Thermoacoustics Analysis in a Rijke Tube

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***Abstract*—This paper deals with the perturbation induced by the gauge pressure alongside a combustion chamber. This perturbation is modeled by acoustic pressure, measured at the end of the combustor. These vibration modes are obtained experimentally using Fourier Analysis. The analytic model of Pressure Distribution Equation (PDE) on can combustor is based on a wave equation. This paper describes the procedure that was carried out to obtain the temperature of a column of hot air inside a simple combustor. The Rijke tube was used with the aim of laying the foundations of a future thermoacoustic phenomenon analysis. The contribution of this work is spatial interpretation of pressure waves and displacement in a chamber during thermoacoustic phenomena.**

Keywords- Acoustics; Rijke tube; Thermodynamics, FFT analisys.

I. INTRODUCTION

The main indicator for acoustics phenomena is the excessive heat released that generates pressure oscillations in the combustion chamber. That relationship between heat and sound was discovered by Rayleigh [1] and formulated by Merkli [2]. Thermoacoustic instabilities can cause several problems in the fuel system such as efficiency degradation, premature wear of its components and even its catastrophic failure [3]. This phenomenon, in combination with high pressures and temperatures (including the incorrect fuel-air mixing process), produces highly polluting particles, such as NOx [4].

Nowadays, the state of the art related to thermoacoustic phenomena affecting gas turbines performance, has been mainly focused on some techniques as well as swirling flows [5] and [6]. Swirling flows provide aerodynamic stability to the combustion process by producing regions of recirculating flows that reduce the flame length and increase the residence time of the reactants in the flame zone [6]. Experimental analysis from combustion test rig using different kinds of injectors, constrictors, air-fuel mixes have been performed to find the best technique for combustion system [7]. Also, another analysis included premixed fluid and Helmholtz resonator [8].

The gas turbine combustion modelling and simulations and the acoustic phenomenon implies the necessity of implementing a control algorithm. The basic gas turbine model equations [9] are important for analysis, design and simulation of a control system especially for Combined Cycle Power Plants (CCPP) [10] and [11]. Focusing on acoustics, new passive and active control techniques for such instabilities have been studied and developed [12] as well as techniques of Adaptive Sliding Phasor Averaged Control (ASPAC: adapted the phase of the valve-commanded fuel flow variations) [13] and Multiscale extended Kalman (MSEK: predict the time-delayed states). These are promising techniques to reduce the energy consumption produced by pressure oscillations [14], but more research is required in this field.

The acoustic model proposed in this work has been planned to be the first part of a control algorithm to diminish the effect of the thermoacoustic phenomena into the can-combustor chamber. This research is carried out in CIDESI Queretaro, for the National Laboratory for Cooling Technologies Research (LaNITeF).

The rest of the paper is structured as follows. Section II presents the analytical model of pressure oscillation trough air columns. Section III presents a Rijke tube experiment. Fourier analysis is required to find the fundamental mode. Speed of sound and properties of the medium, such as density, are required to determine the parameter values of PDE. Section IV describes the results and validation of the model. Conclusions are presented in Section V. Section VI is related to future work.

II. ANALYTICAL MODEL OF PRESSURE OSCILLATION

Direct transformation between thermal energy and acoustic energy is called thermoacoustic phenomenon. Three conditions are required for this transformation to occur: (1) the medium must be a compressible fluid; (2) a temperature

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gradient must exist, and (3) the control volume must be contained by a physical border (chamber).

The wave equation is a linear second-order partial differential equation which describes the propagation of oscillations. The goal of this section is to establish a wave equation formulation in air column when thermoacoustic phenomena appear.

A. Stationary wave in air column

The wave equation for acoustic pressure is given by [15]

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{\kappa}{\rho_0}\frac{\partial^2 \xi}{\partial x^2} \tag{1}$$

where, ξ is the displacement in the air, κ is bulk modulus for the air and ρ_0 is the density of the air under equilibrium conditions. The wave equation solution for acoustic pressure can be solved via separation of variables. A standing wave is created by superposition of two waves which travel in opposite directions in the medium with the same amplitude, see equation (2).

