Analysis of Statistical Time-access Fairness Index of Opportunistic Feedback Fair Scheduler

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***Abstract*—Since the utilization of multiuser diversity in wireless networks can increase the information theoretic capacity of the overall system, much attention has been paid to schedulers exploiting multiuser diversity. However, packet schedulers exploiting multiuser diversity have a disadvantage of consuming the bandwidth for the feedback load. From a view of feedback reduction, the opportunistic feedback fair scheduler is considered as an attractive choice among schedulers exploiting multiuser diversity. In this paper, considering the statistical time-access fairness index (STAFI) as a measure of short term fairness, we study the short term fairness provided by the opportunistic feedback fair scheduler. Numerical results display that the threshold value of the scheduler greatly affects the properties of its short term fairness.**

***Keywords*-Opportunistic feedback fair scheduler; Short term fairness; Statistical time-access fairness index**

I. Introduction

Multiuser diversity [1] is a diversity existing between the channel states of different users in wireless networks. Since packet schedulers exploiting multiuser diversity have an advantage of increasing the information theoretic capacity of the overall system, much attention has been paid to such schedulers (see, e.g., [2], [3], [4], [5], [6], [7], [8] and references therein). However, packet schedulers exploiting multiuser diversity also have a disadvantage of consuming the bandwidth for the feedback load, defined as the amount of channel information that needs to be fed back from MSs (mobile stations) to BS (base station). In addition, it is known that there exists a tradeoff between the information theoretic capacity and fairness achieved by schedulers exploiting multiuser diversity [9]. Therefore, when we consider a scheduler exploiting multiuser diversity, we should take its feedback load and fairness as well as performance gain into account.

To reduce the feedback load while still having the performance gain, several schedulers have been proposed and studied. The one-bit feedback fair scheduler [10], [11], [12], [13] is an example of such schedulers. Under the one-bit feedback fair scheduling, the *normalized* received SNR (Signal-to-Noise Ratio) values of MSs (instead of the received SNR values) are considered. Each MS feeds back one-bit information to BS, only when its *normalized* received SNR is greater than or equal to a predetermined threshold. By doing so, the one-bit feedback fair scheduler can reduce the feedback load from MSs to BS and achieve the ideal *long term fairness*, while having considerable performance gain. However, the one-bit feedback fair scheduler still has a difficulty for the feedback load. The difficulty is that the feedback load of the one-bit feedback fair scheduler linearly increases with the number of MSs, although the performance gain for the capacity also grows as the number of MSs becomes large [14]. This may degrade the scalability of the one-bit feedback fair scheduler. One way to overcome the difficulty against the scalability is to introduce a random access-based feedback scheme. As a scheduler with random access-based feedback scheme, Tang and Heath [7] proposed the opportunistic feedback scheduler. Under the opportunistic feedback scheduling, the feedback resources are random access minislots. MSs transmit feedback information with some probability in each minislot only when their SNR values are greater than or equal to a predetermined threshold. Contrary to the one-bit feedback fair scheduler, the feedback load of the opportunistic feedback scheduler is independent of the number of MSs.

The fairness of scheduler is classified into short term fairness and long term fairness [15], [16]. While long term fairness governs the long run performances such as long run average throughput of individual MSs, short term fairness greatly affects the packet level performances such as delay and loss probability of individual MSs. Since the packet level performances of individual MSs are basic measures of QoS, it is important to examine the short term fairness of scheduler in terms of QoS guarantees.

As a measure of short term fairness, the proportional fairness index is usually considered in wireline networks. The proportional fairness index characterizes the service discrepancy *in bits* between two flows over any time interval during which the two flows are continuously backlogged. However, for the following two reasons, the proportional fairness index is not suitable for wireless networks. First, the proportional fairness index considers the hard deterministic guarantee, and it does not take randomness inherent in the wireless channel conditions into account. Second, the proportional fairness index considers fairness of users' throughputs rather than channel access times, although users can transmit at different rates depending on their current channel quality in wireless

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networks. Liu et al. [17] then consider modifications to the proportional fairness index for short term fairness index in wireless networks. By considering the service in *time* (instead of the service in bits) and a statistical fairness guarantee (instead of the hard deterministic fairness guarantee), they propose a *statistical time-access fairness index* (STAFI) defined as

$$\mathrm{P}\left(\left|\frac{\alpha^{(i)}(t_1,t_2)}{\phi^{(i)}} - \frac{\alpha^{(j)}(t_1,t_2)}{\phi^{(j)}}\right| \geq x\right) \leq f^{(i,j)}(x), \qquad (1)$$

where $\alpha^{(i)}(t_1,t_2)$ denotes the service *in time* that flow i receives during $[t_1,t_2)$, ϕ_i denotes the assigned weight for flow i and $f^{(i,j)}(x)$ is a probability distribution which may depend on i and j.

