

# A Parallel Rolling Horizon Scheme for Large Scale Security Constrained Unit Commitment Problems with Wind Power Generation

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**Abstract**—The Unit Commitment Problem (UCP) is an important category of power planning problems. The purpose of UCP is to determine when to start up and shut down the generator units and how to dispatch the committed units to meet the electricity demands, ancillary services requirements and security constraints. In this paper, we improve the traditional Lagrange Relaxation (LR) approach and analyze the effectiveness of using parallel computing in solving large unit commitment problems with wind penetration and investigate the potential of combining parallel computing with a rolling horizon scheme to improve the solution quality when a large amount of wind power is present. In particular, we first formulate a security constrained unit commitment problem by taking into account power generation costs, ancillary costs, wind power and a variety of security constraints employed in real New York State day-ahead power market. We then propose a parallelized version of the LR method to solve the problem in a single step, analyze the scalability issue of parallel computing, and investigate the impact of increased wind energy penetration. Finally, when a large amount of wind power is present, we further propose an approach that combines parallel computing with a rolling horizon technique to solve the UCP online.

**Index Terms**—unit commitment; ancillary service; wind power; parallel computing; rolling horizon.

## I. INTRODUCTION

The goal of unit commitment problems is to find the optimal production schedule for the power generation units and the production level of each unit over a short term period in order to minimize the operational cost of the power grid [1]. To maintain the security of the electric grid, a variety of security constrained, for example, reserve constraints and transmission constraints, are enforced, and the resulting problem is usually called Security Constrained Unit Commitment (SCUC) problem. In New York State, UCP is solved by New York Independent System Operator (NYISO) in the day-ahead power

market based on the generation and ancillary service bids, which give generation and ancillary service cost of each power generator, from Independent Power Producers (IPP), Loads bid from Load Service Entities (LSE) and Security Constraints set by NYISO and other power regulation authorities. Because of the importance of UCPs, broad and intensive study has been carried out in this field, and many methods have been proposed in literature and used in practice [2].

Depending on the system configuration of a power grid, different optimization objectives and security constraints are considered. For the basic UCP formulation, the objective is simply to minimize the power generation cost subject to the electricity demand. However, as the liberalization of the electricity markets and advancement of optimization techniques, more and more elements are introduced. In [3], a security constrained unit commitment problem (SCUC) with transmission constraints was tackled using a lagrangian relaxation approach, where the transmission and reserve constraints were relaxed to form a dual problem and subsequently solved using subgradient methods. The test result showed that the proposed direct method can reduce the generation cost over the indirect method that does not consider transmission constraint in the dual optimization process. The algorithm was improved in [4] to address the feasibility issue, and a unit de-commitment step was added to achieve a better solution. The AC constraints were considered in [5], and Bender's decomposition technique was used to solve the problem. Furthermore, ancillary services has been gradually introduced into the unit commitment process. In [6], Z. Li and M. Shahidehpour used Lagrangian Relaxation technique to solve the security constrained UCP with the ancillary service constraints and costs; moreover, they also calculated the market clearing price of both generation and ancillary costs. Their work is important because there is a conflict between generation service and ancillary service when a generator is turned on. Additionally, some environmental elements, such as carbon tax, were introduced into the UCP in the past two years [7]. Because of its complexity, it is unusual

The research of Eugene Feinberg was partially supported by NSF grants CMMI-0900206 and CMMI-0928490. The work of Jiaqiao Hu was supported in part by the Air Force Office of Scientific Research under Grant FA95501010340, and by the National Science Foundation under Grants CMMI-0900332 and CMMI-1130761.

for ISOs to solve the problem in a single step. Instead, a multi-step approach is often adopted. For example, in the New York State, different constraints are added at different steps to decrease the complexity of the problem [8]; however, this will decrease the solution quality. Therefore, how to solve a SCUC problem in a single step with a certain time limit becomes a challenging problem.

The presence of renewable energy sources such as wind power can further increase the complexity of the unit commitment problem, and a common method to handle this is to use the scenario tree technique [9] to simulate the uncertainties and dynamics of wind power. However, to make the problem computationally tractable, only a very limited number of scenarios can be used. Many research projects proposed to use a rolling horizon optimization scheme rather than the traditional day-ahead scheme; see for example, the Wilmar project [10][11][12]. An alternative method is to use a probability measure to set up a probability level to limit the probability of power outage within the prescribed threshold [13]. To meet these probability requirements, one needs to set the operation reserve based on the variability of wind power. A rolling horizon approach can also be used to dynamically locate the operation reserve when new wind forecast information is available. Note that the rolling horizon approach is more computationally demanding as compared to traditional UCs in day-ahead scheduling, as decisions need to be made at every time step in an online manner and every decision requires solving a nonlinear optimization problem involving both continuous and discrete decision variables.

In this work, an improved Lagrangian Relaxation (LR) method, which is adapted for parallel computing, is proposed to solve a large scale SCUC problem. Because linear generation cost functions are used, a greedy algorithm is proposed to optimize the generation and ancillary service when a generator is on. By using the proposed algorithm, we expect to solve the large-scale SCUC in a single step and dramatically reduce the computational time. A system based on the features of power system of New York Control Area (NYCA) is simulated to test the significance of our algorithm.

To address the variability of wind power, we follow the idea of [13] and use a probabilistic reserve constraint to describe the uncertainty of wind power. Since the wind power forecast is more accurate over shorter time periods [14][11], the probabilistic reserve constraints method is combined with a rolling horizon scheme to dynamically update the reserve constraints when more accurate wind forecast becomes available. The computational time issue is addressed by implementing our proposed solver on a parallel computing facility, and the research results show that parallel computing has the potential to satisfy this computational speed requirement of Rolling Horizon Scheme.

