Exact Error Probabilities Analysis of Arbitrary 2-D Modulation-OFDM Systems with I/Q Imbalances in Frequency-Flat Rayleigh Fading Channel

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***Abstract*—In OFDM systems, in-phase and quadrature (I/Q) imbalances generated in the analog front-end introduce inter-channel interference and, consequently, error performance degradation. This paper provides an exact expression involving the two-dimensional (2-D) Gaussian Q-function for the error probability of an arbitrary 2-D modulated OFDM signal with I/Q imbalances in frequency-flat Rayleigh fading channel.**

***Index Terms*—I/Q imbalance, Probability of error, Inter-channel interference**

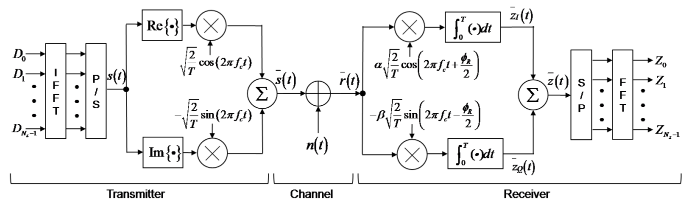

Fig. 1. OFDM transmitter and receiver with I/Q imbalances.

I. Introduction

In-phase and quadrature (I/Q) amplitude and phase imbalances are inevitably caused by signal processing in the analog components such as I/Q mixers, phase shifters, filters, and analog/digital converters within I/Q branches. In the implementation of a modern wireless communication system, I/Q imbalances act as one of the main impairments degrading system performance. Particularly in the orthogonal frequency division multiplexing (OFDM) schemes adapted in a number of wireless communication systems such as DAB, DVB-T, WLAN (802.11 a/g/n), WPAN (802.15.3a), WMAN (802.16 a/d/e), and MBWA (802.20), I/Q imbalances introduce inter-channel interference (ICI) and nonlinearly distort the baseband signals [1], [2]. The effects of I/Q imbalances on OFDM system performance have been analyzed by computer simulations, and several compensation techniques have been reported in many places in the literature [3]-[6].

In general, for a single-carrier system, I/Q imbalances in the receiver lead to a correlation between I/Q branches, and the correlation is revealed as the variations of the received signal points and the noise distribution on the constellation [7], [8]. For a single-carrier system, a method was recently provided to exactly analyze the effect of I/Q imbalances generated in the analog front-end of the receiver on the error performance [9]. In this paper, we derive an exact expression for the error probability of an arbitrary 2-D modulated signal with I/Q imbalances in an OFDM system over frequency-flat Rayleigh fading channel. Through a computer simulation, we verify the validity of the result obtained from the derived expression.

II. System Model

Fig. 1 shows a typical OFDM transceiver where we assume, for simplicity of analysis, I/Q amplitude and phase imbalances take place in the receiver side. As shown in Fig. 1, the OFDM transmitter undergoes an ideal complex up-conversion, but the received signals are affected by I/Q imbalances in the analog front-end of the receiver.

Assume that $D_k \in \mathbf{t\_s} = [t\_s_1 \; t\_s_2 \; ... \; t\_s_M]$ is the complex symbol modulated by an arbitrary 2-D M-ary modulator, and is transmitted on subcarrier k in the OFDM system, where $\mathbf{t\_s}$ is a set of M signal points transmitted through the OFDM system. Then, D_k can be defined as

$$D_k = \zeta_{i,k}\sqrt{E_s}e^{j\psi_{i,k}}, k = 0, 1, ..., N_s - 1, i = 1, 2, ..., M \quad (1)$$

where $\psi_{i,k}$ and $\zeta_{i,k}\sqrt{E_s}$ are the phase and the amplitude of the i-th signal point transmitted on subcarrier k, E_s is the average symbol energy, $\zeta_{i,k}$ is a scale factor which varies with the position of the signal point, and N_s is the number of subcarriers.