In this paper, we focus on the short term fairness of the opportunistic feedback fair scheduler. We study the STAFI of the scheduler to investigate its short term fairness properties. In particular, we consider the STAFI where the assigned weights ϕ_i in (1) are all equal to one. Since the *normalized* SNR processes of MSs are considered and the normalized SNRs of MSs are i.i.d. (independent and identically distributed) under the opportunistic feedback fair scheduling, the opportunistic feedback fair scheduler provides an ideal long term fairness property [5]. However, as far as the author's best knowledge, there is no study on the short term fairness properties of the opportunistic feedback fair scheduler, although the packet level performances of individual MSs are strongly affected by the short term fairness.

The remainder of this paper is organized as follows. In Section II, we describe a system model considered in this paper. We assume that the wireless channel process for each user is modeled by a discrete-time two-state Markov chain. We analyze the STAFI of the opportunistic feedback fair scheduler in Section III. We also develop a numerical method to calculate the exact value of the STAFI by using the inverse discrete FFT method [18]. Section IV provides numerical results to investigate the properties of the short term fairness provided by the opportunistic feedback fair scheduler. Conclusion is drawn in Section V.

II. SYSTEM MODEL

In this paper, we consider a wireless network consisting of a BS and K MSs. We suppose that the BS employs the opportunistic feedback fair scheduler for downlink transmission from the BS to the MSs [7]. In this paper, considering the STAFI as a measure of short term fairness, we study the properties of short term fairness provided by the opportunistic feedback fair scheduler for the downlink transmission.

We assume that the downlink channel of MS i ($i = 1,\ldots,K$) is described by a Rayleigh fading channel model. Time axis is divided into frames of equal size T_f (sec) and time index is given by $t = 0,1,2,\cdots$. The frame duration T_f is considered to be the unit time in our model. Then, the received SNR process $\{z^{(i)}(t)\}$ ($t = 0,1,\ldots$) of MS i ($i = 1,\ldots,K$) is described as a discrete-time stochastic process. We assume

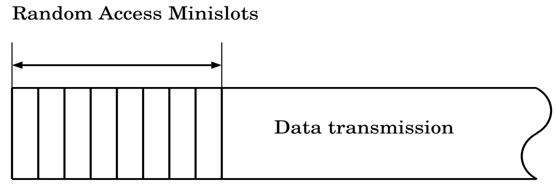

Fig. 1. Uplink frame structure

that the received SNR processes of the K MSs are independent with each other.

Without loss of generality, we consider that MS 1 and MS 2 are tagged users and all the other MSs are background users for the STAFI. More specifically, we assume that for $i = 3,\ldots,K$, the received SNR process $\{z^{(i)}(t)\}$ is a stationary process. But we do not assume the stationarity of $\{z^{(i)}(t)\}$ for $i = 1,2$. When the received SNR process $\{z^{(i)}(t)\}$ is stationary, $z^{(i)}(t)$ at time t is according to the following exponential distribution:

$$\mathrm{P}\{z^{(i)}(t) \leq x\} = 1 - \exp(-x/\bar{z}^{(i)}), \qquad (2)$$

where $\bar{z}^{(i)}$ denotes the average received SNR of MS i and is defined by $\bar{z}^{(i)} = \mathrm{E}[z^{(i)}(t)]$.

A. Opportunistic feedback fair scheduler

Under the opportunistic feedback fair scheduling, the *normalized* SNR processes of MSs are considered, where the normalized SNR process is defined by the process $\{z^{(i)}(t)/\bar{z}^{(i)}\}$ ($i = 1,\ldots,K$). To reduce the feedback load, each MS quantizes or partitions the entire normalized SNR range into two grades with threshold denoted by γ_1. We assume that the threshold γ_1 is *a priori* determined. If $z^{(i)}(t)/\bar{z}^{(i)} < \gamma_1$, we say that the wireless channel state of MS i is in state 0 at time t. If $z^{(i)}(t)/\bar{z}^{(i)} \geq \gamma_1$, we say that the wireless channel state of MS i is in state 1 at time t. We assume that perfect channel estimation is possible at each MS and each MS knows its average SNR $\bar{z}^{(i)}$ ($i = 1,\ldots,K$). Then MS i can determine the grade of its channel to the BS with the knowledge of its normalized SNR.