The organization of this paper is as follows. A security constrained unit commit model is formulated in Section II, and a solution algorithm is given in Section III. Section IV gives the probabilistic formulation of reserve constraints and the method used to handle the constraints. In Section V,

we provide case study to illustrate the performance of our algorithms. The nomenclature is given in the Appendix at the end of this paper.

## II. FORMULATION OF SCUC MODEL

In this work, a SCUC model is formulated based on the realistic problem solved in New York State. Both generation service and ancillary services, including reserve services and regulations services, are optimized to minimize the total operational costs. Realistic security constraints, including balance constraints, ancillary service requirement, load capacity constraint, transmission constraints, etc, are considered in this research. Unlike some other research, which is based on benchmark problems with up to 100 generators and limited security constraints, this work is trying to solve large-scale problem with more than 600 power generators and realistic security constraints enforced in daily power planning process in New York States. The formulation is given in the subsections below.

### A. Objective Function

In this work, the total operational cost, including both the power supply cost and ancillary services cost are optimized. The power supply cost includes power generation cost and start-up cost. The ancillary service cost includes reserve service cost and regulation service cost. Moreover, reserve service is divided into spinning service and non-synchronous service. The formulation is given in (1).

$$\begin{aligned} cost = & \sum_{m=1}^M \sum_{i_m=1}^{I_m} \sum_{t=1}^T (F_{m,i_m,t}(p_{m,i_m,t}) + S_{m,i_m,t}(z_{m,i_m,t})) \\ & + \sum_{m=1}^M \sum_{i_m=1}^{I_m} \sum_{t=1}^T (R_{m,i_m,t}(r_{m,i_m,t})) \\ & + \sum_{m=1}^M \sum_{i_m=1}^{I_m} \sum_{t=1}^T (CReg_{m,i_m,t}(reg_{m,i_m,t})), \end{aligned} \quad (1)$$

where

$$r_{m,i_m,t} = (r10s_{m,i_m,t}, r10ns_{m,i_m,t}, r30s_{m,i_m,t}, r30ns_{m,i_m,t}).$$

The first line in equation (1) includes power generation function, which is formulated as a piecewise linear function, and a startup function, which is formulated as a stepwise linear function. The second line includes the reserve cost function, which is a linear function with respect to the 10-minute and 30-minute spinning reserves and non-synchronous reserves. The third line is the linear regulation cost function. It should be noted that a generator can provide spinning reserve or regulation services only when it is turned on, and provide non-synchronous reserve service only when it is turned off.

### B. Load Balance

In this work, we assume that wind power can be integrated at no additional cost, and that there is no wind curtailment. Thus, wind power will always be delivered to customers. Then, wind power can be considered as negative load in this research.

Here we define the term “net load” as the difference between the electricity demand and the predicted wind power, i.e., the load that needs to be supplied by the traditional generators, including steam generator, gas turbine and hydro power. In day-ahead planning, the hydro and thermal plants should meet the sum of net loads of certain control areas. The mathematical formulation is given in equation (2).

$$\sum_{m=1}^M \sum_{i_m=1}^{I_m} p_{i_m,t} = \sum_{m=1}^M (d_{m,t} - w_{m,t}), \quad \forall t \quad (2)$$

### C. Ancillary service requirements

For the entire control area, there are three kinds of reserve requirements: 10-minute spinning reserve, 10-minute total reserve, and 30-minute total reserve requirements. The mathematical formulations are given in (3), (4), and (5), respectively.

$$\sum_{m=1}^M \sum_{i_m=1}^{I_M} r10s_{m,i_m,t} \geq Res_{10spin}, \quad \forall t \quad (3)$$

$$\sum_{m=1}^M \sum_{i_m=1}^{I_M} (r10s_{m,i_m,t} + r10ns_{m,i_m,t}) \geq Res_{10t}, \quad \forall t \quad (4)$$

$$\sum_{m=1}^M \sum_{i_m=1}^{I_M} (r30s_{m,i_m,t} + r30ns_{m,i_m,t}) \geq Res_{30t}, \quad \forall t \quad (5)$$

In real power market, a control area is often divided into several individual areas, which are usually called “zones”. And for certain collection of zones, there are several location based reserve constraints. Letting  $\Lambda_j$  be the  $j$ th collection of zones, the location based reserve constraints are given by (6), (7), and (8).

$$\sum_{m \in \Lambda_j} \sum_{i_m=1}^{I_M} r10s_{m,i_m,t} \geq ResLB_{j,10spin}, \quad \forall t \quad (6)$$

$$\sum_{m \in \Lambda_j} \sum_{i_m=1}^{I_M} (r10s_{m,i_m,t} + r10ns_{m,i_m,t}) \geq ResLB_{j,10t}, \quad \forall t \quad (7)$$

$$\sum_{m \in \Lambda_j} \sum_{i_m=1}^{I_M} (r30s_{m,i_m,t} + r30ns_{m,i_m,t}) \geq ResLB_{j,30t}, \quad \forall t \quad (8)$$

In a control area operated by an ISO, those collection of zones are called “super-zones”. Equations (6), (7), and (8) should be satisfied for all those super-zones.