The ideally up-converted transmitted signal at the transmitter is formally given by

$$\overline{s}(t) = \frac{1}{\sqrt{2T_s}}\left(s(t)\,e^{j2\pi f_c t} + s^*(t)\,e^{-j2\pi f_c t}\right) \quad (2)$$

where $s(t) = \sum_{k=0}^{N_s-1} D_k e^{j2\pi kt/T_s}$ is a baseband OFDM signal and $s^*(t)$ denotes the complex conjugate of $s(t)$.

After the received bandpass signal, $\overline{r}(t) = \overline{s}(t) + n(t)$, is down-converted by mixing with $x_{LO}^{unbalanced}(t)$, the baseband signal, $\overline{z}(t)$, distorted by I/Q imbalances in time domain, is expressed as

$$\begin{aligned} &\overline{z}(t) = \overline{z}_I(t) + j\,\overline{z}_Q(t) \\ &= K_1 s(t) + K_2 s^*(t) + \sqrt{\tfrac{2}{T_s}}\cdot \\ &\quad \left(\int_0^T K_2 n(t)\, e^{j2\pi f_c t} dt + \int_0^T K_1 n(t)\, e^{-j2\pi f_c t} dt\right) \\ &= K_1 s(t) + K_2 s^*(t) + n_I + j n_Q \end{aligned} \tag{3}$$

where $K_1 = (\alpha e^{-j\frac{\varphi_R}{2}} + \beta e^{j\frac{\varphi_R}{2}})/2$ and $K_2 = (\alpha e^{j\frac{\varphi_R}{2}} - \beta e^{-j\frac{\varphi_R}{2}})/2$ are imbalance coefficients. In (4), $n(t)$ is an additive Gaussian noise with zero mean and variance of σ^2; n_I and n_Q are noise components on I/Q branches, and can be expressed as

$$\begin{aligned} n_I &= \tfrac{1}{\sqrt{T_s}} \alpha \int_0^T n(t)\sqrt{2}\cos\left(2\pi f_c t + \tfrac{\varphi_R}{2}\right) dt \\ n_Q &= \tfrac{1}{\sqrt{T_s}} \beta \int_0^T n(t)\left(-\sqrt{2}\sin\left(2\pi f_c t - \tfrac{\varphi_R}{2}\right)\right) dt \end{aligned} \tag{4}$$

where α and β are the amplitude gains on the I/Q branches, which represent the amplitude imbalances, and φ_R is the deviation from the perfect phase quadrature, which represents the phase imbalance. We assume the condition of $\alpha^2+\beta^2 = 2$ to leave the signal power unchanged, and define gain ratio, γ, and amplitude imbalance, ε, as follows [10]:

$$\gamma = \alpha/\beta\ , \qquad \varepsilon = \gamma - 1. \tag{5}$$

Note that n_I and n_Q have joint Gaussian distribution with zero mean, $E[n_I^2] = \alpha^2\sigma^2$, $E[n_Q^2] = \beta^2\sigma^2$, and $E[n_I n_Q] = \rho_{IQ}\sigma^2$, where $\rho_{IQ} = \alpha\beta\sin\varphi_R$ denotes the correlation coefficient between I/Q axes.

A noise distribution has an elliptical shape before FFT, which means that noise components on the I/Q axes are correlated. After FFT, the complex symbol Z_k, passed through the FFT block, can be expressed as

$$\begin{aligned} Z_k &= FFT\left[K_1 s(t) + K_2 s^*(t) + n_I + j n_Q\right] \\ &= K_1 D_k + K_2 D^*_{N_s-k} + N_{I\_k} + j N_{Q\_k}, \\ &\quad k = 0, 1, ..., N_s - 1 \end{aligned} \tag{6}$$

where N_{k_I} and N_{k_Q} are the noise components of subcarrier k on I/Q branches, which have joint Gaussian distribution with zero mean, $E[N_{k\_I}^2] = E[N_{k\_Q}^2] = \sigma^2$, and $E[N_{k\_I} N_{k\_Q}] = 0$. Note that noise components on the I/Q axes become uncorrelated after FFT. From (6), we also note that D_{N_s-k} causes ICI to D_k [5].

