On Maxmin Active Range Problem for Weighted Consistent Dynamic Map Labeling

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Abstract*—Geographical visualization systems, such as online maps, provide interactive operations on continuous zooming and panning. In consistent map labeling, users can navigate continuously through space without distracting behaviors such as popping and jumping. We study the consistent dynamic map labeling problem: Given a set of labels on the map and each label with a selectable active range and weight, find an appropriate active range for each label such that no two consistent labels intersect at any scale and the minimum weighted active range is maximized. It is named as *maximizing minimum weighted active range problem* (*MMWAR*). This study on MMWAR is of the theoretical and practical significance, since it is common that some labels in practical maps need better visibility than others. We investigate both the *simple* and *general* variants and present several theoretical results. For simple variants, simple 1D-MMWAR and 2D-MMWAR with proportional dilation are optimally solved in $O(n \log n)$ and $O(n^2 \log n)$, respectively. For general variants, we prove that general 1D-MMWAR with constant dilation and 2D-MMWAR with proportional dilation are NP-complete. Moreover, we provide an $O(\log n)$-approximation algorithm for the general 1D-MMWAR with proportional dilation, and an $O(\sqrt{n})$-factor approximation algorithm for the general 2D-MMWAR with proportional dilation. Our experimentation results show that on average, the approximation factors in our algorithms are much smaller than the worst-case upper bounds stated above, and our approximation algorithms run efficiently.

Keywords–Geographic information systems; Dynamic map labeling; NP-hardness; Approximation algorithms.

I. Introduction

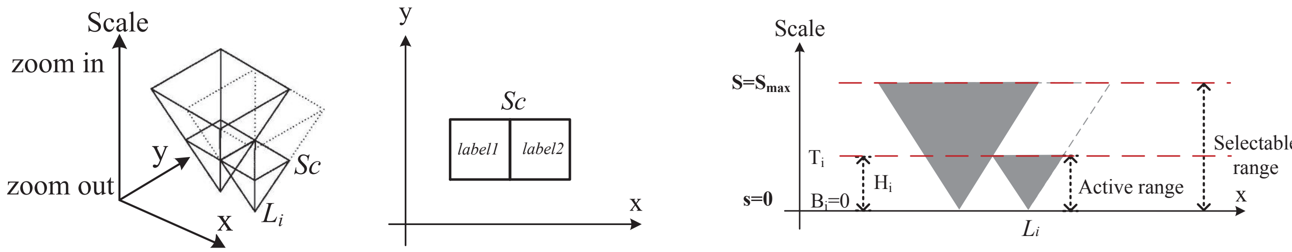
Dynamic map labeling, as a critical problem in cartography and geographic information systems (GIS), provides users interactive operations on zooming and panning maps continually and dynamically. In contrast to the static map labeling problem [1], it can be formulated as a traditional map labeling problem by incorporating *scale* as an additional dimension. Increasing academic concern is aroused to handle these interfaces. Been et al. [2] initially defined the consistent dynamic map labeling problem with a set of consistency desiderata to provide a new and practical framework. According to this framework, during zooming and panning, (a) labels are not allowed to exhibit abrupt change in the position or size; (b) labels should not suddenly disappear and reappear when zooming in or pop up when zooming out; (c) the labeling should be in line with the selected map viewpoint, not be hinged on the navigation history.

Most previous algorithmic studies on consistent dynamic map labeling deal with active range optimization (ARO) problem [3][4], whose objective is to maximize the sum of total active ranges, each of which corresponds to the consistent interval of scales with visible labels. On the other hand, maximizing minimum active range problem is seldom considered, since a few labels may only have a very small selectable range [5]. Nevertheless, the maximizing minimum weighted active range problem, arises in a natural way but with practical importance in situations when, different cities may have different weights in order to reveal the different degrees of importance. For example, on a map of China, attributing Beijing a higher priority (weight) than Tianjin (a nearby city of Beijing) ensures that in case of limited space the capital rather than one of its nearby cities receives a label. Clearly this maximizing minimum weighted active range (MMWAR) problem is equivalent to finding a set of active ranges with different visibility that the overall weighted range assignment is relatively balanced. In particular, none of the existing dynamic map labeling methods provides theoretical studies and related solutions on MMWAR.