We suppose that the opportunistic feedback fair scheduler is employed in a frequency-division-duplex (FDD) system. In the FDD system, at the beginning of the downlink frame, the BS broadcasts a message containing the information for opportunistic feedback to all the MSs. N minislots in an uplink frame for random access feedback follow the downlink message as illustrated in Figure 1. We assume that the number of minislots N is fixed.

The opportunistic feedback fair scheduler then operates as follows:

- At every time t, MS i estimates its received normalized SNR $z^{(i)}(t)/\bar{z}^{(i)}$ and examines if $z^{(i)}(t)/\bar{z}^{(i)}$ is greater than or equal to the threshold γ_1.
- If the normalized SNR of MS i $z^{(i)}(t)/\bar{z}^{(i)}$ ($i = 1,\ldots,K$) is greater than or equal to the threshold γ_1 (i.e., if the wireless channel state of MS i is in state 1), MS i attempts to transmit feedback information to the

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MS with a probability u in every minislot. We hereafter call the probability u the feedback probability.
- Otherwise (i.e., if the wireless channel state of MS i is in state 0), MS i does not feedback any information to the BS in the random access minislots.
- The feedback information can be fed back to the BS if and only if one MS attempts to transmit feedback information in the minislot. Otherwise, either a collision happens or there is no MS to feed back.
- If multiple MSs successfully feedback during the random access period consisting of N minislots, the BS randomly selects one of the successful MSs.
- If there is no successful feedback in all N minislots, the BS randomly selects one MS among all the K MSs.
- The scheduling is performed frame-by-frame.

We assume that the random access attempts are independent among MSs and also independent among random access minislots.

B. Wireless channel model

In this subsection, we consider a wireless channel state process of MS i $(i = 1, \ldots, K)$. Let $\{s^{(i)}(t)\}$ $(t = 0, 1, \ldots; i = 1, \ldots, K)$ denote the wireless channel state process of MS i, where $s^{(i)}(t) = 1$ if $z^{(i)}(t)/\bar{z}^{(i)} \geq \gamma_1$ and $s^{(i)}(t) = 0$ otherwise. We assume that the channel state process $\{s^{(i)}(t)\}$ $(t = 0, 1, \ldots; i = 1, \ldots, K)$ of MS i is well described by a discrete-time 2-state Markov chain [15], [19]. We further assume that for $i = 3, \ldots, K$, the Markov chain $\{s^{(i)}(t)\}$ is stationary from the assumption of the stationarity of the received SNR process $\{z^{(i)}(t)\}$ for $i = 3, \ldots, K$. On the other hand, for $i = 1, 2$, we do not assume the stationarity of $\{s^{(i)}(t)\}$.

Let $\boldsymbol{P} = (p_{i,j})$ $(i, j = 0, 1)$ denote the transition probability matrix of the 2-state Markov chain. The transition probability matrix $\boldsymbol{P}$ is determined as follows (for the detailed derivation of the transition probabilities, see [19]). We first consider the level crossing rate $\chi(\gamma)$ of the received normalized SNR at γ given by [20]

$$\chi(\gamma) = \sqrt{2\pi\gamma} f_d \exp(-\gamma), \tag{3}$$

where f_d denotes the mobility-induced Doppler spread of MSs and we assume that for all the MSs, the mobility-induced Doppler spreads are identical.

For MS i $(i = 3, \ldots, K)$, we next consider the stationary probability vector $\boldsymbol{s} = (s_0, s_1)$ of the 2-state discrete-time Markov chain $\{s^{(i)}(t)\}$. Note here that for the MSs $(i = 3, \ldots, K)$, the channel state processes have the same stationary probability vector due to the normalization of the received SNRs. From (2), the stationary probability vector is given by

$$s_0 = 1 - e^{-\gamma_1}, \quad s_1 = e^{-\gamma_1}. \tag{4}$$

The state transition probabilities are then determined by

$$p_{0,1} = \frac{\chi(\gamma_1)T_f}{s_0}, \quad p_{1,0} = \frac{\chi(\gamma_1)T_f}{s_1}, \tag{5}$$

$$p_{0,0} = 1 - p_{0,1}, \quad p_{1,1} = 1 - p_{1,0}, \tag{6}$$

where s_i $(i = 0, 1)$ and $\chi(\gamma_1)$ are given by (4) and (3), respectively. (5) and (6) determine the transition probability matrix $\boldsymbol{P}$ of the 2-state Markov chain, whose stationary probability vector is given by (4).