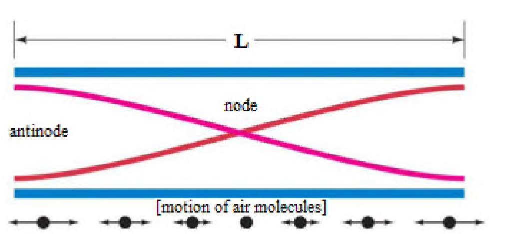
$$\xi(x,t) = 2\xi_0 \sin(kx)\cos(\omega t) \tag{2}$$

Index k corresponds to a wavenumber, ξ_0 is the signal amplitude and ω is the angular frequency. The Taylor'law for fluid [15] is used to formulate a standing waves pressure on air column.

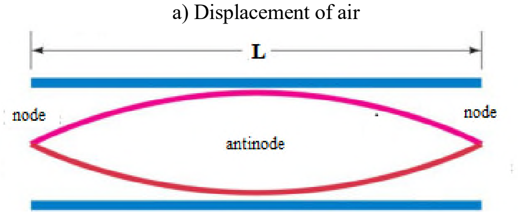
$$p - p_o = \kappa \frac{\partial \xi}{\partial x} \tag{3}$$

$$P(x,t) = -2\xi_0 \kappa k \cos(kx)\cos(\omega t) \tag{4}$$

Equation (4) describes the distribution of pressures with no boundary conditions. The boundary conditions are given by the pressure nodes appearing inside an open tube on both sides.



a) Displacement of air



b) Pressure variation in the air

Figure 1. Pressure and displacement for the air in the columns.

B. Pipe open at both ends

A tube open at both ends provides a physical border from air column. At the open ends, the reflection that occurs is a function of the extension of the tube and the opening, compared to the wavelength that propagates through the tube. The inside of the tube is too narrow, and it is not possible to dissipate all the energy in the open end. This is because the phenomenon of reflection takes place. The reflection causes nodes and antinodes of pressure and displacement, see Figure 1.

Displacement nodes are associated with motion of air molecules, or modal modes. Otherwise, each end of the column must be a pressure node because the atmosphere cannot allow significant pressure change [16]. Also, (3) shows the relationship between pressure and motion of the air.

The general one-dimensional equation that describes the pressure distribution of a gas contained in a cylindrical combustor is obtained considering boundary conditions:

1. $P(0,t) = 0$
2. $P(L,t) = 0$

Form (4), the PDE model has been formulated as:

$$P(x,t) = -2\kappa k\xi \cos(k(x + {}^{\pi}\!/_{2k}))\cos(\omega t) \tag{5}$$

From the boundary conditions evaluation, the obtained PDE coefficients are:

Condition 1:

$$\begin{aligned} P((0),t) &= -2\kappa k\xi \cos(k((0) + {}^{\pi}\!/_{2k}))\cos(\omega t) \\ 0 &= \cos({}^{\pi}\!/_{2})\cos(\omega t) \\ 0 &= 0 \end{aligned}$$

Condition 2:

$$\begin{aligned} P((L),t) &= -2\kappa k\xi \cos(k(({}^{\pi}\!/_{k}) + {}^{\pi}\!/_{2k}))\cos(\omega t) \\ 0 &= \cos({}^{3\pi}\!/_{2})\cos(\omega t) \\ 0 &= 0 \end{aligned}$$

The one-dimension PDE on can combustor is shown in (6).

$$P(x,t) = -2kc^2 \rho\xi \cos(k(({}^{\pi}\!/_{k}) + {}^{\pi}\!/_{2k}))\cos(\omega t) \tag{6}$$

III. RIJKE TUBE EXPERIMENT

Thermoacoustics is concerned on the interactions between heat (thermos) and pressure oscillations in gases (acoustics). A "Rijke Tube", named after its inventor, is a fundamental tool for studying the thermoacoustic phenomenon. Rijke's tube turns heat into sound by creating a self-amplifying standing wave. This open cylinder resonator contains a metallic copper mesh positioned about one-fifth of the way up the tube. When the screen is heated in a burner flame and then moved to one side of the flame, it will produce a strong tone at its resonant frequency for several seconds.