Substituting (1), K_1 and K_2 into (6), we can obtain

$$\begin{aligned} Z_k &= S_{k\_I} + j S_{k\_Q} + N_{k\_I} + j N_{k\_Q}, \\ &\quad k = 0, 1, \ \dots\ , N_s - 1 \end{aligned} \tag{7}$$

where S_{k_I} and S_{k_Q} are the received signal components on I/Q axes, expressible as

$$\begin{aligned} &S_{k\_I} \\ &= \tfrac{\zeta_{i,k}\sqrt{E_s}}{2}\left(\alpha\cos\left(\psi_{i,k} - \tfrac{\varphi_R}{2}\right) + \beta\cos\left(\psi_{i,k} + \tfrac{\varphi_R}{2}\right)\right) \\ &\quad + \tfrac{\zeta_{m,N_s-k}\sqrt{E_s}}{2}\left(\alpha\cos\left(\psi_{m,N_s-k} - \tfrac{\varphi_R}{2}\right)\right. \\ &\qquad \left. -\beta\cos\left(\psi_{m,N_s-k} + \tfrac{\varphi_R}{2}\right)\right) \\ &S_{k\_Q} \\ &= \tfrac{\zeta_{i,k}\sqrt{E_s}}{2}\left(\alpha\sin\left(\psi_{i,k} - \tfrac{\varphi_R}{2}\right) + \beta\sin\left(\psi_{i,k} + \tfrac{\varphi_R}{2}\right)\right) \\ &\quad + \tfrac{\zeta_{m,N_s-k}\sqrt{E_s}}{2}\left(-\alpha\sin\left(\psi_{m,N_s-k} - \tfrac{\varphi_R}{2}\right)\right. \\ &\qquad \left. +\beta\sin\left(\psi_{m,N_s-k} + \tfrac{\varphi_R}{2}\right)\right), \\ &\qquad i = m = 1, 2, ..., M \end{aligned} \tag{8}$$

where ψ_{m,N_s-k} and $\zeta_{m,N_s-k}\sqrt{E_s}$ are the phase and the amplitude of the m-th signal point on subcarrier $(N_s - k)$, respectively. Note that the second terms of S_{k_I} and S_{k_Q} induce the ICI, that is, the complex symbol transmitted on subcarrier $(N_s - k)$ interferes with the complex symbol transmitted on subcarrier k.

III. Exact Error Probability Expression for an Arbitrary 2-D Modulated OFDM Signal with I/Q Imbalances

In this section, we derive the exact SER/BER expressions for an arbitrary 2-D modulated OFDM signal with I/Q imbalances in AWGN, frequency flat Rayleigh fading, and frequency selective Rayleigh fading channels, respectively.

A. AWGN Channel

Fig. 2 shows the geometry of the correct decision region R_{s_1} for the received signal, $r_{-}s_1$, when the signal transmitted on subcarrier k is $t_{-}s_1$. This geometry acts as a basic shape for the evaluation of exact error probability for I/Q unbalanced case [9]. As shown in Fig. 2, the distortion due to ICI which arises from I/Q imbalances results in a shift of the received signal point on the constellation. In Fig. 2, $S_k|t_{-}s_i$ denotes the transmitted signal on subcarrier k interfered by the signal $t_{-}s_i$ transmitted on subcarrier $(N_s - k)$. I/Q noise components (N_{k_I}, N_{k_Q}) added to the transmitted signal also change the position of the received signal points, $r_{-}s_i,\ \ i = 1, 2, ..., M$.