III. Analysis

In this section, we analyze the STAFI between MS 1 and and MS 2, which are tagged users.

Let $c^{(i)}(t)$ $(i = 1, \ldots, K; t = 0, 1, \ldots)$ denote a random variable representing the amount of service of MS i at time t, i.e., $c^{(i)}(t) = 1$ when the opportunistic feedback fair scheduler selects MS i for downlink transmission at time t, and $c^{(i)}(t) = 0$ otherwise. The amount service $\alpha^{(i)}(t_0 t_0 + n)$ for MS i in $[t_0, t_0 + n)$ is then expressed as

$$\alpha^{(i)}(t_0, t_0 + n) = \sum_{t=t_0}^{t_0+n-1} c^{(i)}(t).$$

In this paper, we hereafter consider only the cases where $t_0 = 0$, because we focus on the transient properties of the short term fairness of the scheduler. Let $\beta^{(i,j)}(n)$ $(i, j = 1, \ldots, K; n = 0, 1, \ldots)$ denote the difference between the amount service for MS i and that for MS j in $[t_0, t_0 + n)$. $\beta^{(i,j)}(n)$ is given by

$$\beta^{(i,j)}(n) = |\alpha^{(i)}(0, n) - \alpha^{(j)}(0, n)|.$$

We are now ready to provide an expression of the STAFI of the scheduler. Let $G_n(x)$ $(n = 1, 2, \ldots)$ denote the STAFI. $G_n(x)$ is defined by

$$\begin{aligned} G_n(x) &= \mathrm{P}(\beta^{(1,2)}(n) \geq x) \\ &= \mathrm{P}(|\alpha^{(1)}(0, n) - \alpha^{(2)}(0, n)| \geq x). \end{aligned}$$

We further define the probability mass function $g_n(x)$ $(n = 1, 2, \ldots)$ by

$$g_n(x) = \mathrm{P}(\beta^{(1,2)}(n) = x) = \mathrm{P}(|\alpha^{(1)}(0, n) - \alpha^{(2)}(0, n)| = x).$$

In what follows, we analyze the STAFI $G_n(x)$. For this purpose, we define some matrices and vectors. We first define a $(K - 1) \times (K - 1)$ matrix $\boldsymbol{R}$ by

$$\begin{aligned} [\boldsymbol{R}]_{i,j} &= \sum_{k=\max(0,i+j-K+2)}^{\min(i,j)} \binom{i}{k} p_{1,1}^{k} p_{1,0}^{i-k} \\ &\quad \cdot \binom{K-2-i}{j-k} p_{0,1}^{j-k} p_{0,0}^{K-2-i-j+k}, \end{aligned} \tag{7}$$

where $[\boldsymbol{R}]_{i,j}$ $(i, j = 0, \ldots, K-2)$ denotes the (i, j)th element of $\boldsymbol{R}$. Note that $\boldsymbol{R}$ is a transition probability matrix of the Markov chain $\{r(t)\}$ $(t = 0, 1, \ldots)$, where $r(t)$ is defined by $r(t) = \sum_{k=3}^{K} I(s^{(k)}(t) = 1)$. Thus, $[\boldsymbol{R}]_{i,j}$ denotes the conditional probability that j MSs among the $(K - 2)$ MSs excluding MS 1 and MS 2 are in state 1 at time t given that i MSs among the $(K-2)$ MSs was in state 1 at time $t-1$. Let $\boldsymbol{r}$ denote the stationary probability vector of $\boldsymbol{R}$. The stationary probability vector $\boldsymbol{r}$ is given by

$$[\boldsymbol{r}]_j = \binom{K-2}{j} s_0^{K-2-j} s_1^{j}, \tag{8}$$

where $[\boldsymbol{r}]_j$ $(j = 0, \ldots, K-2)$ denotes the jth element of $\boldsymbol{r}$, and s_0 and s_1 are given by (4). Note here that since we assume that $\{s^{(i)}(t)\}$ is stationary for $i = 3, \ldots, K$, the stationary probability vector $\boldsymbol{r}$ is also the initial state probability vector of the Markov chain $\{s^{(i)}(t)\}$ for $i = 3, \ldots, K$.