In this paper, we study the problem MMWAR and propose a suite of algorithms. The present paper is structured as follows. Section II describes some preliminary concepts. The related work is presented in Section III. The complexity of MMWAR is investigated in Section IV. An algorithmic study of MMWAR is presented in Section V. We give exact algorithms for simple 1D-MMWAR and 2D-MMWAR with proportional dilation. The general 1D-MMWAR and 2D-MMWAR with proportional dilation are provided with approximation algorithms, whose performance is evaluated by experiments in Section VI. Section VII concludes this paper and discusses some open problems.

II. Preliminaries

In this study, we adopt the model of consistent dynamic map labeling and all above mentioned desiderata to our problem [2]. Each label is represented by a three dimensional (3D) solid. It is formed by extruding the label shape through the vertical dimension (*zooming scale*). Each solid can be truncated to a single scale interval, named its active range (or height, for short), corresponding to the scale selected by the label. The labels are assumed not to slide and rotate. See Figure 1, we consider invariant point placements with axis-aligned square labels. The output of our proposed algorithm is a set of disjoint active ranges. See Figure 1(a), at any zooming scale Sc, we obtain a set of disjoint labels at the cross section at scale Sc. This set of labels represents the labeling of the points we considered at this specific scale Sc. See Figure 1(b) as an example. In the dynamic setting of zooming in and out



(a) Front view of square frusta (b) Placement of labels at scale Sc (c) Side view of square labels

Figure 1. Two square labels with selectable ranges and active ranges

of the scene, different labeling at different zooming scales of the solution will be shown to the users.

We give several notations on the MMWAR problem. In principle, the input is a set of extrusions, each extrusion with a selectable interval and weight, for which we intend to assign an active range. Assume that we are given a set of extrusions $\Gamma = (L_1, L_2, ..., L_n)$, each L_i with a selectable range (s_i, S_i) and weight w_i, where $s_i, S_i \geq 0$ are at most the global maximum scale $S_{\max}$. Our objective is to calculate an active range assignment A for Γ. More specifically, for each extrusion $L_i \in \Gamma$, a contiguous active range $(B_i, T_i) \subseteq (s_i, S_i)$ is assigned. $H_i = T_i - B_i$ indicates the height of active range. A is a feasible solution if the resulting extrusions are pairwise disjoint. For a solution A, we say that an extrusion L_i is active in A at scale Sc if $Sc \in (B_i, T_i)$; otherwise L_i is inactive in A at scale Sc. The goal is to maximize the minimum weighted height of A, i.e., $MaxMin\frac{H_i}{w_i}$.

Following the work [5], we consider two variants in this problem, i.e., *simple* and *general*. The problem stated in the previous paragraph is the *general* variant. For the simple variant, it is of importance to theory and practice to consider the scenario in which all labels are selectable at all scales and all labels are selected when zooming in. Specifically, in simple MMWAR variant, $(s_i, S_i) = (0, S_{\max})$, $B_i = 0$ for all $L_i \in \Gamma$; see Figure 1(c). Moreover, we study two kinds of dilation cases in this paper, i.e., *proportional dilation* and *constant dilation*. We say that labels have proportional dilation if their sizes could change with scale proportionally. In contrast, if the sizes of labels are fixed at every scale, we say that labels have constant dilation. For the simple MMWAR problem with proportional dilation, the shapes of extrusions appear to be rectangular pyramids by extruding rectangular labels. For the general MMWAR problem with proportional dilation, the shapes of extrusions appear to be congruent square pyramids or frusta segments of congruent square cones.

III. Related Work

Map labeling is essential for a wide range of applications and becomes the focus of research [6][7]. Before the proposal of dynamic labeling problem, there was a large number of studies on automated label placement dealing with *static* fixed position [5]. The research outputs covered various settings and NP-hardness proofs [1][8]. A typical task of these works was to select and place labels without intersection so as to maximize the number of selected labels. Exact and approximation algorithms [9] [10][11] are known for several types of the static label optimization problem. Among them, Agarwal et al. [12] proposed a PTAS for the unit-width rectangular label placement problem and a $\log n$-approximation algorithm for the arbitrary rectangle case. Then, the improvement was obtained in [10]. Chalermsook and Chuzhoy [11] studied the *Maximum Independent Set of Rectangles* problem and presented an $O(\log\log n)$-approximation algorithm.

