fading, i.e. no LOS), and  $K = \infty$  the usual Additive White Gaussian Noise (AWGN) channel.

At the receiver,  $N_R$  different receiving chains are followed as shown in Fig. 3. Perfect channel estimation and synchronization avoid interference from other users. The cyclic prefix is suppressed and an M-DFT operation converts each time domain symbol in to a frequency-domain symbol at the receiver. After demapping, the received symbols at each antenna  $Y_n$  can be expressed as

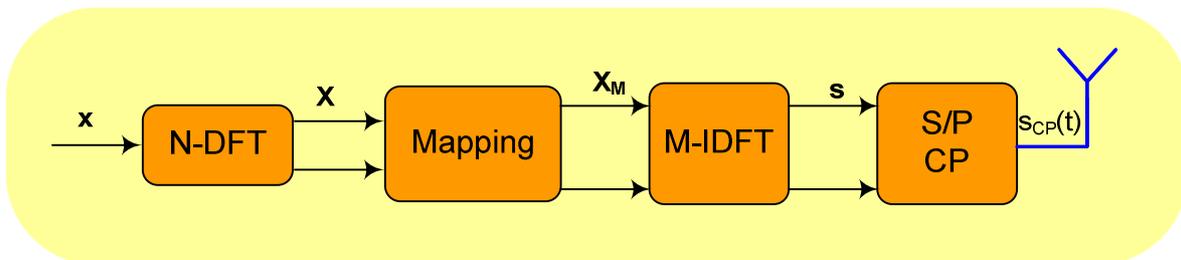


Figure 2. SC-FDMA transmitter scheme

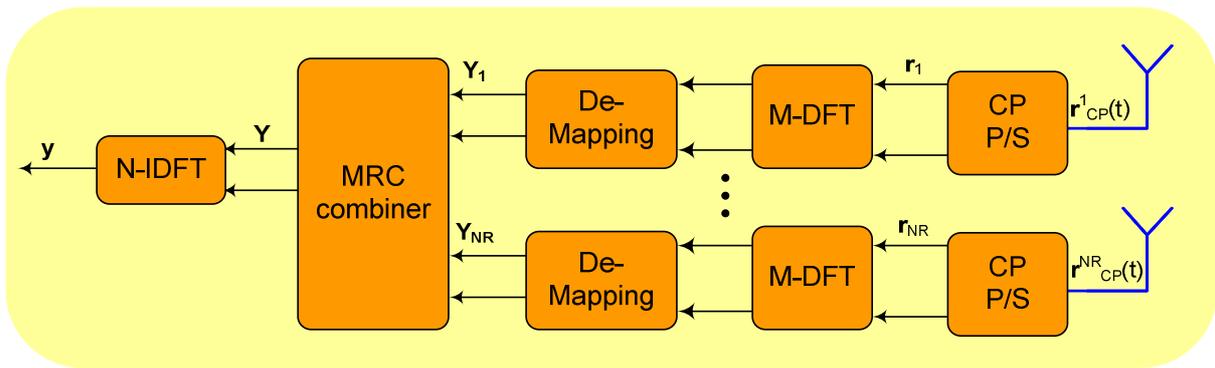


Figure 3. MRC SC-FDMA receiver scheme

$$\mathbf{Y}_n = \mathbf{H}_n \mathbf{X} + \boldsymbol{\eta} \quad (11)$$

where  $\boldsymbol{\eta}$  is a noise vector whose entries are i.i.d. complex Gaussian  $\mathcal{CN}(0, N_0)$  and  $\mathbf{H}_n$  represents the  $N \times N$  diagonal matrix whose entries are the channel frequency response as seen by antenna  $n$  for each allocated subcarrier [10].

These  $N_R$  signals are frequency combined using MRC:

$$\mathbf{Y} = \frac{\sum_{n=1}^{N_R} \mathbf{Y}_n \mathbf{H}_n^*}{\sum_{n=1}^{N_R} |\mathbf{H}_n|^2} \quad (12)$$

After taking IDFT of  $\mathbf{Y}$ , signal is given to detector. The output of detector is the estimated input bit sequence.