We next define a $4(K-1) \times 4(K-1)$ matrix $\boldsymbol{Q}$ by

$$\boldsymbol{Q} = \boldsymbol{P} \otimes \boldsymbol{P} \otimes \boldsymbol{R}, \tag{9}$$

where $\otimes$ denotes the Kronecker product, $\boldsymbol{P}$ is determined by (5) and (6), and $\boldsymbol{R}$ is defined by (7). Note that the matrix $\boldsymbol{Q}$ is a transition probability matrix for the Markov chains $\{s^{(i)}(t)\}$ for $i = 1, \ldots, K$.

Let $\psi(k, n, x)$ denote the probability that given that k MSs is in state 1, the number of minislots is equal to n and the feedback probability is equal to x, the k MSs fail to feed back. For $k = 0, \ldots, K-2$, $n = 1, 2, \ldots$ and $0 \leq x \leq 1$, $\psi(k, n, x)$ is given by

$$\psi(k, n, x) = [1 - k(1-x)^{k-1}x]^n.$$

We then define a $4(K-1) \times 4(K-1)$ diagonal matrix $\boldsymbol{D}(z)$ by

$$\boldsymbol{D}(z) = \mathrm{diag}(\boldsymbol{d}_{0,0}(z), \boldsymbol{d}_{0,1}(z), \boldsymbol{d}_{1,0}(z), \boldsymbol{d}_{1,1}(z)), \tag{10}$$

where $\boldsymbol{d}_{i,j}(z)$ $(i, j = 0, 1)$ is a $1 \times (K-1)$ vector given by

$$\begin{aligned} [\boldsymbol{d}_{0,0}(z)]_k &= \psi(k, N, u)\frac{z + z^{-1} + K - 2}{K} \\ &\quad +1 - \psi(k, N, u), \end{aligned}$$

$$\begin{aligned} [\boldsymbol{d}_{0,1}(z)]_k &= \psi(k+1, N, u)\frac{z + z^{-1} + K - 2}{K} \\ &\quad +(1 - \psi(k+1, N, u))\frac{z^{-1} + k}{k+1}, \end{aligned}$$

$$\begin{aligned} [\boldsymbol{d}_{1,0}(z)]_k &= \psi(k+1, N, u)\frac{z + z^{-1} + K - 2}{K} \\ &\quad +(1 - \psi(k+1, N, u))\frac{z + k}{k+1}, \end{aligned}$$

$$\begin{aligned} [\boldsymbol{d}_{1,1}(z)]_k &= \psi(k+2, N, u)\frac{z + z^{-1} + K - 2}{K} \\ &\quad +(1 - \psi(k+2, N, u))\frac{z + z^{-1} + k}{k+2}, \end{aligned}$$

for $k = 0, \ldots, K-2$. We further define $4(K-1) \times 4(K-1)$ matrix $\boldsymbol{C}(z)$ by

$$\boldsymbol{C}(z) = \boldsymbol{D}(z)\boldsymbol{Q}, \tag{11}$$

where $\boldsymbol{D}(z)$ and $\boldsymbol{Q}$ are defined by (10) and (9), respectively. Finally, we define $\eta_n(z)$ $(n = 1, 2, \ldots)$ by

$$\eta_n(z) = (\boldsymbol{r}^{(1)} \otimes \boldsymbol{r}^{(2)} \otimes \boldsymbol{r})\boldsymbol{C}(z)^n \boldsymbol{e},$$

where $\boldsymbol{r}^{(i)}$ denotes the initial state probability vector of the Markov chain $\{s^{(i)}(t)\}$ for $i = 1, 2$, respectively, $\boldsymbol{r}$ denotes the initial state probability vector of the Markov chain for MS i $(i = 3, \ldots, K)$, which is given by (8), $\boldsymbol{e}$ denotes a $4(K-1) \times 1$ vector whose elements are all equal to one, and $\boldsymbol{C}(z)$ is defined by (11).