Additionally, due the fluctuation of power demand and wind power, ISOs has certain requirements for regulation services, which are expressed in (9).

$$\sum_{m=1}^M \sum_{i_m=1}^{I_M} r10s_{m,i_m,t} \geq Reg_t, \quad \forall t \quad (9)$$

### D. Transmission constraints

Transmission constraints between different zones are also considered. The modeling of transmission constraints follows the method used in [4], and is given in (10).

$$\sum_{m=1}^M \Gamma_{l,m} \left( \sum_{i_m=1}^{I_m} p_{m,i_m,t} + w_{m,t} - d_{m,t} \right) \leq Tram_{l,max}, \quad \forall l, t \quad (10)$$

where  $\Gamma_{l,m}$  is the line flow distribution factor for the transmission line  $l$  due to the net power injection of zone  $m$ .

### E. Single generator capacity constraints

A generator that has been turned on might provide generation and reserve simultaneously. In practice, the sum of these services should be within the maximum power output limit. These requirements are given in (11), (12), (13), and (14). When a generator is off-line, it might be used to provide non-synchronous reserve services, the sum of which should also be within the maximum output limit. The formulations are given in (16), (17), and (18). In addition, regulation service should also be within a certain limit, which is given in equation (15).

$$p_{m,i_m,t} + r10s_{m,i_m,t} + r30s_{m,i_m,t} \leq p_{m,i_m,max} z_{m,i_m,t}, \quad \forall m, i_m, t \quad (11)$$

$$p_{m,i_m,min} z_{m,i_m,t} \leq p_{m,i_m,t} \leq p_{m,i_m,max} z_{m,i_m,t}, \quad \forall m, i_m, t \quad (12)$$

$$r10s_{m,i_m,t} \leq r10s_{m,i_m,max} z_{m,i_m,t}, \quad \forall m, i_m, t \quad (13)$$

$$r30s_{m,i_m,t} \leq r30s_{m,i_m,max} z_{m,i_m,t}, \quad \forall m, i_m, t \quad (14)$$

$$reg_{m,i_m,t} \leq reg_{m,i_m,max} z_{m,i_m,t}, \quad \forall m, i_m, t \quad (15)$$

$$r10ns_{m,i_m,t} + r30ns_{m,i_m,t} \leq p_{m,i_m,max} (1 - z_{m,i_m,t}), \quad \forall m, i_m, t \quad (16)$$

$$r10s_{m,i_m,t} \leq r10s_{m,i_m,max} (1 - z_{m,i_m,t}), \quad \forall m, i_m, t \quad (17)$$

$$r30s_{m,i_m,t} \leq r30s_{m,i_m,max} (1 - z_{m,i_m,t}), \quad \forall m, i_m, t \quad (18)$$

### F. Other constraints

Some other constraints such as minimum up time and down time constraints are also considered in our model. The detailed formulation can be found in, e.g., [15] and [16]. In addition, the maximum number of stops will be considered in our model, which means that a generator can only be turned off for a limited number of times on a single day. This constraint has never been considered in previous studies.

### III. SOLUTION ALGORITHM

As proposed in [4], a direct method is used to solve the SCUC problem. Compared with the work in [3], we introduced the ancillary service costs in the objective function, and single generator capacity constraints are added to the model. Moreover, a parallel computing scheme is developed to enhance the computational speed.

#### A. Lagrangian Relaxation Algorithm

To solve this problem, Lagrangian Relaxation method is used to relax the demand, reserve, transmission, and regulations constraints. A dual problem is thus obtained, and its objective function is given in equation (19).

$$\begin{aligned}
 Dual\ Cost = Cost + \sum_{t=1}^T \{ & \\
 \lambda_{d,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} p_{m,i_m,t} - \sum_{m=1}^M (d_{m,t} - w_{m,t}) ] & \\
 + \lambda_{10s,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} r10s_{m,i_m,t} - Res_{10spin} ] & \\
 + \lambda_{10t,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} (r10s_{m,i_m,t} + r10ns_{m,i_m,t}) & \\
 - Res_{10t} ] & \\
 + \lambda_{30t,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} (r30s_{m,i_m,t} + r30ns_{m,i_m,t}) & \\
 - Res_{30t} ] & \\
 + \sum_{j=1}^J \{ \lambda_{j,10s,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} r10s_{m,i_m,t} \mathcal{I}(m \in \Lambda_j) & \\
 - Res_{j,10spin} ] & \\
 + \lambda_{j,10t,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} (r10s_{m,i_m,t} + r10ns_{m,i_m,t}) \mathcal{I}(m \in \Lambda_j) & \\
 - Res_{j,10t} ] & \\
 + \lambda_{j,30t,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} (r30s_{m,i_m,t} + r30ns_{m,i_m,t}) \mathcal{I}(m \in \Lambda_j) & \\
 - Res_{t,10t} ] \} & \\
 + \lambda_{reg,t} [ \sum_{m=1}^M \sum_{i_m=1}^{I_m} regs - Reg_{10spin} ] & \\
 + \sum_{t=1}^T \sum_{l=1}^L \{ \lambda_{tran,l,t} [ Tran_{l,max} & \\
 - \sum_{m=1}^M \Gamma_{l,m} ( \sum_{i_m=1}^{I_m} p_{m,i_m,t} + w_{m,t} - d_{m,t} ) ] \}, & \quad (19)
 \end{aligned}$$

where "Cost" equals the cost function in equation (1). The second line of the equation (19) is due to the relaxation of demand constraints; lines 3 – 7 are due to