#### IV. SIMULATION RESULTS

In this section, simulation results are given in order to evaluate BER performance for MRC SC-FDMA. LTE settings are fixed to those in Table I. Channel profiles described in Table II are adopted from ITU-R\_M.1225 specs [16] with an added direct LOS with Rice factor  $K$ . Following figures give the BER performance of MRC SC-FDMA for two receiving antenna with different antenna correlation  $\rho$ , number of allocated subcarrier  $N$  and Rice factor  $K$ . Results are given for Vehicular B (VB) and Pedestrian A (PA) channels. For

TABLE I. SIMULATION PARAMETERS

FFT size	2048
Modulation Techniques	BPSK, 16QAM
Carrier frequency	2.00 GHz
System Bandwidth	20 MHz
Sampling Frequency	30.72 MHz
Number of used subcarriers	4, 32
Antenna configuration	$1 \times \{1, 2, 4\}$
Channel model	ITU-R VA & PA channel
Equalizers	MMSE, ZF & MRC
Number of receiving antenna	1, 2, 4

TABLE II. DELAY SPREAD AND COHERENCE BANDWIDTH FOR CONSIDERED CHANNELS

Channel model	Delay spread (r.m.s.)	Coherence Bandwidth (50%)
PA	46 ns	4.35 MHz
VB	4001 ns	50 KHz

comparison, ZF or MMSE frequency equalized single antenna detection [9] are also included in figures, as well as several results for OFDM.

Figs. 4 and 5 show BER of MRC without antenna correlation over VB channel for Localized and Interleaved SC-FDMA, respectively. As it is known, in ZF single antenna reception, OFDMA determines the lower bound for SC-FDMA, while in MMSE. SC-FDMA results are better. It is shown that under MRC, SC-FDMA results are also better than those of OFDM. Improvement for MRC Localized SC-FDMA is slightly higher than that of Interleaved SC-FDMA.

The effect of channel correlation among antenna can be inspected in Fig. 6. Results for Interleaved SC-FDMA over channel were as expected: the more antenna correlation, the more errors at detection. However, performance under Rice fading correlation is less affected by correlation value as direct path affect both antenna anyway. Due to the same reason, specific frequency correlation function influences less SC-FDMA performance under a LOS [9]. However, Fig. 7 shows that VB channel results are better than those of PA channel for MRC Localized SC-FDMA.

Fig. 8 gives results equivalent to those of Figs. 6 and 7 but for OFDM. Note that no effect due to PA or VB channel model or localized/interleaved mode exists in OFDM. Effect of correlation factor is more noticeable than in SC-FDMA. Expected 3dB gain for Rayleigh coherent MRC combination ( $\rho = 1$ ) can be found.

In Fig. 9, results for VB and PA channels are given for two different numbers of allocated subcarriers in Localized and Interleaved SC-FDMA. Over VB channel, improvement is better for a higher number of subcarriers as the probability of at least one very faded pair of carriers is lower. In general, PA channel performance is worse as its coherence bandwidth is higher.

#### V. CONCLUSION

In this paper, we have investigated BER performance of MRC SC-FDMA over Rayleigh and Rice fading channels. It is known that ZF SC-FDMA behavior is worse than that of OFDM for single antenna systems. However, coherent combination of signal received on two antennas improves SC-FDMA up to overtake OFDM. Influence of fading frequency correlation function on SC FDMA performance is kept under MRC. Lower correlation among allocated

subcarriers (i.e., higher frequency diversity) improves performance of SC-FDMA (VB channels vs. PA fading; Interleaved mode vs. Localized).

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support of the European Commission through Erasmus Mundus External Cooperation Window “Mobility for Life” and express their gratitude to Junta de Andalucía (Proyecto de Excelencia P07-TIC-03226), to the Spanish Government (Plan Nacional de I+D+I, TEC2010-18451), and to Sinhgad Technical Education Society (India).

