On Throughput Characteristics of Type II Hybrid-ARQ with Decode and Forward Relay using Non-Binary Rate-Compatible Punctured LDPC Codes

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Abstract— **In this paper, an NB RCP LDPC (Non-Binary Rate-Compatible-Punctured Low Density Parity Check) code is designed over the extended Galois Field. The designed NB RCP LDPC code is applied to the type II HARQ (Hybrid Automatic Repeat reQuest) with Decode and Forward (DF) relay using MIMO-OFDM modulation. The designed code enables us to decrease the coding rate with incremental redundancy for each retransmission in HARQ. The retransmission is made by the DF relay after its successful decoding. We have verified through computer simulations that the proposed type II HARQ scheme with DF relay greatly improves the throughput and average retransmission characteristics compared with the scheme without DF relay.**

Keywords-NB RCP LDPC code; Hybrid-ARQ; Decode and Forward Relay; MIMO-OFDM; Symbol-LLR.

I. INTRODUCTION

An LDPC code which suits the flexible coding rate design and has the high error correcting capability through iterative decoding can be constructed on arbitrary extended Galois field. The Non-Binary (NB) LDPC code constructed on extended Galois field generally exhibits the better BER performance than the binary LDPC codes [1],[2]. There exist also Rate-Compatible-Punctured (RCP) LDPC codes with variable coding rate obtained by properly puncturing the mother LDPC code. The RCP LDPC codes enable us to use the same decoder as the mother code [3] and suit the ARQ (Automatic Repeat reQuest) error correcting schemes [4],[5] with the incremental redundancy. When comparing the HARQ using NB RCP LDPC codes with the existing RCPT (Rate Compatible Punctured Turbo) HARQ using binary Turbo codes [6], the HARQ with NB LDPC codes can cope with flexible coding rates, code word lengths and NB symbol LLR additions without using inter-leavers for burst errors on the channel. By combining the NB LDPC codes with the RCP codes, the NB RCP LDPC codes were designed and the designed NB RCP LDPC codes were applied to the type II HARQ [7],[8]. In this paper, the NB RCP LDPC coded type II HARQ with the MIMO-OFDM modulation is used for the Decode and Forward (DF) relay scheme [9],[10]. By using the DF relay, the source node can be replaced by the relay, once the relay correctly decodes the LDPC encoded packet from the source. This replacement from the source to the relay effectively reduces the number of retransmissions and improves the throughput performance very much. We have verified through computer simulations that the proposed DF relaying scheme with type II HARQ and RCP LDPC code greatly improves the throughput and average retransmission characteristics compared with the case without DF relay.

The paper is organized as follows. In Section II, the RCP LDPC code is introduced. In Section III, NB LDPC coded Type II HARQ scheme is described. In Section IV, we propose decode and forward relaying scheme. In Section V, we present the symbol LLR generation in OFDM demodulation. In Section VI, the computer simulation results are shown. The paper concludes with Section VII.

II. RCP LDPC CODE

The encoding and decoding procedure of RCP LDPC code is as follows. We call the code before puncture and the code after puncture as the mother code and the efficient code, respectively. In RCP LDPC code, the encoder and decoder of mother code can also be applied to the efficient code. When the parity check matrix of mother code is given by $\boldsymbol{H}_M\,(M \times N)$ and the generator matrix by $\boldsymbol{G}_M\,(N \times K)$ with $K = (N - M)$, the coding rate of mother code becomes $R_M = (1 - M/N) = K/N$. The coding rate after the puncture of p symbols from the mother code is given by $R_E = K/(N-P)$. We denote the message vector as $\boldsymbol{m} = (m_1, m_2, \ldots, m_K)$, the code word of mother code as $\boldsymbol{C_M} = (C_{M1}, C_{M2}, \ldots, C_{MN})$, the index of position to be punctured as $\boldsymbol{P} = (p_1, p_2, \ldots, p_p)$ and the code word of efficient code as $\boldsymbol{C}_E = (C_{E1}, C_{E2}, \cdots, C_{EN})$. The encoding procedure is first to generate the mother code by $\boldsymbol{C_M} = \boldsymbol{mG_M}$ and next to puncture the position using $\boldsymbol{P}$ to obtain $\boldsymbol{C}_E$. The decoding procedure is to produce the symbol LLR from the receive signal and it is fed to the mother code decoder as the initial value for the sum-product algorithm. The symbol LLR for the position $\boldsymbol{P}$ is initially set to 0, because there is no available symbol LLR corresponding to the position $\boldsymbol{P}$.