the relaxation of reserve constraints of the whole control area, while lines 8 – 13 are due to the relaxation of location reserve constraints, line 14 is due the relaxation or regulation constraints, and lines 15 – 16 are the relaxation of transmission constraints.  $\lambda_{d,t}$ ,  $\lambda_{10s,t}$ ,  $\lambda_{10t,t}$ ,  $\lambda_{30t,t}$ ,  $\lambda_{j,10s,t}$ ,  $\lambda_{j,10t,t}$ ,  $\lambda_{j,30t,t}$ ,  $\lambda_{reg,t}$ , and  $\lambda_{tran,l,t}$  are the corresponding lagrangian multipliers. For simplicity, we define  $\lambda_t = \{ \lambda_{d,t}, \lambda_{10s,t}, \lambda_{10t,t}, \lambda_{30t,t}, \lambda_{j,10s,t}, \lambda_{j,10t,t}, \lambda_{j,30t,t}, \lambda_{reg,t} \}$ . The dual problem is to find  $\max_{\lambda_t} [\min(Dual\ Cost)]$ . A single generator problem is defined in equation (20).

$$\begin{aligned}
 DualCost_{m,i_m} = \sum_{t=1}^T \{ & \\
 F_{i,i_m,t}(P_{m,i_m,t}) + S_{m,i_m,t}(z_{m,i_m,t}) & \\
 + R_{m,i_m,t}(r_{m,i_m,t}) & \\
 + CReg_{m,i_m,t}(reg_{m,i_m,t}) & \\
 - p_{m,i_m,t} (\lambda_{d,t} + \sum_{l=1}^L \lambda_{tran,l,t} \Gamma_{l,m}) & \\
 - r10s_{m,i_m,t} [ \lambda_{10s,t} + \lambda_{10t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j) (\lambda_{j,10s,t} + \lambda_{j,10t,t}) & \\
 - r10ns_{m,i_m,t} [ \lambda_{10t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j) \lambda_{j,10s,t} ] & \\
 - r30s_{m,i_m,t} [ \lambda_{30t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j) \lambda_{j,30t,t} ] & \\
 - r30ns_{m,i_m,t} [ \lambda_{30t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j) \lambda_{j,30t,t} ] & \\
 - reg_{m,i_m,t} \lambda_{reg,t} & \quad (20)
 \end{aligned}$$

Then the dual cost can be express as follows.

$$Dual\ Cost = \sum_{m=1}^M \sum_{i_m=1}^{I_m} DualCost_{m,i_m} + Extra \quad (21)$$

where *Extra* is the difference between equations (19) and (21). It could be seen that the term *Extra* does not depend on the status of power generators, and is a constant if the values of multipliers are given.

A subgradient method is used to solve the dual problem. The value of  $\lambda_t$  is initialized first, and its value can be used to minimize dual cost function. Because the term *Extra* is a constant, we just need to minimize the term  $DualCost_{m,i_m}$  individually. Dynamic problem is used to solve the single generator problem. To calculate the one-step reward, we need to optimize the allocation of generation services and ancillary services when a generator is on or off. When a generator is on, it could provide generation services, spinning reserve services, and regulation services. The one-step cost optimization problem (22) should be solved subject to constraints (11), (13), (14), and (15). On the other hand, when a generator is off, it could provide non-synchronous reserve services. Another one-step cost optimization problem (23) should be solved subject to constraints (17) and (18).

$$\begin{aligned}
 \min \{ & F(p_{m,i_m,t}) + R_{m,i_m,t}(r_{m,i_m,t}) \\
 & + CReg_{m,i_m,t}(reg_{m,i_m,t}) \\
 & - p_{m,i_m,t}(\lambda_{d,t} + \sum_{l=1}^L \lambda_{tran,l,t} \Gamma_{l,m}) \\
 & - r10s_{m,i_m,t}[\lambda_{10s,t} + \lambda_{10t,t} \\
 & + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)(\lambda_{j,10s,t} + \lambda_{j,10t,t})] \\
 & - r30ns_{m,i_m,t}[\lambda_{30t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)\lambda_{j,30t,t}] \\
 & - reg_{m,i_m,t}\lambda_{reg,t} \quad (22)
 \end{aligned}$$

$$\begin{aligned}
 \min \{ & R_{m,i_m,t}(r_{m,i_m,t}) \\
 & - r10ns_{m,i_m,t}[\lambda_{10t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)\lambda_{j,10s,t}] \\
 & - r30ns_{m,i_m,t}[\lambda_{30t,t} + \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)\lambda_{j,30t,t}] \quad (23)
 \end{aligned}$$

General linear programming techniques can be used to solve these two problems; however, it is time consuming. Since piecewise generation cost functions and linear ancillary service cost functions are used in this work, we propose to use a greedy algorithm, which can significantly reduce the computational time. Let

$$F(p) = b_0 + \begin{cases} b_1 p & a_0 \leq p < a_1 \\ (b_1 - b_2)a_1 + b_2 p & a_1 \leq p < a_2 \\ \dots & \dots \\ \sum_{k=2}^K (b_{k-1} - b_k)a_{k-1} + b_k p & a_{K-1} \leq p \leq a_K, \end{cases}$$

where we have  $b_1 < b_2 < \dots < b_k$  and  $0 = a_0 < a_1 < \dots < a_K$ . We can transform it into another formula

$$F(p) = G(x_1, x_2, \dots, x_K) = b_0 + \sum_{k=1}^K b_k x_k,$$

where  $p = \sum_{k=1}^K x_k$ ,  $0 \leq x_1 \leq a_1 - a_0$  and  $0 \leq x_k \leq (a_k - a_{k-1}) \cdot \mathcal{I}(x_{k-1} = a_{k-1} - a_{k-2})$ ,  $k \geq 2$ . The ancillary service costs are formulated as below:

$$\begin{aligned}
 R(r) = & rc10s \cdot r10s + rc10ns \cdot r10ns \\
 & + rc30s \cdot r30s + rc30ns \cdot r30ns
 \end{aligned}$$

$$CReg(reg) = creg \cdot reg,$$

where  $rc10s$ ,  $rc10ns$ ,  $rc30s$ ,  $rc30ns$ , and  $creg$  are constant cost coefficients. Then equation (22) can be equivalently written as follows.