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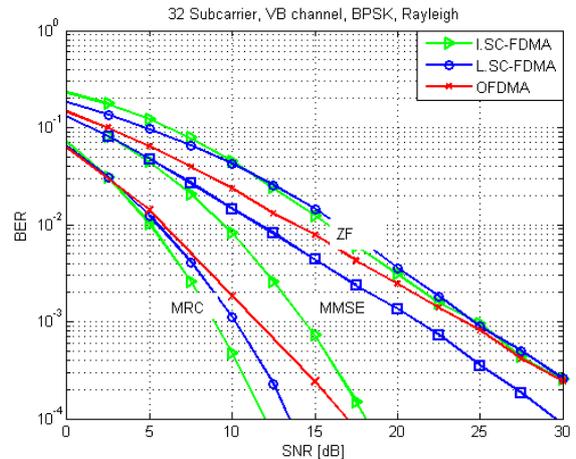


Figure 4. BER of MRC SC-FDMA over Rayleigh fading VB channel

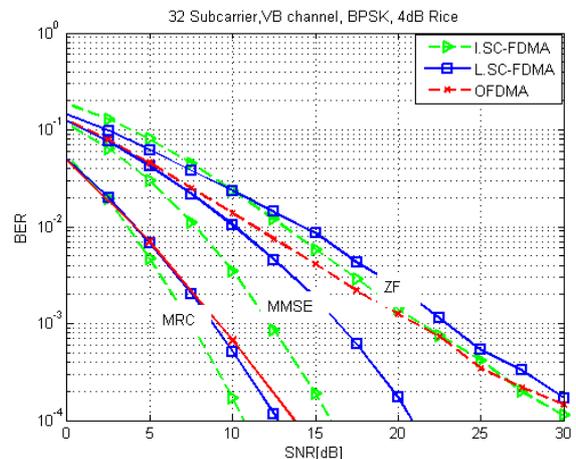


Figure 5. BER of MRC SC-FDMA over Rice fading VB channel

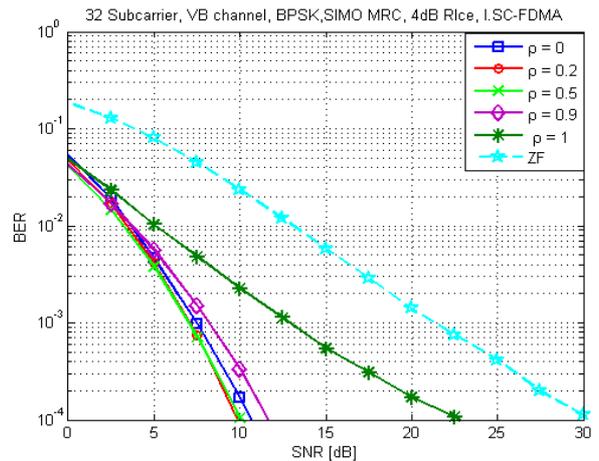


Figure 6. BER of Interleaved SC-FDMA with MRC for different antenna correlation factor over Rice fading VB channel

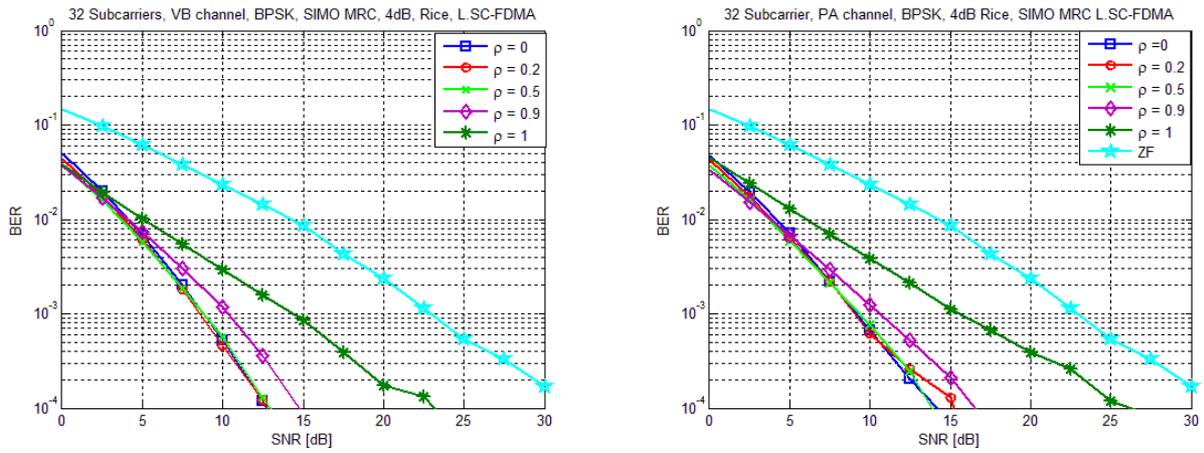


Figure 7. BER performance of localized VB (left) and PA (right) SC-FDMA with MRC for different antenna correlation factor over Rice fading