An experiment was carried out using the Rijke tube [11] which shows the presence of a thermoacoustic phenomenon, as depicted in Figure 2. A steel pipe of 0.6 m long and a diameter of 0.04445 m (1 ¾ in) and thickness 0.00121 m (18 gauge) was used, as illustrated in Figure 1. x_m is the distance (0.12m) where a metallic mesh was placed.

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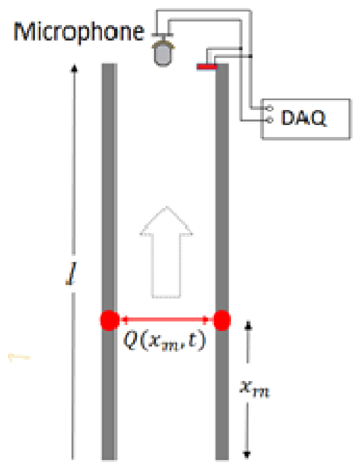
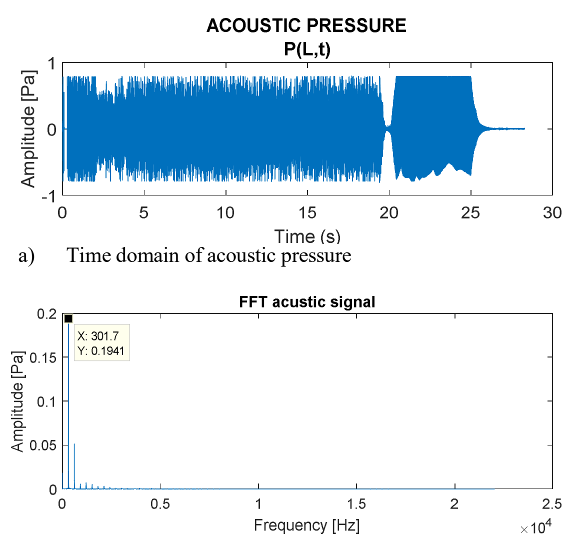


Figure 2. Data adquisition for Rijke tube experiment.

Rijke tube turns heat into sound by creating a self-amplifying standing wave. The heat source was provided by a gas thorn. The gas thorn adds energy to the system until the metallic mesh temperature rises to 600 °C. Then, the heat source is removed and the thermoacoustic phenomenon appears. It happens when the position of the pipe is vertical.

A. *Fourier analysis*

Time domain is the analysis physical signals of information with respect to time. It is beneficial when observing data such as temperature. However, some applications require analyzing the frequency components of signals, such as pressure oscillation of the air.



a) Time domain of acoustic pressure

b) Frequency domain of acoustic pressure

Figure 3.Signal analysis of acoustic pressure.

The acoustic pressure was measured using a microphone during the Rijke tube experiment. The same performance as in [17] was present. Time domain acoustic signal does not allow to see much information for analysis. A Fourier analysis was required. A Fast Fourier Transform (FFT) is an algorithm that samples a signal over a time (or space) period and divides it into its frequency components. FFT helps to find the fundamental frequency of the produced air column resonance.

The sampling frequency for the thermoacoustic test was 44100 samples per second. Fast Fourier Transform was applied with a resolution of 15 bits. Frequency resolution was about 1.34 Hz, which means that every 743 ms the algorithm takes a package sample to analyze.