$$\min F'_{m,i_m,t}(p_{m,i_m,t}) + R'_{m,i_m,t}(r_{m,i_m,t}) + CReg'_{m,i_m,t}(reg_{m,i_m,t}) \quad (24)$$

where

$$\begin{aligned}
 F'_{m,i_m,t}(p_{m,i_m,t}) & = G'(x_1, x_2, \dots, x_K) \\
 & = b_0 + \sum_{k=1}^K (b_k - \lambda_{d,t} - \sum_{l=1}^L \lambda_{tran,l,t} \Gamma_{l,m}) x_k
 \end{aligned}$$

$$\begin{aligned}
 R'_{m,i_m,t}(r_{m,i_m,t}) & = [rc10s_{m,i_m,t} - \lambda_{10s,t} - \lambda_{10t,t} \\
 & - \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)(\lambda_{j,10s,t} + \lambda_{j,10t,t})] \cdot r10s_{m,i_m,t}
 \end{aligned}$$

$$+ [rc30s_{m,i_m,t} - \lambda_{30t,t} - \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)\lambda_{j,30t,t}] \cdot r30s_{m,i_m,t}$$

$$CReg'_{m,i_m,t}(reg_{m,i_m,t}) = (creg_{m,i_m,t} - \lambda_{reg,t}) \cdot reg_{m,i_m,t}$$

and equation (23) is equivalent to (25):

$$\min R'_{m,i_m,t}(r_{m,i_m,t}), \quad (25)$$

where

$$\begin{aligned}
 R'_{m,i_m,t}(r_{m,i_m,t}) & = [rc10ns_{m,i_m,t} - \lambda_{10t,t} - \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)\lambda_{j,10t,t}] \cdot r10ns_{m,i_m,t} \\
 & + [rc30ns_{m,i_m,t} - \lambda_{30t,t} - \sum_{j=1}^J \mathcal{I}(m \in \Lambda_j)\lambda_{j,30t,t}] \cdot r30ns_{m,i_m,t}
 \end{aligned}$$

When a generator is "on", the greedy algorithm works as follows:

- 1) Initialize the power generation and ancillary service levels to 0.
- 2) Sort the linear cost coefficients, including those in generation cost function  $F'$  and ancillary cost functions  $R'$  and  $CReg'$ , in equation (24), to form a non-decreasing list  $\{c_h\}$ , where  $h \in \{1, 2, \dots, K+3\}$ .
- 3) Let  $p_{m,i_m,t} = p_{m,i_m,min}$ .
- 4) From  $h = 1$  to  $h = K+3$ , consider the following cases:
  - a)  $c_h$  is a generation cost coefficient; if  $c_h > 0$ , stop; else,  $c_h$  should be the generation cost coefficient of the  $k$ th segment and

$$a_k + r10s_{m,i_m,t} + r30s_{m,i_m,t} \leq p_{m,i_m,max},$$

then  $p_{m,i_m,t} = \max(a_k, p_{m,i_m,min})$ .

- b)  $c_h$  is a 10-minute spinning reserve cost coefficient; if  $c_h > 0$ , stop; else,

$$r10s_{m,i_m,t} = \min(r10s_{m,i_m,max},$$

$$p_{m,i_m,max} - p_{m,i_m,t} - r30s_{m,i_m,t}).$$

- c)  $c_h$  is a 30-minute spinning reserve cost coefficient;

if  $c_h > 0$ , stop; else

$$r30s_{m,i_m,t} = \min(r30s_{m,i_m,max}, p_{m,i_m,max} - p_{m,i_m,t} - r10s_{m,i_m,t}).$$

- d)  $c_h$  is a regulation cost coefficient; if  $c_h > 0$ , stop; else,  $r30s_{m,i_m,t} = reg_{m,i_m,max}$ .

When a generator is “off”, a similar algorithm can be applied:

- 1) Initialize ancillary service levels to 0.
- 2) Sort the linear cost coefficients of ancillary cost function  $R'$  to form a non-decreasing list  $\{c_1, c_2\}$ .
- 3) For  $h = 1$  or 2, consider the following cases:
  - a)  $c_h$  is a 10-minute non-synchronous reserve cost coefficient; if  $c_h > 0$ , stop; else,

$$r10s_{m,i_m,t} = \min(r10ns_{m,i_m,max}, p_{m,i_m,max} - r30ns_{m,i_m,t}).$$

- b)  $c_h$  is a 30-minute non-synchronous reserve cost coefficient; if  $c_h > 0$ , stop; else,

$$r30s_{m,i_m,t} = \min(r30ns_{m,i_m,max}, p_{m,i_m,max} - r10ns_{m,i_m,t}).$$

### B. A Parallel computing scheme

In the LR algorithm, the dual problem is decomposed into identical single generator problems. Therefore, it is natural to assign those problems to individual CPU's and solve them simultaneously. In every iteration, the root CPU will “broadcast” the values of lagrangian multipliers to the branch CPU's, where the single generator sub-problems are solved, and the solutions are “collected” to the root CPU to update the value of the multipliers. The computing scheme is illustrated in figure III-B. Suppose there are  $N$  power generators, then the computational time required to solve the dual problem can be estimated by equation (26) if the computational load is equally distributed to each branch CPU.