III. NB RCP LDPC CODED TYPE II HARQ SCHEME

In Fig. 1, we show the block diagram of NB RCP LDPC coded Type II HARQ scheme using MIMO-OFDM modulation. At the transmitter, the data bits are firstly encoded by the CRC-16 error detecting code and secondly encoded by the NB LDPC code on GF(4) or GF(16). The encoded LDPC code word is divided into the OFDM frames for making a packet for each transmission using the predetermined puncture table introduced in [3]. In Fig. 2, we show how to divide the coded symbols of an LDPC code to the OFDM frames. The encoded NB alphabets are mapped to QPSK signal points for GF(4) or 16QAM for GF(16). These signal points are then modulated by OFDM with guard interval insertion. The OFDM signal is then transmitted to the quasi-static frequency selective channel from each transmit antenna. At the receiver, for each antenna, the guard interval is first removed and then OFDM demodulation is made using FFT. By demodulating each subcarrier QPSK modulated or 16QAM modulated, the symbol LLR (Log Likelihood Ratio) is calculated. The symbol LLR will be defined in Section V. The symbol LLR values are then fed to the LDPC decoder and the iterative decoding using sum-product algorithm is made. The decoded information bits are error-detected by the CRC-16 code. If error is not detected, the data bits are fed to the data sink and the ACK is returned to the source node (transmitter) to finish the transmission. But if errors are detected, the NACK is returned and the retransmission is requested. As the type II Hybrid ARQ (HARQ) scheme is employed, at the first transmission, only the data symbols without encoding are sent to the receiver. After the 2nd transmission, as shown in Fig. 2, the parity symbols are sent several times with the incremental redundancy depending on the error detection status at the receiver. When the channel quality is good, the uncoded data packet for the first transmission succeeds with high probability leading to the high throughput performance. On the other hand, when the channel quality is bad, the parity packets are retransmitted several times till the LDPC code rate reaches the lowest one half resulting in enough error correction capability.

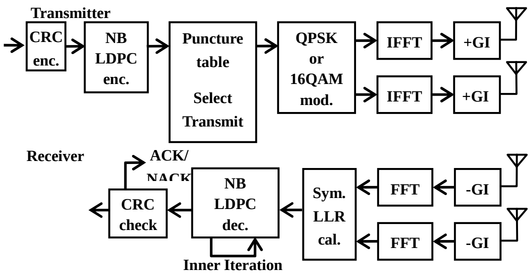

Fig. 1 Transmitter and receiver configuration of MIMO NB RCP LDPC coded type II HARQ scheme

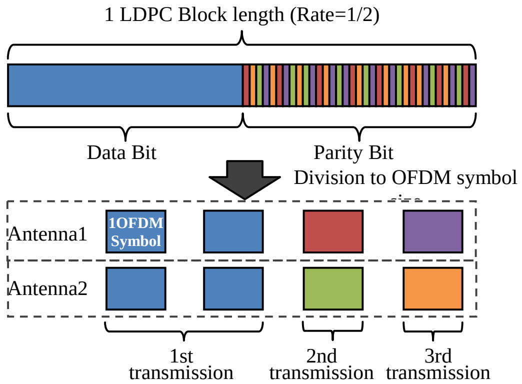

Fig. 2 OFDM frames obtained from the division of an LDPC code word

At the transmitter side, only one time of encoding process of RCP LDPC code is enough and there is no need of re-encoding process for decreasing the coding rate thereafter. Accordingly the complexity of encoding process of RCP LDPC code does not increase compared with the fixed code rate LDPC code. At the receiver side, the same decoder can be used for each coding rate, so there is no increase of complexity when compared with the fixed coding rate.