Time-Frequency Signal Analysis Acoustic pressure was measured using a microphone. The signal for amplitude vs time is shown in Figure 3. The first modal mode occurs at 301 Hz with an amplitude of 0.1941 Pa.

B. *Speed of sound of the air*

The speed of sound is only affected by changes in the medium (density, humidity, temperature, etc.). For our case study, the element that changed significantly was the temperature.

On the other hand, the geometry of the combustor determines related terms like frequency and wavelength. The symbol λ is the length of the wave, that is, the space that the wave travels in a cycle. Figure 4 illustrates a reflection wave on pipe open ends. As both the output wave (green) and the reflected wave (red) only perform half a cycle inside the tube, the tube length is half the wavelength.

Consider just the motion of perturbation without the properties of the medium. The fundamental frequency is inversely proportional to the length of the tube.

$$c = \lambda f \tag{7}$$

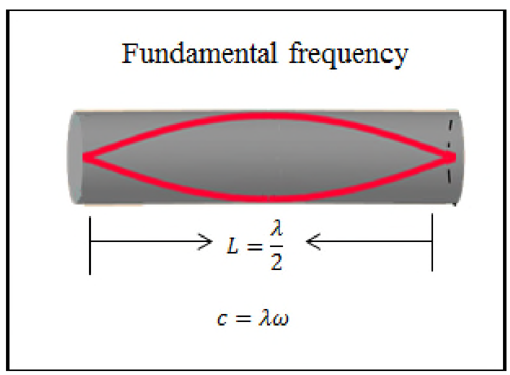


Figure 4. First vibration mode in air column.

In an ideal tube, the wavelength of the sound produced is directly proportional to the length of the tube. A tube which is open at one end and closed at the other produces sound with a wavelength equal to four times the length of the tube. In acoustics, end correction is a short distance applied or added to the actual length of a resonance pipe, to calculate the precise resonance frequency of the pipe. The pitch of a

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real tube is lower than the pitch predicted by the simple theory.

Equation (3) describe the end correction for pipe given in (8).

$$\lambda = 2(L + 0.61D) \tag{8}$$

The speed of sound result was 379.18 m/s.

C. *Temperature of the air*

Temperature is a physical quantity produced by motion of the molecules, which gives a perception of hot and cold. Temperature is associated with friction. Molecular friction is associated with pressure and density in the air. The ideal gas law is a good approximation of the behavior of many gases under many conditions, although it has several limitations.

The most frequently form of a state equation is given in (9).

$$\frac{P}{\rho} = \frac{RT}{M} \tag{9}$$

The acoustic wave velocity, c, depends on the material properties. Speed of sound is proportional to bulk modulus and inversely proportional to density of the air as dictated in (1).

$$\frac{\kappa}{\rho} = c^2$$

where $\kappa = \gamma P$ and γ is the heat capacity ratio. There is a strong relationship between pressure of the air, density of the air, temperature of the air and speed of sound [18] shown in (10).

$$\frac{\gamma P}{\rho} = c^2 = \frac{\gamma RT}{M} \tag{10}$$

Equation (10) infers that temperature can be expressed in terms of speed of sound.

$$T_{air} = \left(\frac{c_{air}}{20.055 \frac{m}{s \cdot K^{1/2}}} \right)^2 \tag{11}$$

Equation (11) was defined using nominal values of air.

The density of a substance is its mass per unit volume. The density can be changed by changing either the pressure or the temperature. Increasing the temperature generally decreases the density.

There are table properties of the air which have the density of the air in terms of temperature.

TABLE I. TEMPERATURE AND DENSITY OF THE AIR

Velocidad del sonido (m/s)	T (°K)	T(°C)	ρ [Kg/m^3]
379.18	357.48	84.48	0.9870

Table 1 shows the density of the air due to gradient of temperature. PDE variables are found.