$$t_{dual} \approx \left( \frac{N}{\# \text{ of CPUs}} \times t_{single} + t_{update} + t_{comm} \right) \times n_{iteration} \quad (26)$$

where  $t_{dual}$  is the computational time needed to solve the dual problem,  $t_{single}$  is the computational time in solving a single generator problem,  $t_{comm}$  is the communication time between the root CPU and branch CPU's, and  $n_{iteration}$  is the number of iterations in the subgradient search process.

## IV. PROBABILISTIC UNIT COMMITMENT MODEL AND ROLLING HORIZON (RH) SCHEME

For simplicity, we only consider the probabilistic reserve constraints. Without loss of generality, we can replace the reserve constraint in Section II-C with the following constraint (27).

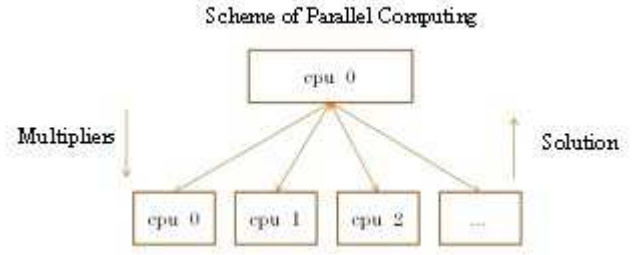


Figure 1. Parallel Computing Scheme to Solve UCP

$$\mathcal{P}\left\{ \sum_{m=1}^M \sum_{i_m=1}^{I_m} p_{m,i_m,t} + \sum_{m=1}^M \sum_{i_m=1}^{I_m} res_{m,i_m,t} \geq \sum_{m=1}^M d_{m,t} - \sum_{m=1}^M w_{m,t}, \forall t \right\} \geq 1 - \alpha, \quad (27)$$

where  $res_{m,i_m,t}$  is the general reserve service level. Using Bonferroni's inequality, we can transform equation (27) to another equation (28).

$$\mathcal{P}\left\{ \sum_{m=1}^M \sum_{i_m=1}^{I_m} p_{m,i_m,t} + \sum_{m=1}^M \sum_{i_m=1}^{I_m} res_{m,i_m,t} \geq \sum_{m=1}^M d_m - \sum_{m=1}^M w_{m,t}, \right\} > 1 - \frac{\alpha}{T} \quad (28)$$

Here, we assume that  $w_{m,t}$  follows a normal distribution  $N(\mu_{m,t}^w, (\sigma_{m,t}^w)^2)$ . Thus, the above equation can be written as (29).

$$\sum_{m=1}^M \sum_{i_m=1}^{I_m} p_{m,i_m,t} + \sum_{m=1}^M \sum_{i_m=1}^{I_m} res_{m,i_m,t} \geq \sum_{m=1}^M d_m - \sum_{m=1}^M \mu_{m,t}^w + z_{1-\frac{\alpha}{T}} \sum_{m=1}^M \sigma_{m,t}^w \quad (29)$$

Equation (29) may not accurately describe the reserve requirement, because the wind power in different zones are in general correlated. Nevertheless, for simplicity we assume that they are independent, and the general correlated case will be considered in our future research. This reserve constraints is very similar to the constraints in Section II-C, and a similar LR algorithm can be applied to solve the problem.

If we assume that wind power forecasts are updated every hour, we can update equations (2) and (29) and solve the corresponding UCP on a hourly basis. The RH scheme considers the updated wind power information in both unit commitment and economic dispatch processes, while in traditional day-ahead scheme, the updated wind power information is only considered in economic dispatch process. Thus, by involving more accurate information in the optimization process, we expect to get better solutions with decreased operational costs.

## V. CASE STUDIES

### A. New York Control Area

To study the effectiveness of our approach on large scale problems, we simulated a SCUC problem based on the characteristics of New York State control Area. NYCA is divided into 11 sub-zones with transmission interface between adjacent sub-zones. The detailed zone map is given in figure V-A. One feature of power grid in New York State is that most of the electricity demand comes from the southeast area of New York state, i.e., Long Island and New York City, while a large portion of the power resources are located in the west and north parts of the state. Additionally, in the near future, most of the power farms will be located in zones *A – E* [17], and will bring more burden to transmission lines. This uneven distribution of power generation sources and power demand makes transmission constrained unit commitment an important problem in NYCA. Moreover, locational reserve requirements are enforced to maintain the safety operation of the power grid.

We follow the practice of NYISO and divide NYCA into two super-zones, where west super-zones include load zones *A – E* and east super-zones include zones *F – G*. Additional reserve requirements are enforced for east super-zones. In addition, similar reserve requirements are also enforced on zone *K*, which is Long Island. The reserve requirements can be formulated in the forms of equations (6), (7), and (8). The transmission constraints are formulated in the form of equation (10).