IV. DECODE AND FORWARD RELAYING SCHEME

The Decode and Forward relay model is shown in Fig. 3 and is composed of source, relay and destination. In this model, we assume the relay is located at the middle point on the straight line between the source and destination. When the receive power at the receiver attenuates in proportion to $1/d^{\alpha}$ where d is the distance between transmitter and receiver, the receive power from source to relay and the one from relay to destination become 2^{α} times larger than the one from source to destination. Next, we will illustrate the operation at each node in Fig. 3. At the source node, the encoding and modulation process using the NB RCP LDPC coded type II HARQ with MIMO OFDM is made, and uncoded and parity check packets are generated using the division of an LDPC code word as shown in Fig. 2. At the 1st transmission, the source broadcasts the uncoded data packet to the destination and the relay simultaneously. The relay and the destination receive the packet and they make the error detection independently using CRC-16 code. Then the destination and relay send the ACK or NACK back to the source, and the error detection results are shared among the source, relay and destination. When the destination returns ACK, the data are received successfully at the destination through only one transmission and this situation is equivalent to the case without relay. On the other hand, when the destination returns NACK, retransmission must be made. Moreover, when the relay also returns NACK, the parity check packet with incremental redundancy is retransmitted from the source. At the destination and the relay, the received parity check packet is combined with the data packet for the 1st transmission, and the combined LDPC code word is decoded. Then the decoded information bits are CRC checked and ACK or NACK is returned to the source. If the destination returns NACK but the relay dose ACK, then the relay receives the correct information bits. Accordingly, the relay can encode the received information bits to the same RCP LDPC code word with the source, i.e., we can replace the source by the relay for the subsequent ARQ retransmission. The parity check packet with incremental redundancy is generated at the relay and is sent from the relay afterwards. Once the relay successfully receives the packet from the source, then the relay performs the function of source and the source stops any further retransmission. This strategy is quite useful because the relay is closer to the destination than the source, thus the transmission error does not occur so frequently compared with the source.

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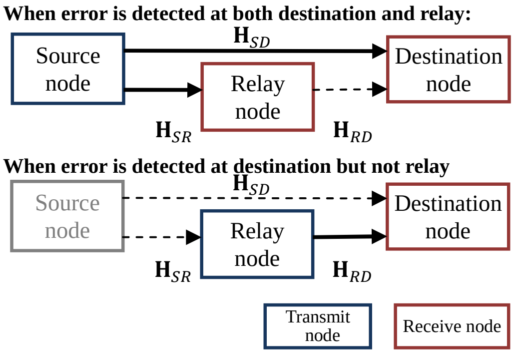

Fig. 3 DF (Decode and Forward) relaying model

Another point to be noticed is that the consumption of total transmit power remains the same as the one without the relay, because the source is replaced by the relay and the source does not consume any transmission power after the replacement.

V. SYMBOL LLR GENERATION IN OFDM DEMODULATION

As an example of symbol LLR calculation in OFDM demodulation, we show the case where the NB LDPC code on GF(4) is used and the QPSK modulation is employed for each subcarrier of OFDM. When the transmit signal, receive signal, signal points of QPSK and the subcarrier channel fading value are denoted as x, r, s_0, s_1, s_2, s_3 and h respectively, the symbol LLR for the alphabets $a = 0,1,2,3$ on GF(4) is defined as

$$
\begin{aligned}
LLR_a &= \log_e\left\{\frac{P(x=a|r\Delta r)}{P(x=0|r\Delta r)}\right\} = \log_e\left\{\frac{P(s_a, r\Delta r)/P(r\Delta r)}{P(s_0, r\Delta r)/P(r\Delta r)}\right\} \\
&= \log_e\left\{\frac{P(s_a, r\Delta r)}{P(s_0, r\Delta r)}\right\} = \log_e\left\{\frac{P(s_a)\,p(r\Delta r|s_a)}{P(s_0)\,P(r\Delta r|s_0)}\right\} \qquad (1) \\
&= \log_e\left\{\frac{p(r|s_a)\Delta r}{P(r|s_0)\Delta r}\right\} = \log_e\frac{p(r|s_a)}{P(r|s_0)}
\end{aligned}
$$

where the priori probabilities are set to $P(s_0)=P(s_1)=P(s_2)=P(s_3)=1/4$, i.e., equal probabilities. In (1), $P(x=a|r\Delta r)$ denotes the probability that the transmit symbol x equals a when the receive signal point r falls in the small area $r\Delta r$ centered at r. $p(r|s_a)$ is the transition probability density function from $s_a \to r$ and is expressed as

$$p(r|s_a) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{|r-hs_a|^2}{2\sigma^2}\right) \qquad (2)$$