We are now ready to present the analysis of the STAFI $G_n(x)$. Note that $\eta_n(z)$ can also be expressed in the power series of z as $\eta_n(z) = \sum_{l=-n}^{n} c_l z^l$, where c_l $(l = -n, \ldots, n)$ is a (unknown) real constant satisfying $0 \leq c_l \leq 1$ and $\sum_{l=-n}^{n} c_l = 1$. Then the probability mass function $g_n(x)$ is expressed as $g_n(x) = c_x + c_{-x}$. Thus, if we determine the unknown real constants $\{c_l\}_{l=-n}^{n}$, we obtain the probability mass function $g_n(x)$. The STAFI $G_n(x)$ is then given by $G_n(x) = \sum_{l=x}^{n} g_n(l) = 1 - \sum_{l=0}^{x-1} g_n(l)$.

There are several possible methods to determine the unknown real constants $\{c_l\}_{l=-n}^{n}$. In this paper, we use the *inverse discrete FFT method* [18] to determine them. Since $g_n(x)$ has a finite support, i.e., $g_n(x) = 0$ for $x > n$, we can calculate the exact value of $g_n(x)$ by using the inverse discrete FFT method.

For comparison, we consider a random scheduler which randomly selects a MS among K MSs irrespective of their received SNRs. For the STAFI of the random scheduler, we define $\tilde{\eta}_n(z)$ $(n = 1, 2, \ldots)$ by

$$\tilde{\eta}_n(z) = \left(\frac{z + z^{-1} + K - 2}{K}\right)^n,$$

which corresponds to $\eta_n(z)$ of the opportunistic feedback fair scheduler. Similar to the case of the opportunistic feedback fair scheduler, from $\tilde{\eta}_n(z)$, we can calculate the exact value the STAFI $\tilde{G}_n(x)$ and the probability mass function $\tilde{g}_n(x)$ for the random scheduler.

IV. Numerical results

In this section, we provide numerical results to investigate the properties of the STAFI of the opportunistic feedback fair scheduler. Throughout numerical results provided in this subsection, we set the parameters as $f_d = 10$ Hz and $T_f = 1$ msec where we decided these parameter values according to [15], [19]. In the numerical results provided in this paper, we also set the initial state probability vectors $\boldsymbol{r}^{(1)}$ and $\boldsymbol{r}^{(2)}$ of MS 1 and MS 2 to the stationary probability vector $\boldsymbol{s}$.

First, we observe the effect of the threshold γ_1 on the STAFI $G_n(x)$. Figure 2 displays the STAFI $G_{256}(x)$ of the opportunistic feedback fair scheduler as a function of x. In Figure 2, we set the number of MSs K, the number of minislots N and the feedback probability u to 30, 5 and 0.80, respectively. For comparison, Figure 2 also shows the STAFI $G_{256}(x)$ of the random scheduler. In the figures, "OFF(x dB)" means the opportunistic feedback fair scheduler whose threshold γ_1 is equal to x, and "RS" means the random scheduler.

In Figure 2, we observe the following. For whole range of x, the STAFIs $G_{256}(x)$ of the opportunistic feedback fair schedulers are greater than the STAFI $G_{256}(x)$ of the random scheduler. In other words, the short term fairness provided by the opportunistic feedback fair schedulers is worse than that provided by the random scheduler. This is due to the positive correlation of the normalized SNR process $\{z^{(i)}(t)/\bar{z}^{(i)}\}$ in