In accordance with the day-ahead power market in New York State [18], piecewise linear generation cost functions and stepwise startup cost functions are used. Each generation cost function can have up to 12 pieces. A total of 641 power generators, including nuclear plants, hydro plants, steam plants, and gas turbines, are simulated in this work. The net load for each zone is calculated by subtracting the forecasted wind power from the forecasted electricity demands. Four different wind penetration level cases are used:  $1275MW$ ,  $4250MW$ ,  $6000MW$ , and  $8000MW$ . According to the penetration level, the regulation requirement is adjusted as proposed in [17]. A single day (24 hour period) in August is used for the study. Because currently, the report [17] by NYISO indicates that the current reserve level is enough for the  $8000MW$  penetration of wind power, so we will not consider the probabilistic reserve level management in this case.

The LR algorithm was coded in C++ and implemented on New York Blue Gene, a distributed-memory supercomputing cluster. Up to 50 nodes were used, while each node has two  $700MHz$  PowerPC processors and  $1G$  DDR memory. Figure 3 shows the computational time required to execute the algorithm v.s. the number of CPUs used. The minimum computational time is around 180 seconds, which is much less than the 1600 seconds computational time when 1 CPU is used. The total operation costs, which is the sum of generation costs and ancillary costs, are given in table I. It is interesting to note that the costs are all negative. This is reasonable because

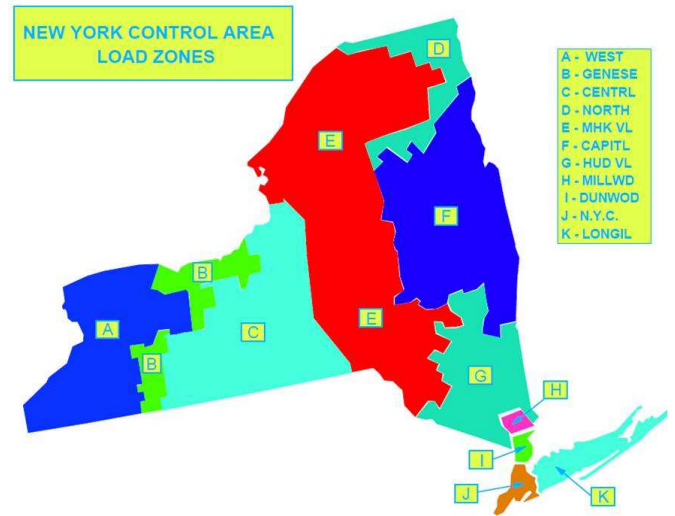


Figure 2. New York State Control Area

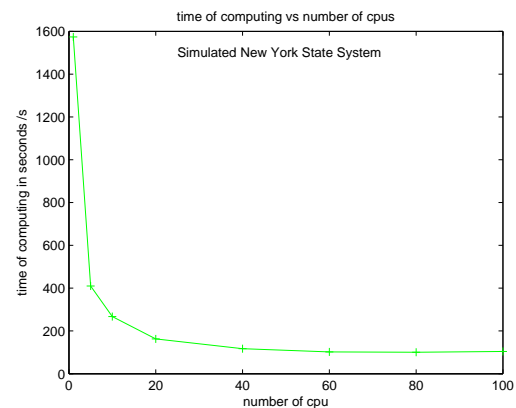


Figure 3. Computational time in seconds *vs* number of CPU's for NYCA case

for some IPP's, they want to assure that some plants will be selected for generation, for example, nuclear plants and some coal steam plants. Because whatever they bid for generation, they will be paid by the positive market clearing price. Thus, they have the incentive too keep those cheap power sources online. From the table, it is obvious that high penetration of wind power will save money for the New York control area. The plots for marginal regulation costs are given in figure 4. Because the increment in regulation requirements, the marginal costs for regulation generally increase. Because of the location-based reserve services, different sub-zones might have different marginal reserve cost services. Besides, we note that the Lagrangian multiplier for transmission constraints increases as the penetration level of wind energy grows. This is reasonable for New York state because most of the wind power resources in NYCA are located in north parts while most of electricity consumption are located in southeast regions. Thus, the increased power penetration will bring more pressure on transmission lines in New York State.

Penetration Level (MW)	Operation Costs
1275	$-1.12 \times 10^7$
4250	$-1.23 \times 10^7$
6000	$-1.27 \times 10^7$
8000	$-1.30 \times 10^7$

Table I

TOTAL OPERATION COSTS FOR DIFFERENT PENETRATION LEVEL OF WIND POWER

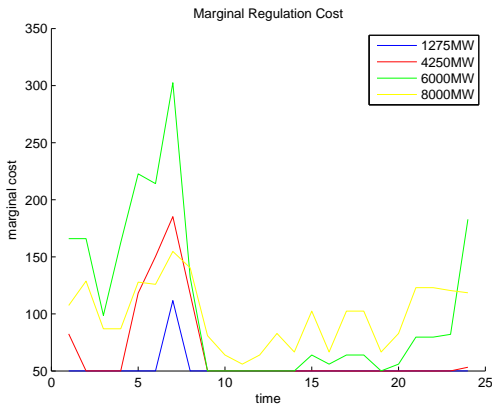


Figure 4. Marginal Regulation Costs for different wind penetration level

B. Rolling horizon study

The proposed method has been applied to solving large sized problems based on the ten-unit system of [19], which has been repeated 100 times so that the problem comprises 1000 units. The generator parameters are slightly perturbed because it is unrealistic to have so many identical generators. The load profile is based on the System D in [20], which has been multiplied by 100 accordingly. We assume 25% of wind penetration level, which is close to the 8000MW case in NYCA. Figure 5 shows that the computational time decreases dramatically when the number of processors increases from 1 to 20, and the minimum computational time is about 20% of the sequential computational time. This result is not as good as that for the NYCA case, which involves much more constraints. For NYCA case, the computational time decreases from 1600 seconds (around 0.5 hours ) to 3 minutes, which make it reasonable to restart the UCP solver every hour when new information is available.