In recent years, dynamic map labeling has become a new bright spot. Petzold et al. [13] generated a reactive conflict graph storing all potential conflicts information by using preprocessing phase. Poon and Shin [14] presented algorithms for labeling points that precomputed a hierarchical data structure to store solutions at different scales. For consistent dynamic map labeling, Been et al. [2] initially presented several consistency desiderata and formulated a new algorithmic framework for fast and consistent labeling. They also showed the NP-hardness of optimal active range selection problem. In addition, several approximation algorithms were given for 1D and 2D labeling problems [3]. Nöllenburg et al. [15] explored three extensions of the one-sided boundary labeling model allowing continuous zooming and panning. Moreover, Gemsa et al. [16] gave an FPTAS for the continuous sliding model of the ARO problem. Yap [17] summarized a few open ARO problems in dynamic map labeling. More recently, Liao Chung-Shou et al. [4] considered the dynamic map labeling problem with a set of rectangles and gave several approximations to maximizing the sum of total visible ranges.

When considering the objective of maximizing the minimum active ranges, Gemsa et al. [18] not only considered *MaxTotal* ARO problem, but also studied *MaxMin* ARO problem. They showed both problems are NP-complete. However, they only considered the continuous map rotations. None of the existing research about *MaxMin* ARO studies the zooming setting. Furthermore, few researches are incorporating weights into consideration in the map labeling problem formulation. Poon et al. [19] first defined static label-placement models for labeling static points with weights and presented several corresponding algorithms. Schwartges [20] assigned labels to map objects like cities or streets and used the weights to determine the importance of a label. This is a good attempt to reduce the gap between theory and practice. Since dynamic map labeling is still an active research line, some unsolved problems remain, such as MMWAR.

IV. Complexity

In this section, we prove that two variants of MMWAR are NP-complete by reductions from the NP-hard problem *Planar 3SAT* [21]. An instance of *Planar 3SAT* is a *3SAT* formula Φ whose variable-clause $Graph_{\Phi}$ is planar.

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A. General 1D-MMWAR with constant dilation

We start by considering the NP-hardness proof of the general 1D-MMWAR with constant dilation.

Theorem 1: General 1D-MMWAR with constant dilation is NP-complete; i.e., given a set of n axis-aligned rectangular extrusions $\Gamma = (L_1, L_2, ..., L_n)$ with weight w_i for each L_i and a real number $K > 0$, it is NP-complete to decide whether there is a valid assignment *A* of active ranges to Γ with $Min\frac{H_i}{w_i} \geq K$. The problem is still NP-complete even when all extrusions are squares of two different sizes and with the same weight.

B. General 2D-MMWAR with proportional dilation

In the general 2D variant of MMWAR, we give the NP-hardness proof by assuming that all extrusions are congruent square pyramids with two different weights. We note that for the variant with pyramids of only one weight, whether it is NP-complete is still open.

Theorem 2: General 2D-MMWAR with proportional dilation is NP-complete; i.e., given a set of n axis-aligned rectangular cones $\Gamma = (P_1, P_2, ..., P_n)$ with different weights w_i and a real number $K > 0$, it is NP-complete to decide whether there is a valid assignment *A* of active ranges to Γ with $Min\frac{H_i}{w_i} \geq K$. The problem is still NP-complete even all extrusions are congruent square pyramids with two different weights.

V. TRIANGLES 1D-MMWAR & CONES 2D-MMWAR

A suite of algorithms are devised to solve several variants of 1D- and 2D-MMWAR problems. In the simple variants, the active ranges start from *Zero* scale. On the other hand, the active ranges start from any scale in general version, which is closer to the reality with practical significance.