To obtain the conditional probability $P_k\{r_{-}s_1 \in R_{s_1} | S_t = t_{-}s_1\}$ that the signal point $r_{-}s_1$ received through subcarrier k falls into a shaded region $R_{s_1} = X_1^- C X_2^+$, which represents the correct region for $t_{-}s_1$ in Fig. 2, we use the coordinate rotation and shifting technique well explained in [9] as follows:

$$\begin{bmatrix} X_j \\ Y_j + d_j \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix}, \quad j = 1, 2 \tag{9}$$

where $d_j,\ j = 1, 2$ are the distances from the origin to the $X_j,\ j = 1, 2$ axes which denote the decision boundaries; I and Q have joint Gaussian distribution with $E[I] = S_{k\_I}$, $E[Q] = S_{k\_Q}$, $Var[I] = Var[Q] = \sigma^2$, and $COV[I\,Q] = 0$. After rotational transformation (9), Y_1 and Y_2 which are axes newly made by the coordinate rotations with $-\theta_1$ and θ_1 from Q axis have joint Gaussian probability density function (pdf)

 ISBN:978-1-61208-113-7

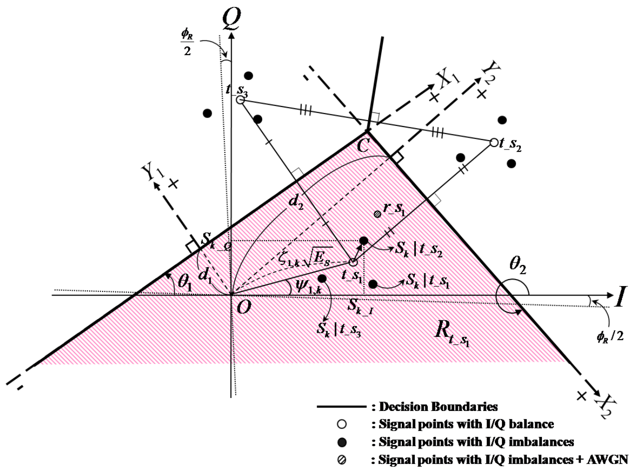

Fig. 2. Decision region and boundaries for ternary signal points.

$f(y_1, y_2, \rho_{Y_1Y_2})$ with

$$\begin{cases} E[Y_j] = S_{k\_Q}\cos\theta_j - S_{k\_I}\sin\theta_j - d_j, \quad j = 1, 2 \\ Var\left[Y_j\right] = \sigma^2 \\ \rho_{Y_1Y_2} = \cos\left(\theta_1 - \theta_2\right) \end{cases} \tag{10}$$

where $Var[Y_j]$ is the variance of Y_i; $\rho_{Y_1Y_2}$ is the correlation coefficient between Y_1 and Y_2.

Consequently, the conditional probability $P_k\{r\_s_1 \in R_{t\_s_1}|S_t = t\_s_1\}$ in Fig. 2 can be obtained as

$$\begin{aligned} &P_k\left\{r\_s_1 \in R_{t\_s_1}|S_t = t\_s_1\right\} = P_k\left\{Y_1 \leq 0,\, Y_2 \leq 0\right\} \\ &= \int_{-\infty}^{0}\int_{-\infty}^{0} f\left(y_1, y_2, \rho_{Y_1Y_2}\right) dy_2 dy_1 \\ &= \int_{-\infty}^{-\frac{E[Y_1]}{\sqrt{Var[Y_1]}}}\int_{-\infty}^{-\frac{E[Y_2]}{\sqrt{Var[Y_2]}}}\left[2\pi\sqrt{1-\rho_{Y_1Y_2}^2}\right]^{-1}\cdot \\ &\qquad \exp\left[-\frac{1}{2}\left(\frac{u^2 - 2\rho_{Y_1Y_2}uv + v^2}{1-\rho_{Y_1Y_2}^2}\right)\right] dvdu \\ &= Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_2]}{\sqrt{Var[Y_2]}}; \rho_{Y_1Y_2}\right). \end{aligned} \tag{11}$$