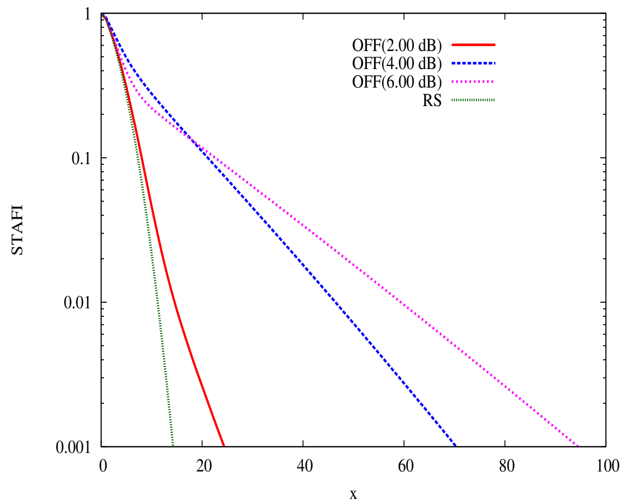

Fig. 2. Effect of γ_1 on STAFI $G_{256}(x)$ $(u = 0.80)$

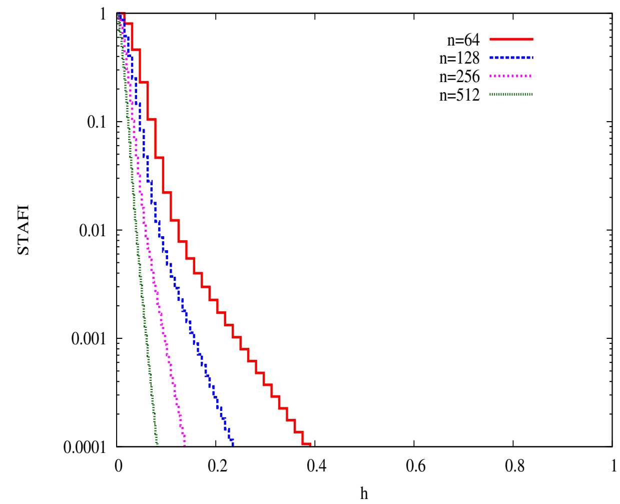

Fig. 4. STAFI $G_n(hn)$ as a function of h (γ_1 = 2.00 dB)

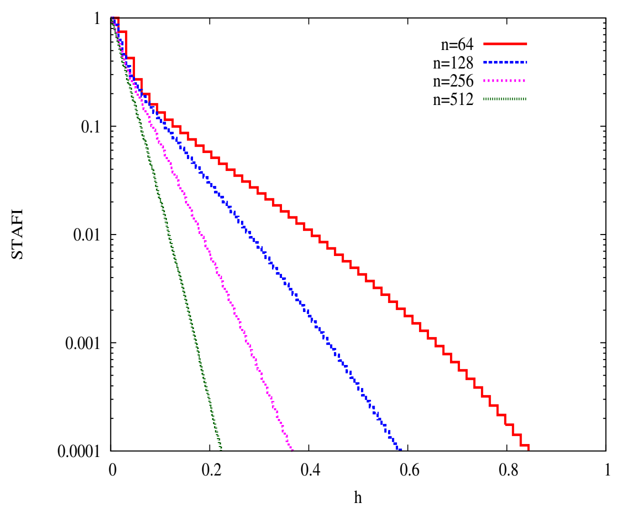

Fig. 3. STAFI $G_n(hn)$ as a function of h (γ_1 = 4.00 dB)

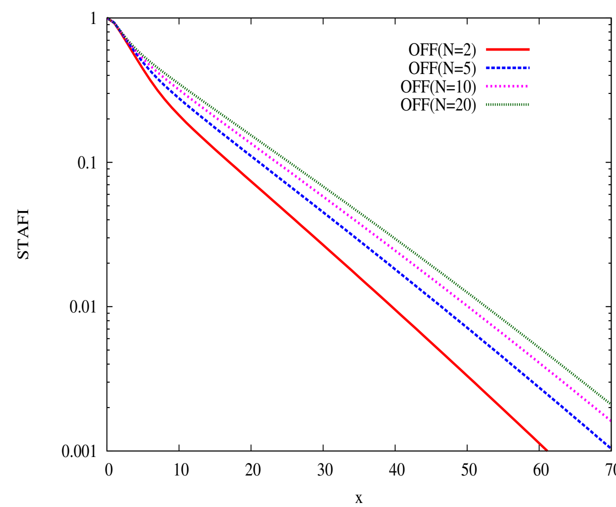

Fig. 5. Effect of number of minislots N on STAFI $G_{256}(x)$

time. We also see that for whole range of x, the OFF (2.00 dB) yields better short term fairness than the OFF (4.00 dB) and the OFF (6.00dB). Comparing the OFF (4.00 dB) and the OFF (6.00dB), we observe that for small x of $G_{256}(x)$, the OFF (6.00 dB) provides better fairness than the OFF (4.00 dB) However, the situation is converse for large x of $G_{256}(x)$. Thus, the OFF (6.00 dB) can keep the probability of moderate unfairness lower, but it can cause serious unfairness with higher probability, compared to the OFF (4.00 dB). A similar non-monotonous property about the threshold value has been observed for the one-bit feedback fair scheduler, too [12].