A. Simple 1D-MMWAR

In simple 1D-MMWAR with proportional dilation, each extrusion is an inverted triangle with top edge attached to the horizontal line $s = S_{max}$ and apex located on the x-axis, i.e., $(B_k, T_k) \subseteq (0, S_{max})$ and $B_k = 0$, thus active range height $H_k = T_k$. The truncated extrusions differ only by heights of top edges. Figure 1(c) shows an example of active ranges assignment for the labels. Observe that the objective of the problem is $MaxMin\frac{H_i}{w_i}$ for each extrusion L_i.

Let $\Gamma = (L_1, L_2, ..., L_n)$ be the set of extrusions, and let p_i be the apex of triangle-shaped extrusion L_i on the x-axis. Assume that $p_1, ..., p_n$ are arranged from left to right. For each extrusion L_k, $k < n$, we define the left side edge and right side edge as E_k^l and E_k^r. Without loss of generality, we assume that, for every two adjacent extrusions L_k and L_t $(t > k)$, E_k^r and E_t^l intersect at h_{kt}. We denote the scale of h_{kt} as H_{kt}. They are stored as a *Doubly Linked List DuLinkList[n-1]*, in which, E_k^r and E_{k+1}^l are the left pointer field and the right pointer field, and $H_{k(k+1)}$ are the value field. Then, we construct a *RB-Tree* $\Re$ to store $(H_{12}, H_{23}, ..., H_{(n-1)n})$.

Hence, we give the exact algorithm with low time complexity, as shown in Algorithm 1.

Theorem 3: Simple 1D-MMWAR with proportional dilation can be solved in $O(n \log n)$ time.

Proof: For each pair of adjacent extrusions L_s and L_t, only one of them is assigned the active range height H_{st} in simple 1D-MMWAR. By handling intersecting points of all pairwise extrusions in Algorithm 1, none of them intersect after range assignments.

Algorithm 1 Compute the maximum minimum weighted active range for simple 1D-MMWAR with proportional dilation

Input: $\Gamma = (L_1, L_2, ..., L_n)$, a selectable range $(0, S_{max})$ and weight $\pi = (w_1, w_2, ..., w_n)$
Output: $MaxMin\frac{H}{w}$, *HList*
for each extrusion L_i in Γ **do**
  DuLinkList $\leftarrow L_i$
end for
Construct *RB-Tree* $\Re$ from *DuLinkList*
for the minimum element $H_{ij} \in \Re$ **do**
  find two corresponding extrusions (L_i, L_j) with weights (w_i, w_j)
  if $\frac{H_{ij}}{w_i} \geq \frac{H_{ij}}{w_j}$ **then**
    add L_i with H_{ij} to the list *HList*
    delete L_i and update *DuLinkList* and $\Re$
  else
    add L_j with H_{ij} to the list *HList*
    delete L_j and update *DuLinkList* and $\Re$
  end if
end for
$MaxMin\frac{H}{w} \leftarrow min\{\frac{H}{w}\}, \{H \in \textit{HList}, w \in \pi\}$
Return $MaxMin\frac{H}{w}$

Optimality. For the smallest $H_{\alpha\beta}$ in the RB-Tree, it denotes the lowest intersecting point of two extrusions L_α and L_β with weights w_α and w_β. The value $H_{\alpha\beta}$ must be assigned to either L_α or L_β. Assume that $w_\alpha > w_\beta$, thus $\frac{H_{\alpha\beta}}{w_\alpha} < \frac{H_{\alpha\beta}}{w_\beta}$ (weighted active ranges). It shows that $\frac{H_{\alpha\beta}}{w_\alpha}$ is the smaller one. The only way to increase $\frac{H_{\alpha\beta}}{w_\alpha}$ is to assign $H_{\alpha\beta}$ to L_β and remove it to a candidate set $HList$, since the remaining values of *H* are larger than $H_{\alpha\beta}$. Thus, the weighted active range of L_α can be increased to be larger than $\frac{H_{\alpha\beta}}{w_\alpha}$. That is to say, whenever we select one of two intersecting extrusions, we select the one whose weighted active range is larger and add it to the candidate set $HList$. By this means, all the smaller weighted active ranges can be maximized and restored to $HList$.