Finally, since the baseband signal transmitted on subcarrier $(N_s - k)$, which causes ICI to the signal transmitted on subcarrier k in the receiver, is one of the M signals, $t\_s_i,\ i = 1, 2, ..., M$, the average SER for a signal point t_s_1 transmitted on subcarrier k can be written as

$$\begin{aligned} &P_{ser\_t\_s_1} \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[\left(1 - P_k\left\{r\_s_1 \in R_{t\_s_1}|S_t = t\_s_1\right\}\right)P\{t\_s_1\}\right] \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[\left(1 - Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_2]}{\sqrt{Var[Y_2]}}; \rho_{Y_1Y_2}\right)\right)P\{t\_s_1\}\right] \end{aligned} \tag{12}$$

where $P\{t\_s_1\}$ is *a priori* probability for the transmitted signal point, and m denotes an index of the signal point transmitted on subcarrier $(N_s - k)$ which causes interference, as in (8).

In general, the decision region of a transmitted signal point is a polygon that may be either closed or open [12], and the decision region can be expressed as a linear combination of the basic shapes [9], [13]. Therefore, an exact error probability expression for the signal point with the polygonal decision region can be obtained by using the probability of (12).

When a signal point t_s_1 that has a closed region case with u-sided polygonal shape is transmitted, the SER for the signal point is derived as

$$\begin{aligned} &P_{ser\_t\_s_1}^{c} \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[\left(1 - P_k\left\{r\_s_1 \in R_{t\_s_1^c}|S_t = t\_s_1^c\right\}\right)P\left\{t\_s_1^c\right\}\right] \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[\left\{1 - \left(Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_2]}{\sqrt{Var[Y_2]}}; \rho_{Y_1Y_2}\right)\right.\right.\right. \\ &\qquad + Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_4]}{\sqrt{Var[Y_4]}}; \rho_{Y_1Y_4}\right) \\ &\left.\left.\left. - \sum_{i=2}^{u-1} Q\left(\frac{E[Y_i]}{\sqrt{Var[Y_i]}}, -\frac{E[Y_{i+1}]}{\sqrt{Var[Y_{i+1}]}}; -\rho_{Y_iY_{i+1}}\right)\right)\right\}P\left\{t\_s_1^c\right\}\right]. \end{aligned} \tag{13}$$

And, when a signal point t_s_1 that has a open region case with v-sided polygonal shape is transmitted, the SER for the signal point is derived as

$$\begin{aligned} &P_{ser\_t\_s_1}^{o} \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[\left(1 - P_k\left\{r\_s_1 \in R_{t\_s_1^o}|S_t = t\_s_1^o\right\}\right)P\left\{t\_s_1^o\right\}\right] \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[\left\{1 - \left(Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_2]}{\sqrt{Var[Y_2]}}; \rho_{Y_1Y_2}\right)\right.\right.\right. \\ &\left.\left.\left. - \sum_{i=2}^{v-1} Q\left(\frac{E[Y_i]}{\sqrt{Var[Y_i]}}, -\frac{E[Y_{i+1}]}{\sqrt{Var[Y_{i+1}]}}; -\rho_{Y_iY_{i+1}}\right)\right)\right\}P\left\{t\_s_1^o\right\}\right]. \end{aligned} \tag{14}$$

Finally, the average SER of an arbitrary 2-D modulated OFDM signal with I/Q imbalances on k-th subcarrier is obtained as

$$\begin{aligned} &P_k^{ser} \\ &= \sum_{i=1}^{U}\left[\frac{1}{M}\sum_{m=1}^{M}\left[\left(1 - P_k\left\{r\_s_i^c \in R_{s_i}^c|S_t = t\_s_i^c\right\}\right)P\left\{t\_s_i^c\right\}\right]\right] \\ &+ \sum_{i=1}^{V}\left[\frac{1}{M}\sum_{m=1}^{M}\left[\left(1 - P_k\left\{r\_s_i^o \in R_{s_i}^o|S_t = t\_s_i^o\right\}\right)P\left\{t\_s_i^o\right\}\right]\right] \end{aligned} \tag{15}$$

where U is the number of signal points with the closed correct region, V is the number of signal points with the open correct region, and $M = U + V$.