Accordingly, the symbol LLR is calculated as

$$LLR_a = \log_e\left\{\frac{p(r|s_a)}{p(r|s_0)}\right\} = \log_e\left\{\frac{\frac{1}{2\pi\sigma^2}\exp\left(-\frac{|r-hs_a|^2}{2\sigma^2}\right)}{\frac{1}{2\pi\sigma^2}\exp\left(-\frac{|r-hs_0|^2}{2\sigma^2}\right)}\right\}$$

$$= \left(-\frac{|r-hs_a|^2}{2\sigma^2}\right) - \left(-\frac{|r-hs_0|^2}{2\sigma^2}\right) = \frac{|r-hs_0|^2 - |r-hs_a|^2}{2\sigma^2} \qquad (3)$$

VI. COMPUTER SIMULATION RESULTS

The BER characteristics of RCP LDPC code on AWGN channel are examined when the rate 1/2 mother code on GF(4) or GF(16) is punctured to change the coding rate. The simulation condition is listed in Table I and the simulation results are shown in Figs. 4 and 5. From the simulation results, we know that the efficient codes on GF(4) or GF(16) with different coding rates are obtained from a mother code and the error correction capability corresponding to each coding rate is achieved. Next, the throughput performance and the average number of retransmission characteristic for 4×4 MIMO-OFDM NB RCP LDPC coded Type II Hybrid-ARQ with GF(4) and QPSK modulation are investigated. We compared the proposed relay model with the one without relay. The simulation condition is listed in Table II. The simulation results for throughput characteristic are shown in Fig. 6 and Fig. 7. The simulation results for average number of retransmission are shown Fig. 10 and Fig. 11.

TABLE I Simulation condition of RCP LDPC code

Channel		AWGN	
Modulation		QPSK	16QAM
Size of Galois field		GF(4)	GF(16)
Mother code	Size of parity check matrix	(256,512)	(128,256)
	Average weight	(2.66,5.32)	(2.41,4.82)
	Coding rate	4/8	2/4
Efficient code	Information bit length	512	
	Coding rate	4/8,4/7,4/6,4/5, 4/4	2/4,2/3,2/2
Max SPA iteration		20	

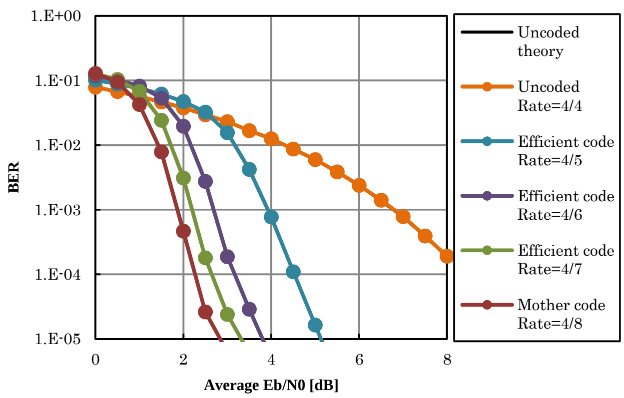

Fig. 4 BER characteristics of RCP-LDPC code on AWGN channel (QPSK)

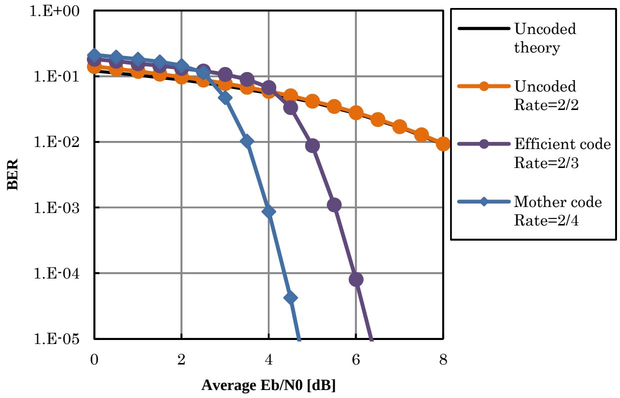

Fig. 5 BER characteristics of RCP-LDPC code on AWGN channel (16QAM)