Complexity. For the running time, we construct the *RB-Tree* by using $T_1(n)$. Thus, we have

$$T_1(n) = O(\log n!) < O(n \log n).$$

For each operation on the *RB-Tree*, the running time $T_2(n) = O(\log n)$. Hence, the overall time complexity of Algorithm 1 follows.

$$T(n) = T_1(n) + T_2(n) = O(n \log n).$$

Therefore, we can obtain the optimal solution of $MaxMin\frac{H}{w}$ in $O(n \log n)$ time. ■

B. Simple 2D-MMWAR

The idea of Algorithm 1 for 1D-MMWAR can be easily extended to solve the simple 2D-MMWAR.

In contrast to 1D-MMWAR, we need to construct a RB-Tree to store all pairs of the lowest intersection of 3D cones, whose amount is $O(n^2)$. Similar to Theorem 3, each time we choose the smallest H_s from the RB-tree for comparing the

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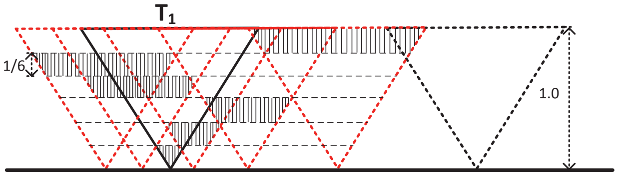

Figure 2. An illustration of the intersection degree and active ranges assignment.

weighted active range $\frac{H_s}{w}$ of two intersecting cones. Then, we remove those cones involving with the larger weighted active range from the RB-Tree. There are $O(n)$ 3D cones, which implies we need to delete those H_s from RB-tree in total $O(n \log n)$ time. Hence, the time complexity is $O(n^2 \log n)$.

Theorem 4: Simple 2D-MMWAR with proportional dilation can be solved in $O(n^2 \log n)$ time.

C. General 1D-MMWAR

In this subsection, we give an approximation algorithm that yields $\log n$-approximation for the general 1D-MMWAR with proportional dilation.

Suppose that we are given a set of congruent triangles or trapezoidal segments of congruent triangles with apexes on the *x*-axis. They are assumed to be with the same weight. As described in Algorithm 2, the idea is to choose the triangle intersecting with the maximum number of other triangles, then assign the active ranges to these triangles as evenly as possible. In addition, we need a new piece of notation Δ, i.e., *intersection degree* of triangle T, which denotes the number of triangles that intersect with triangle T in the graph. As shown in Figure 2, there are $eight$ congruent triangles, whose selectable ranges are assume to be one unit, i.e., $S_{max} = 1$. If triangle T_1 is intersecting with $five$ triangles (red triangles), we say that the intersection degree of triangle T_1 is $five$, which is denoted as $\Delta = 5$.

See Algorithm 2 for the pseudo-code of our algorithm. In the following theorem, we show that such a solution is in fact an $O(\log n)$-approximation for the general 1D-MMWAR problem with the same weight.

Algorithm 2 Compute the maximum minimum weighted active range for general 1D-MMWAR with proportional dilation

Input: a set Γ of n congruent triangles, a selectable range $(0, S_{max})$ and weight w
Output: $MaxMin\frac{H}{w}$
for each triangle $T_i \in \Gamma$ **do**
  $\Delta_i \leftarrow$ the number of intersecting triangles
end for
$\Delta_{max} \leftarrow Max\{\Delta_1, \cdots, \Delta_n\}$
for each triangle $T_i \in \Gamma$ **do**
  $H \leftarrow \frac{S_{max}}{\Delta_{max}+1}$
end for
Return $\frac{H}{w}$

Theorem 5: Given a set of n congruent triangles with the same weight, a $\log n$-approximation for the general 1D-MMWAR with proportional dilation of *General Congruent Triangles* can be computed in $O(n^2)$ time.