The result of rolling horizon approach was compared with the traditional day-ahead planning method. In current market, the stochastic problem was solved once every day, and only dispatch problem was solved when the real data was available. The operation costs of the next 24 hours of both approaches were compared. The result is shown in Figure 6. A significant reduction of cost is observed when applying rolling horizon approach. For stochastic problem with wind energy, the cost reduction (approximately 3%) is more significant.

VI. CONCLUSION

In this paper, we have formulated a security constrained unit commitment problem by incorporating complex ancillary

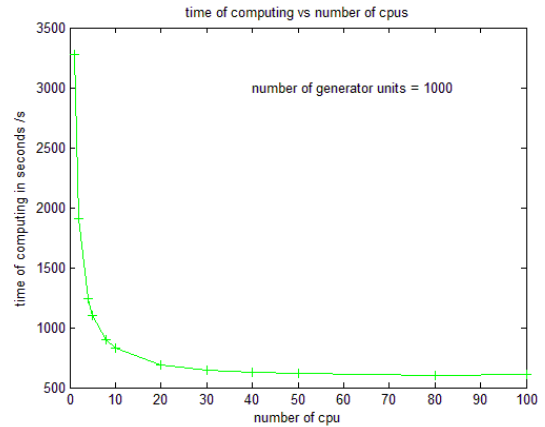


Figure 5. Relationship between computational time (in seconds) and number of CPU's

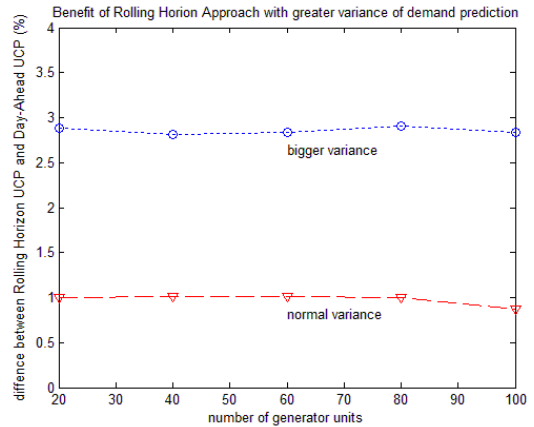


Figure 6. Comparison between Rolling Horizon Approach and Day-ahead Approach

services, security, and local reserve constraints, and applied this model to the New York Control Area. We investigated the impact of the increasing penetration of wind power on the New York state day-ahead power market. Additionally, the test results show that parallel computing can significantly reduce the computational time, which makes it possible for rolling horizon implementation of the algorithm. Our testing results on a standard test system show that the rolling horizon approach may lead to significant cost reduction over the traditional day-ahead approach.

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- $r30ns_{m,i_m,t}$  30-minute non-synchronous reserve service level of generator  $i$  in zone  $m$  at time period  $t$ .
- $r_{m,i_m,t}$  reserve service vector defined as  $(r10s_{m,i_m,t}, r10ns_{m,i_m,t}, r30s_{m,i_m,t}, r30ns_{m,i_m,t})$ .
- $CReg_{m,i_m,t}$  regulation cost function of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $reg_{m,i_m,t}$  regulation service level of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $d_{t,m}$  prediction of electricity demand of time period  $t$  in zone  $m$ .
- $w_{t,m}$  prediction of wind power of time period  $t$  in zone  $m$ .
- $Res_{10s}$  10-minute spinning reserve requirement for the whole ISO control area.
- $Res_{10t}$  10-minute total reserve requirement for the whole ISO control area .
- $\Lambda_j$  super-zone  $j$  or the  $j$ th collection of zones.
- $ResLB_{j,10s,t}$  10-minute spinning reserve requirement for sub control area  $j$  at time  $t$ .
- $ResLB_{j,10t}$  10-minute total reserve requirement for sub control area  $j$  at time  $t$ .
- $p_{m,i_m,max}$  maximum output when generator  $i_m$  in zone  $m$  is on.
- $p_{m,i_m,min}$  minimum output when generator  $i_m$  in zone  $m$  is on.
- $\Gamma_{l,m}$  line flow distribution factor for the transmission line  $l$  due to the net power injection of zone  $m$
- $Tran_{i,m,max}$  maximum transmission capacity of transmission line  $l$  in designate direction.
- $Ms_{i_m,m}$  maximum number of times that generator  $i_m$  in zone  $m$  is allowed to be shut down.

## APPENDIX

## NOMENCLATURE

- $p_{m,i_m,t}$  electricity output level of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $z_{m,i_m,t}$  binary variable that is 1 if generator  $i_m$  in zone  $m$  is on during time period  $t$ ; 0 otherwise.
- $F_{m,i_m,t}$  fuel cost function of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $S_{m,i_m,t}$  startup cost function of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $R_{m,i_m,t}$  reserve service cost function of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $r10s_{m,i_m,t}$  10-minute spinning reserve level of generator  $i$  in zone  $m$  at time period  $t$ .
- $r10ns_{m,i_m,t}$  10-minute non-synchronous reserve service level of generator  $i_m$  in zone  $m$  at time period  $t$ .
- $r30s_{m,i_m,t}$  30-minute spinning reserve level of generator  $i$  in zone  $m$  at time period  $t$ .