In the simulation, an LDPC code word is divided into 32 OFDM symbols. As the coding rate of mother LDPC code is 1/2, the former 16 OFDM symbols are the information data symbols and the latter 16 OFDM symbols are the parity check symbols. For the 1st transmission, 16 OFDM symbols made from information data are transmitted from 4 transmit antennas simultaneously using 4 OFDM symbol duration. For the 2nd transmission and thereafter, i.e., retransmission, 4 OFDM symbols made from the parity check symbols are transmitted from 4 transmit antennas simultaneously. In each retransmission, 1 parity check OFDM symbol is transmitted from each antenna using 1 OFDM duration. The coding rate is decreased gradually from 4/5, 4/6, 4/7 to 4/8 for each retransmission. After all the parity check OFDM symbols are transmitted and the coding rate reaches 4/8=1/2, if the errors are still detected at the destination, the whole transmission of the same RCP LDPC code word in the same manner is repeated up to 3 times. The symbol LLR combining is used at the destination for the repeated reception of the same RCP LDPC code word. For the comparative scheme, we considered the type I HARQ with the fixed coding rate

TABLE II Simulation condition of NB GF(4) RCP LDPC coded type II Hybrid ARQ scheme with 4 × 4 MIMO-OFDM

Channel		Quasi-static equal power Rayleigh fading channel with 16 delay paths
Number of transmit and receive antennas		4×4
Power attenuation constant of channel		α = 3
Modulation		QPSK-OFDM
Size of Galois field		GF(4)
Mother code	Size of parity check matrix	(1024,2048)
	Average weights	(2.66,5.32)
	Coding rate	4/8
Efficient code	Length of information bits	2048
	Coding rates	4/4,4/5,4/6,4/7,4/8
Max SPA iteration		20
Number of OFDM subcarriers		64
GI length		16 (=*T*/4)
Delay interval		1 (=*T*/64)
Channel State Information (CSI)		Perfect at receiver
Error detection code		CRC-16 code

LDPC code and set the maximum number of repetition also to be 3. The packet combining at the destination is also used through the symbol LLR addition. Next, using the NB RCP LDPC code on GF(16) and with 16QAM modulation, we made the similar simulation to the above GF(4) with QPSK modulation. The simulation condition is given in Table III. The simulation results for throughputs are shown in Fig. 8 and Fig. 9. The simulation results for average number of retransmission are shown in Fig. 12 and Fig. 13. In these simulations, an LDPC code word is divided into 4 information data packets and 4 parity check packets. For each retransmission, the coding rate is decreased from 2/2, 2/3 to 2/4. When the all OFDM symbols are transmitted and the coding rate reaches 2/4=1/2, and if the error is still detected, the same RCP LDPC code word is repeatedly transmitted up to 3 times.

TABLE III Simulation condition of NB GF(16) RCP LDPC coded type II Hybrid ARQ scheme with 2 × 2 MIMO-OFDM

Channel		Quasi-static equal power Rayleigh fading channel with 16 delay paths
Number of transmit and receive antennas		2×2
Power attenuation constant of channel		α = 3
Modulation		16QAM-OFDM
Size of Galois field		GF(16)
Mother code	Size of parity check matrix	(128,256)
	Average weights	(2.41,4.82)
	Coding rate	2/4
Efficient code	Length of information bits	512
	Coding rates	2/2,2/3,2/4
Max SPA iteration		20
Number of OFDM subcarriers		64
GI length		16 (=*T*/4)
Delay interval		1 (=*T*/64)
Channel State Information (CSI)		Perfect at receiver
Error detection code		CRC-16 code

We compare the type II HARQ with type I HARQ in Fig. 6, 7, 8, and 9. As the type I HARQ scheme has the fixed coding rate, the throughput for each coding rate saturates to the certain value less than the maximum in high average receive E_b/N_0 region, while the throughput of type II HARQ approaches almost the maximum value of 8 (bits/sec/Hz) by adaptively changing the coding rate. The reason why the final throughput for type II HAQR is slightly less than 8 (bits/sec/Hz) is due to the use of CRC-16 code to detect the errors in information data. In type II HARQ, however, the parity check packet is sequentially retransmitted in responding to the NACK, so the number of retransmission becomes large compared with the type I HARQ. Also in type II HARQ, the iterative decoding of LDPC code is done for each retransmission of parity check packet, thus the decoding time tends to increase.

Next, we compare the cases with and without relay. When the average receive E_b/N_0 is high, the throughputs with and without relay are almost equal, but when the average receive E_b/N_0 is low, the throughput with relay is higher than without relay.