Proof: Given n congruent triangles with the same weight, whose selectable ranges are assumed to be one unit, i.e., $S_{max} = 1$. Considering the triangle T_m intersecting with the maximum number of triangles Δ_{max}, Let X_T be the set containing T_m and all the triangles intersecting with T_m. Thus, $|X_T| = \Delta_{max} + 1$. According to Algorithm 2, each triangle $T \in X_T$ is assigned an active range height $\frac{1}{\Delta_{max}+1}$. Obviously, none of the active ranges of the triangles in X_T conflicts with each other after range assignment, since they are assigned evenly. Then, considering the case that triangle $T' \notin X_T$ with intersection degree Δ', observe that $\Delta' \leq \Delta_{max}$. $X_{T'}$ contains T' and all the triangles intersecting with T', in which we assume there are k common triangles in $X_{T'}$ and X_T, $k \leq \Delta'$. Observing that each of those k common triangles has been assigned an active range of height $\frac{1}{\Delta_{max}+1}$, which occupy a range of total height $\frac{k}{\Delta_{max}+1}$. For the remaining $\Delta' + 1 - k$ triangles, they have the total range of height $\frac{\Delta_{max}+1-k}{\Delta_{max}+1}$ to be assigned. We assign each triangle an active range of height $\frac{1}{\Delta_{max}+1}$ for these remaining $\Delta' + 1 - k$ triangles. Since $\Delta' \leq \Delta_{max}$, all triangles in $X_{T'}$ and X_T can be assigned the active ranges without conflict. Note that when $k = 0$, $X_{T'}$ and X_T are disjoint subsets of triangles.

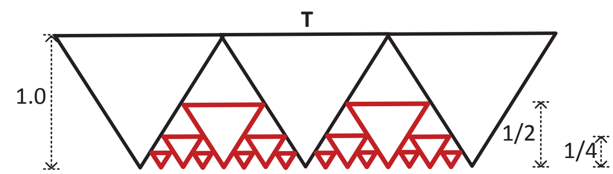

Figure 3. An illustration of finding the optimal solution for general 1D-MMWAR with proportional dilation.

When $n = 1$, the active range is S_{max}. When $n = 2$ or 3, the solutions are $\frac{S_{max}}{2}$ and $\frac{S_{max}}{3}$, respectively. As for any case that X_T with maximum intersecting degree contains m $(3 < m \leq n)$ triangles, we consider the optimal solution S^* for m triangles in this case. The solution is illustrated as *bold red triangles* in Figure 3. The gaps between disjoint triangles can be extracted up to $m - 3$ triangles in $O(m \log m)$ time. Then, we calculate the summation of all selected active range $A(t)$ for each $t \in X_T$ in the solution from the bottom to top, i.e.,

$$\sum_{t \in X_T} A(t) = 3 + 2 \times \frac{1}{2} + 2^2 \times \frac{1}{2^2} + 2^3 \times \frac{1}{2^3} + \ldots + 2^i \times \frac{1}{2^i}.$$

With the purpose of reaching the amount of triangles m, we have

$$i = \log(m-1) - 1 \Rightarrow \sum_{t \in X_T} A(t) = 2 + \log(m-1)$$

It indicates that the optimal solution follows.

$$S^* \leq \frac{\sum_{t \in X_T} A(t)}{m} = \frac{2 + \log(m-1)}{m}$$

In this case, we obtain the solution $S = \frac{1}{m}$.

Hence, Algorithm 2 achieves the approximation factor $\frac{S^*}{S} = 2 + \log(m-1) \leq 2 + \log(n-1)$. Thus, we obtain an $O(\log n)$-approximation for the general 1D-MMWAR with proportional dilation. ■

We can extend our algorithm to handle triangles with two weights. Hence, we obtain the following Corollary 1.

Corollary 1: Given n congruent triangles with the two weights w_1 and w_2 such that w_1 is a constant factor of w_2, Algorithm 2 computes an $O(\log n)$-factor approximation for the general 1D-MMWAR with proportional dilation.

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D. General 2D-MMWAR

Suppose that we are given a set of congruent square pyramids or frusta segments of congruent square cones with apexes on the horizontal plane. For simplicity, we set $S_{max} = 1$. Our algorithm, say Algorithm 3, for the general 2D-MMWAR runs greedily using the same method as Algorithm 2. The only difference is that we are now considering congruent square frustra instead of congruent triangles. The pseudo-code of new algorithm is given in Algorithm 3.