IV. RESULTS

The one-dimension PDE on can combustor was formulated. The determination of parameter values was described in this paper.

$$P(x,t) = -2kc^2 \rho \xi \cos(k((\pi/k) + \pi/2k)) \cos(\omega t)$$

Pressure distribution along the can combustor is shown in Figure 5. This paper becomes useful for several combustor chambers with similar geometries.

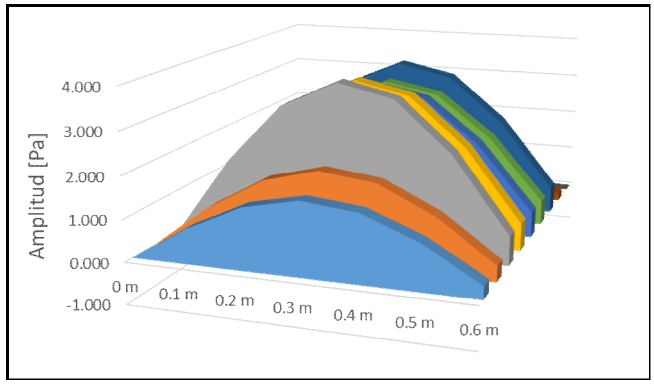

Figure 5. Fundamental mode of vibration in air column.

The result of the gradient of air temperature, within a finite volume and under conditions of natural convection, was the manifestation of the thermoacoustic phenomenon. The phenomena presence was used to validate the PDE model for the pressure oscillation inside the Rijke tube.

Advanced FFT Spectrum Analyzer was used to verify in real-time how the signal has grown. This is depicted in Figure 6.

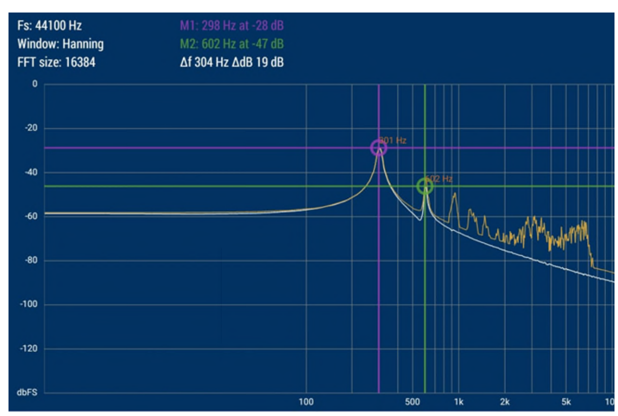

Figure 6. Fundamental frecuency measure on Rijke tube experiment.

It is demonstrated with the gas, due to its expansion and compression ratio, it will generate high pressure areas whose

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quality is directly proportional to the temperature of the medium. The melting temperature of the metal gauze oscillates around 600 °C at t=0 so convectively transferred the thermal energy and change the properties of the medium. The homologated temperature of the column was 78 ° C.

It was found experimentally that the speed of sound has been affected by the temperature change in the medium that propagates the disturbance, which was 379 m / s. Following the relationship in (1), we have the fundamental frequency of the gaseous fluid contained in a cylindrical geometry.

V. CONCLUSION

The propagation of acoustic oscillation in a compressive medium (air in this case), can be induced by applying thermal energy to the chambers' surface. The thermodynamic relationship between pressure, density and temperature has been verified for a single geometry combustor chamber, as well for this study in case of the Rijke tube.

The PDE model for the dynamic behavior of the pressure oscillation into a Rijke tube synthesized in this work was validated by predicting the first mode frequency of the air column pressure inside.

For thermoacoustic refrigeration, it is very important to know fluid dynamics and thermal behavior due to pressure oscillation.

VI. FUTURE WORK

The PDE model synthetized for the pressure distribution dynamics inside the chamber complemented in (11) to know the temperature behavior in the same volume, will be the basis of a control algorithm to regulate the thermal energy in the chamber surface to avoid or mitigate the effect of thermoacoustics.

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