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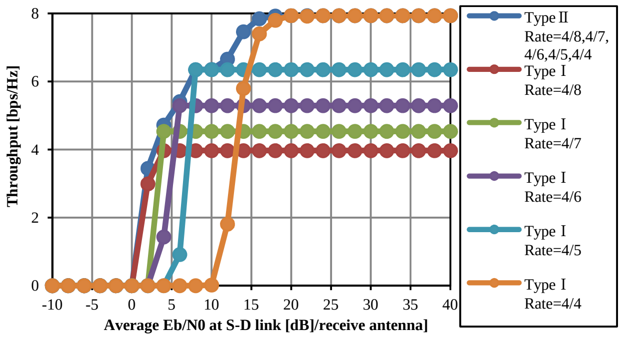

Fig. 6 Throughput characteristics of NB GF(4) LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 4×4, QPSK)

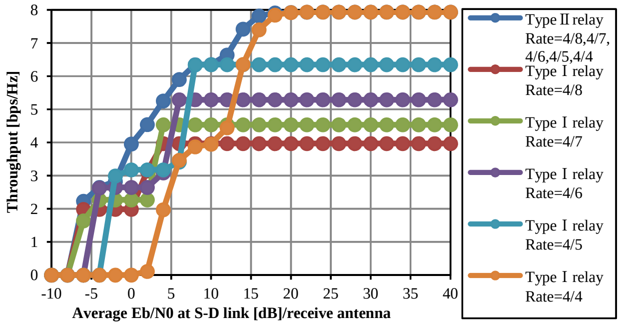

Fig. 7 Throughput characteristics of NB GF(4) LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with relay, 4×4, QPSK)

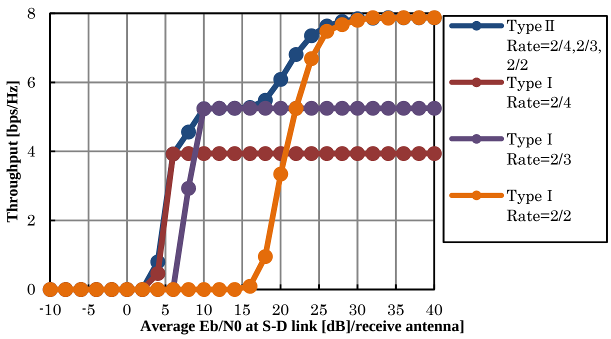

Fig. 8 Throughput characteristics of NB GF(16) LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, 16QAM)

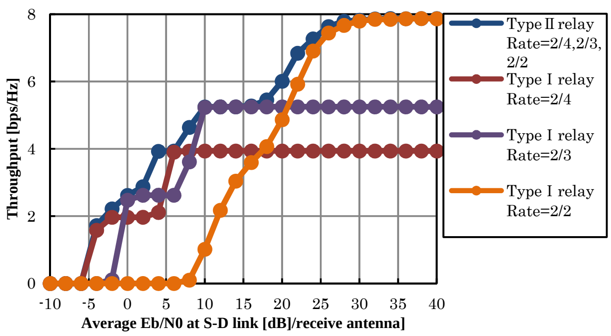

Fig. 9 Throughput characteristics of NB GF(16) LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with relay, 2×2, 16QAM)

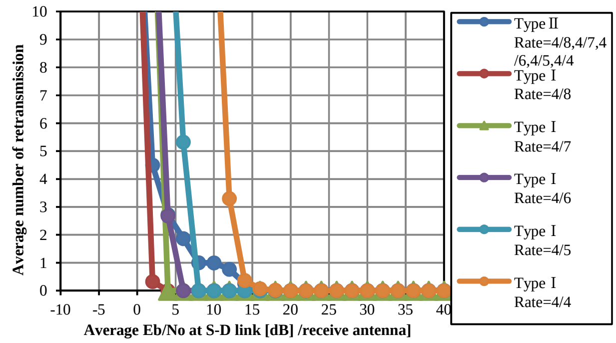

Fig. 10 Average number of retransmission of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 4×4, QPSK)

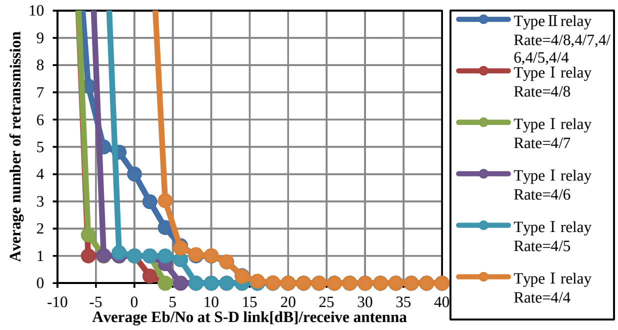

Fig. 11 Average number of retransmission of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with relay, 4×4, QPSK)