Algorithm 3 Compute the maximum minimum weighted active range for general 2D-MMWAR with proportional dilation

Input: a set Γ of n congruent frusta, a selectable range $(0, S_{max})$ and weight w
Output: $MaxMin\frac{H}{w}$
for each frustum $T_i \in \Gamma$ **do**
  $\Delta_i \leftarrow$ the number of intersecting frusta
end for
$\Delta_{max} \leftarrow Max\{\Delta_1, \cdots, \Delta_n\}$
for each frustum $T_i \in \Gamma$ **do**
  $H \leftarrow \frac{S_{max}}{\Delta_{max}+1}$
end for
Return $\frac{H}{w}$

Theorem 6: Given a set of n axis-aligned congruent square frusta with the same weight, an $O(\sqrt{n})$-approximation algorithm for the maximum minimum weighted active range can be computed in $O(n^2)$ time.

Proof: The correctness proof uses the similar approach as the proof in Theorem 5. Recall the notation of intersection degree Δ, each extrusion is assigned only one active range of height $\frac{1}{\Delta_{max}+1}$, none of which intersect with each other after range assignments. So, we start by considering the cone T intersecting with the maximum number of cones, denoted as X_T. We show that the cone T is intersecting with other eight cones to formulate a *square* in 2D plane. For i-th cutting procedure, the amount of the cones reaches $(2^i + 1)^2$. As illustrated in Figure 4, when $i = 2$, we extract one cone with height $\frac{1}{2}$ from each pairwise disjoint cones with height 1. Thus, when the amount of the cones reaches the number n, we obtain that $i = \log(\sqrt{n}-1)$. Then, we calculate the summation of all selected active range $A(t)$ for X_T in the solution from bottom to top.

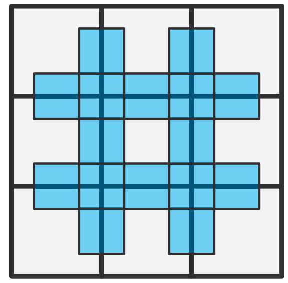
$$\sum_{t \in X_T} A(t) = 3\sqrt{n} + 2\log(\sqrt{n}-1) - 2$$

which implies the optimal solution S^* as follows.

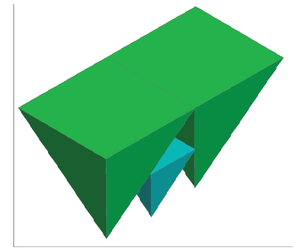
$$S^* \leq \frac{\sum_{t \in X_T} A(t)}{n} = \frac{3\sqrt{n} + 2\log(\sqrt{n}-1) - 2}{n}$$

Furthermore, Algorithm 3 gives a solution $\frac{1}{\Delta_{max}+1} \geq \frac{1}{n}$. Overall, Algorithm 3 achieves the approximation factor $\frac{S^*}{S} \leq 3\sqrt{n} + 2\log(\sqrt{n}-1) - 2$. Thus, we obtain an $O(\sqrt{n})$-approximation for the general 2D-MMWAR with proportional dilation. ∎

We can extend our algorithm to handle congruent square frusta with two weights. Hence, we obtain the following Corollary 2.



(a) An example of the top-down view.



(b) 3D model of cutting the gap between each pairwise disjoint cones.

Figure 4. Illustration of the cutting procedure for general 2D-MMWAR with proportional dilation

Corollary 2: Given n congruent frusta with two weights w_1 and w_2 such that w_1 is a constant factor of w_2, an $O(\sqrt{n})$-approximation for the general 2D-MMWAR with proportional dilation can be computed.

VI. Experiments and Evaluation

In this section, we evaluate the performance of Algorithm 2 for general 1D-MMWAR and Algorithm 3 for general 2D-MMWAR. Since these problems are proved to be NP-hard, we compare the results obtained by the proposed algorithms with the theoretical bounds. The experiments are conducted on a 3.4GHz Intel PC with 4GB RAM. The programming language is MATLAB(R2013a).

A. Approximation ratio for general 1D-MMWAR

For Algorithm 2, we uniformly distributed 1,000 congruent triangles along a straight-line segment. For each triangle set of size $n = 50, 100, ..., 1,000$, we randomly generate 1,000 cases. The average performance ratio is recorded in Figure 5, where the horizontal axis represents input size of congruent triangles and the vertical axis represents the average approximation ratio.