We next examine how the STAFI of the opportunistic feedback fair scheduler changes as the increase of observation period n. Figures 3 and 4 exhibit the STAFI $G_n(hn)$ as a function of h for $n = 64, 128, 256, 512$. In the figures, we set the number of MSs K, the number of minislots N and the feedback probability u to 30, 5, 0.8, respectively. We set the threshold γ_1 to 4.00 dB in Figure 3 and to 2.00 dB in Figure 4. In Figures 3 and 4, we observe that the STAFI $G_n(hn)$ of the opportunistic feedback fair scheduler rapidly decreases with increase of the observation period n for every h. In other words, the STAFI of the opportunistic feedback fair scheduler rapidly approaches to the ideal long term fairness as the progress of time.

Next, we observe the effect of the number of minislots N on the STAFI $G_n(x)$. Figure 5 displays the STAFI $G_{256}(x)$ of the opportunistic feedback fair scheduler as a function of x. In Figure 5, we set the number of MSs K, the threshold γ_1 and the feedback probability u to 30, 4.00 dB and 0.80, respectively. In the figures, "OFF(N=x)" means the opportunistic feedback fair scheduler where the number of minislots N is equal to x. In Figure 5, we observe that with the increase in the number of minislots N, the short term fairness of the

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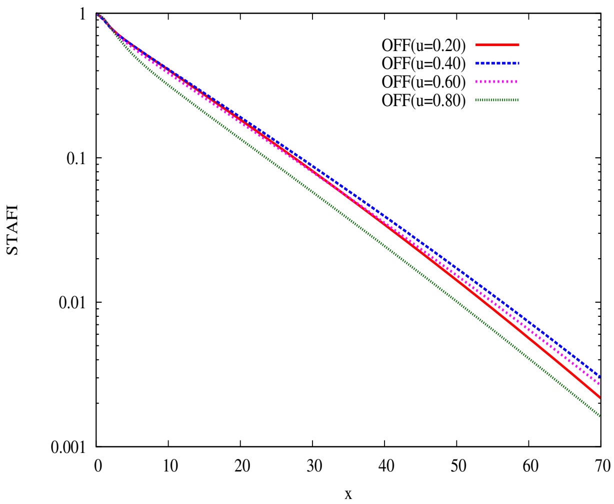

Fig. 6. Effect of feedback probability u on STAFI $G_{256}(x)$

opportunistic feedback fair schedulers becomes worse.

Finally, we observe the effect of the feedback probability u on the STAFI $G_n(x)$. Figure 6 displays the STAFI $G_{256}(x)$ of the opportunistic feedback fair scheduler as a function of x. In Figure 6, we set the number of MSs K, the threshold γ_1 and the number of minislots N to 30, 4.00 dB and 10, respectively. In the figure, "OFF(u=x)" means the opportunistic feedback fair scheduler where the feedback probability is equal to x. In Figure 6, we observe that among the four opportunistic feedback fair schedulers for $u = 0.2, 0.4, 0.6, 0.8$, the scheduler for $u = 0.4$ yields the worst short term fairness. When the feedback probability u is small, the short term fairness of the opportunistic feedback fair scheduler becomes worse with the increase in the feedback probability u. However, if the feedback probability is greater than a certain value, the short term fairness becomes better with the increase in the feedback probability.

V. Conclusion

In this paper, considering the STAFI as a measure of short term fairness, we studied the short term fairness provided by the opportunistic feedback fair scheduler. We developed a numerical method to to calculate the exact value of the STAFI by using the inverse discrete FFT method. In the numerical results, we observed that the threshold γ_1 strongly affects the properties of the short term fairness provided by the opportunistic feedback fair scheduler. The opportunistic feedback fair scheduler with larger threshold γ_1 can keep the probability of moderate unfairness lower, but is can cause serious unfairness with higher probability, compared to the opportunistic feedback fair scheduler with smaller threshold. The impacts of the number of minislots N and the feedback probability u on the properties of short term fairness do not seem to be so strong, compared to the effect of the threshold γ_1. We also observed that the STAFI of the opportunistic feedback fair scheduler approaches to the ideal fairness in a relatively short time period. However, if rigorous fairness is required even in a relatively short time period, we should carefully determine the threshold value γ_1 by considering the short term fairness of the scheduler as well as its information theoretic capacity.

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