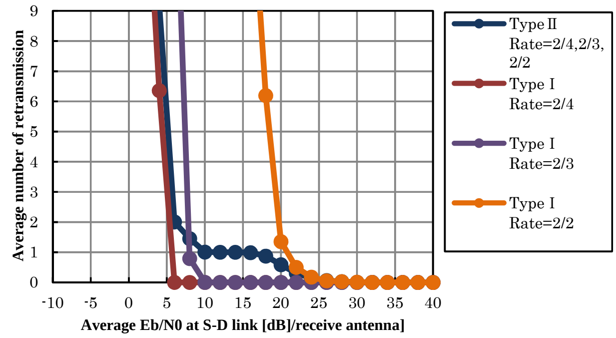

Fig. 12 Average number of retransmission of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, 16QAM)

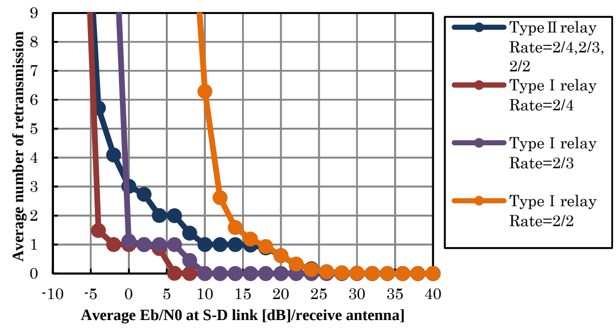

Fig. 13 Average number of retransmission of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with relay, 2×2, 16QAM)

This is because for the high average receive E_b / N_0 region, the destination can receive the packet correctly without retransmission. Accordingly, the relay is not used for this high E_b / N_0 region, so there is no difference between with and without relay. On the other hand, for the region where the average receive E_b / N_0 is low, the transmission from source to destination often fails, but the transmission from relay to destination succeeds with high probability, thus the retransmission is switched from the source to the relay for this low E_b / N_0 region. For the type I HARQ schemes in Fig. 7, Fig. 11, Fig. 9 and Fig. 13, we know that the throughput with relay is largely improved compared with the one without relay for the region where the average number of retransmission is 1. For this region the throughput of type I HARQ is almost one half of the throughput for high E_b / N_0 region. This means that for this region the transmission is switched from the source to relay and the retransmission from the relay to destination is almost successful. This observation proves that the use of relay is quite effective in HARQ.

As for the proposed type II HARQ, the throughput is larger than all the type I HARQ schemes and is optimum for all average receive E_b / N_0 values. However, the average number of retransmission becomes larger than the type I ARQ schemes because of the incremental redundancy retransmissions.

VII. CONCLUSIONS AND FUTURE WORKS

In this paper, we applied the NB RCP LDPC code to the type II HARQ scheme with Decode and Forward relay. We simulated the throughput and average retransmission characteristics and showed the effectiveness of NB RCP LDPC coded type II HARQ with relay. As for the modulation scheme, we considered MIMO-OFDM with QPSK and 16QAM. Quasi-static frequency selective Rayleigh fading channel is considered between each transmit and receive antenna. The relay is located in the middle between source and destination. In the proposed type II HARQ scheme, the first transmission is made without error correcting code, i.e., the information packet with CRC check is broadcasted to the relay and the destination. If the error is detected at destination, the parity check packet is retransmitted from the source. At the relay and the destination, the information packet and the parity check packet are combined and the LDPC decoding is done. If the error is not detected at the relay, but is detected at the destination, then the relay retransmits the parity check packet in place of the destination. This means that the source is replaced by the relay. If the error is still detected at the destination, the parity check packets are retransmitted from the relay several times with the incremental redundancy till the coding rate reaches 1/2. We clarified that by using the proposed HARQ relaying scheme, the higher throughput and the fewer average number of retransmissions are achieved for the low average receive S/N region comparing to without relay.

In the future study, we will investigate the scheme in which the transmitter knows the CSI, thus the redundancy of the parity check packet can be controlled by the transmitter, leading to fewer retransmissions.

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