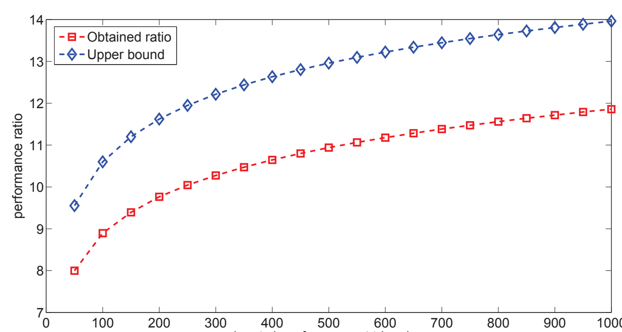


Figure 5. Comparing the approximation ratio of Algorithm 2 with its theoretical upper bound.

B. Approximation ratio for general 2D-MMWAR

For Algorithm 3, we uniformly distributed 1,000 congruent frusta in the unit square. For each frustum set of size $n = 50, 100, ..., 1,000$, we randomly generate 1,000 cases, and record the average approximation ratio in Figure 6.

Summarizing and evaluating our results, we have observed that the proposed approximation algorithms have much smaller approximation ratios than the worst-case theoretical upper bounds. Besides, it seems that, as the problem scale increases, the real approximation ratio increases little.

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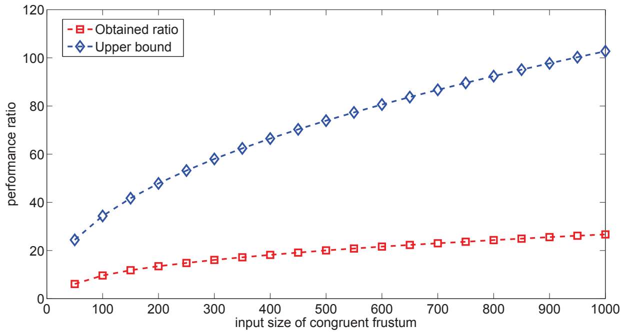

Figure 6. Comparing the approximation ratio of Algorithm 3 with its theoretical upper bound.

C. Running time for Algorithms 2 and 3

For the running time, we averaged the running time of the 1,000 cases on both algorithms with input size from 50 to 1,000 and showed the results in Figure 7. It indicates that the running times of the approximation algorithms follow the theoretical complexity bounds, and both algorithms run efficiently.

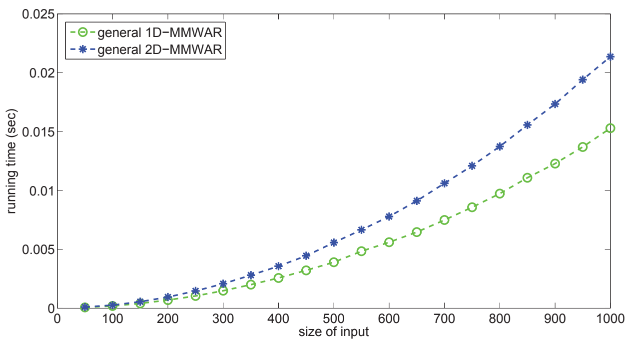

Figure 7. Average running time in seconds

VII. Conclusion and Discussion

The weighted active range optimization problem is of great theoretical and practical importance in map labeling, and this is the first work with the objective of maximizing the minimum weighted active range. We prove that general 1D-MMWAR with constant dilation and general 2D-MMWAR with proportional dilation are NP-complete. We have proposed two exact algorithms for simple 1D-MMWAR and 2D-MMWAR with proportional dilation and two approximation algorithms for general 1D-MMWAR and 2D-MMWAR with proportional dilation. For the complexity analysis, there are still several open problems. The complexity of general 2D-MMWAR with constant dilation is still unknown. For proportional dilation, since we assume that the input extrusions have two different weights in the NP-hardness proof of general 2D-MMWAR, the complexity of the problem with only one weight remains as an open problem. Furthermore, we believe that the approximation factor and time complexity of the approximation algorithms and corollaries for general 1D-MMWAR and 2D-MMWAR with proportional dilation could be further improved.

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