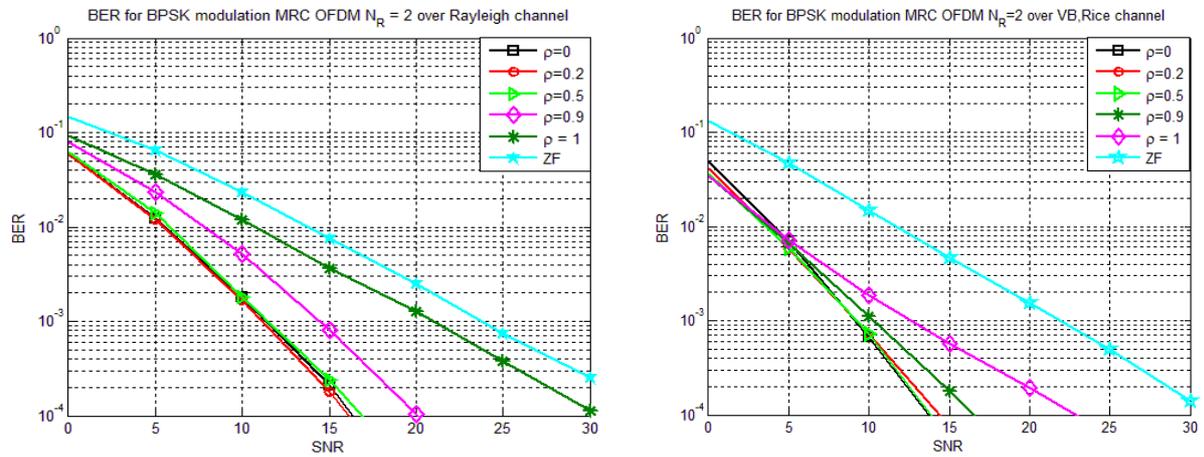


Figure 8. BER of OFDM with MRC for different antenna correlation factor over Rayleigh (left) and Rice 4dB (right) fading

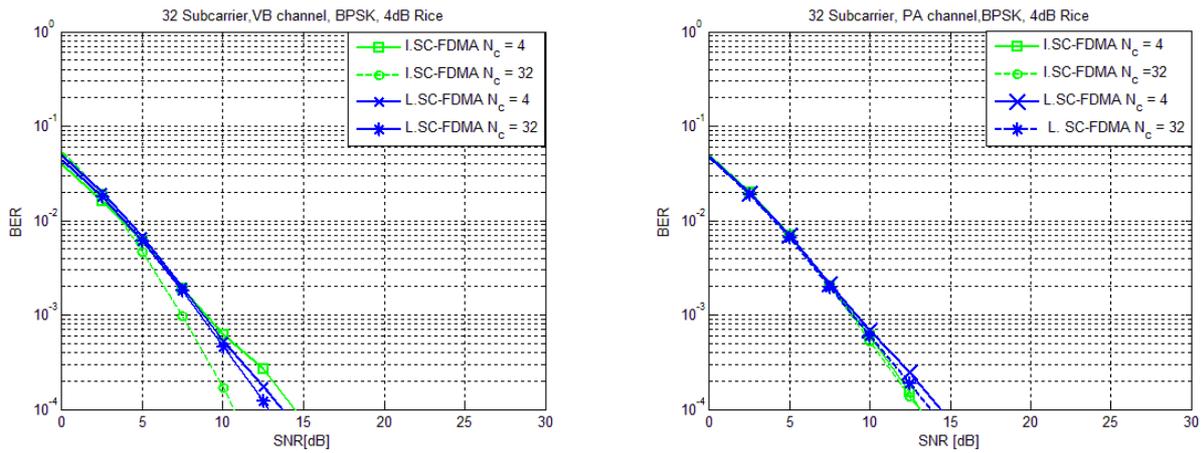


Figure 9. BER performance of SC-FDMA with MRC for different number of allocated subcarriers over VB and PA Rice fading channels