Obtaining the exact BER expression for an arbitrary 2-D modulated OFDM signal with I/Q imbalances on k-th subcarrier is very tedious work, but exact BER performance is obtained by using the result of [14] in the form

$$\begin{aligned} P_k^{ber} = \frac{1}{\log_2 M}\sum_{l=1}^{M}\sum_{\substack{h=1 \\ h\neq l}}^{M}\left[\frac{1}{M}\sum_{m=1}^{M}\left[H\_d\left(t\_s_l,\ t\_s_h\right)\cdot\right.\right. \\ \left.\left. P_k\left\{r\_s_l \in R_{s_h}\left|S_t = t\_s_l\right.\right\}\cdot P\left\{t\_s_l\right\}\right]\right] \end{aligned} \tag{16}$$

where $t\_s_i,\ i = 1, 2, ..., M$ are the transmitted symbol, R_{s_h} represents the decision region for the symbol t_s_h , and $H\_d(t\_s_l,\, t\_s_h)$ denotes the Hamming distance between t_s_l and t_s_h .

B. Frequency-Flat Rayleigh Fading Channel

To obtain the exact error probability for the arbitrary 2-D modulated OFDM system in frequency-flat Rayleigh fading channel, we first should derive a closed-form solution

 ISBN:978-1-61208-113-7

of $\int_0^\infty Q\left(\Delta_1\sqrt{\gamma},\, \Delta_2\sqrt{\gamma};\, -\rho\right) \cdot f_\gamma\left(\gamma\right)d\gamma$ where $f_\gamma(\gamma) = \exp(-\gamma/\bar{\gamma})/\bar{\gamma},\ \gamma \geq 0$. In the 2-D Gaussian Q-function, $Q(x,\, y;\, \rho)$, of (15) and (16), the range of x and y is $-\infty < x, y < \infty$. Therefore, rewriting the 2-D Gaussian Q-function by using the Crag representation [15] and its alternative expression [16], $\int_0^\infty Q\left(\Delta_1\sqrt{\gamma},\, \Delta_2\sqrt{\gamma};\, -\rho\right) \cdot f_\gamma\left(\gamma\right)d\gamma$ is expressed as

$$\begin{aligned}&\int_0^\infty Q\left(\Delta_1\sqrt{\gamma},\, \Delta_2\sqrt{\gamma};\, -\rho\right) f_\gamma\left(\gamma\right)d\gamma\\ &= \tfrac{1}{2}\mathrm{sgn}\left[\mathrm{sgn}\left(\Delta_1\right) + \mathrm{sgn}\left(\Delta_1\Delta_2\right)\right] - \tfrac{1}{2}\mathrm{sgn}\left(\Delta_1\right)\\ &+ \sum_{i=1}^{2}\mathrm{sgn}\left(\Delta_i\right)\tfrac{1}{2\pi}\int_0^{\omega_i}\int_0^\infty \exp\left(-\tfrac{\Delta_i^2\gamma}{2\sin^2\theta}\right) f_\gamma\left(\gamma\right)d\gamma d\theta,\\ &\qquad -\infty < \Delta_i < \infty \text{ and } 0 \leq \omega_i \leq \pi\end{aligned} \tag{17}$$

where $\omega_1 = \pi/2 + \sin^{-1}((\rho\Delta_1 - \Delta_2)/\sqrt{(\Delta_1^2 - 2\rho\Delta_1\Delta_2 + \Delta_2^2)})$; $\omega_2 = \pi/2 + \sin^{-1}((\rho\Delta_2 - \Delta_1)/\sqrt{(\Delta_1^2 - 2\rho\Delta_1\Delta_2 + \Delta_2^2)})$. And then, applying the moment generating function (MGF) of γ, *i.e.* $M_\gamma(s) = (1 - \bar{\gamma}s)^{-1}$, to (17), we can straightforwardly obtain the following closed-form expression from [17, eq. (5A.24)] as follows:

$$\begin{aligned}&\int_0^\infty Q\left(\Delta_1\sqrt{\gamma},\, \Delta_2\sqrt{\gamma};\, -\rho\right) \cdot f_\gamma\left(\gamma\right)d\gamma\\ &= \tfrac{1}{2}\mathrm{sgn}\left[\mathrm{sgn}\left(\Delta_1\right) + \mathrm{sgn}\left(\Delta_1\Delta_2\right)\right] - \tfrac{1}{2}\mathrm{sgn}\left(\Delta_1\right)\\ &+ \tfrac{1}{2}\sum_{i=1}^{2}\mathrm{sgn}\left(\Delta_i\right)\left[\tfrac{\omega_i}{\pi} - \tfrac{1}{\pi}\beta_i\left(\tfrac{\pi}{2} + \tan^{-1}\alpha_i\right)\right]\end{aligned} \tag{18}$$

where $\beta_i = \sqrt{c_i/1 + c_i}\mathrm{sgn}\omega_i$, $\alpha_i = -\beta_i\cot\omega_i$, and $c_i = \Delta_i^2\bar{\gamma}$.

Finally, applying the result of (18) to (15) and (16), the exact symbol and bit error probabilities for an arbitrary 2-D modulation based OFDM system with I/Q imbalances over frequency-flat Rayleigh fading channel can be derived in exact closed-form expressions.

IV. Numerical Results and Conclusions

The 2-D modulation format considered in this section is (4+12)-APSK, which have been adopted as standard modulation techniques for the DVB-S2 system and space data system [18], [19] for their robust performance in nonlinear high power amplifiers (HPA). Because the SIR for the practical imbalance values is in the order of 20-30dB [11], we consider the imbalance values of $\varphi_R = 3°,\ \gamma = 1.1\ (SIR \approx 25dB)$ and $\varphi_R = 5°,\ \gamma = 1.2\ (SIR \approx 20dB)$ in this paper. Fig. 3 shows the SER and BER curves for (4+12)-APSK based OFDM system over frequency-flat Rayleigh fading channel. From the figure we can see excellent matches between the results obtained from our exact expressions and computer simulations. We can also verify that the gap of error rate increases as the effect of I/Q imbalances becomes greater.

In this paper, we have provided an exact closed-form expression involving the 2-D Gaussian Q-function for the error probabilities of an arbitrary 2-D modulated OFDM signal with I/Q imbalances in frequency-flat Rayleigh fading channel, and analyzed the effect of I/Q imbalances on error performance. The result can be readily applied to numerical evaluation for various cases of practical interest involving unbalanced I/Q modulation in OFDM systems.

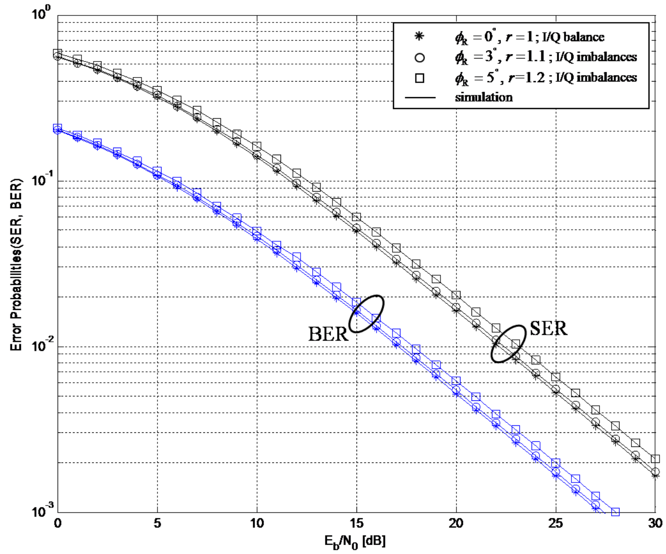

Fig. 3. SER and BER for (4+12)-APSK-based-OFDM system.

Acknowledgment

This research was supported by NSL program through the Korea Science and Engineering Foundation funded by the Ministry of Education, Science and Technology (2010-0015083).

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