

Controlling a group of microCHPs: planning and realization

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Abstract—This paper discusses the planning problem of a group of domestic Combined Heat and Power (microCHP) appliances, which together form a Virtual Power Plant (VPP). To act on an electricity trading market, this VPP has to specify a production plan for electricity for given times of the day to offer to this market. These amounts have to be delivered exactly when these times arrive; moreover, deviations from these contracts are penalized for. We focus on the planning of individual microCHPs for one day ahead, given that the aggregated output of the group should fulfill a desired production pattern that the VPP wants to offer on the market. The contribution in this context is twofold. Firstly, we present a planning approach based on column generation which calculates for all individual appliances production patterns. The production patterns are calculated such that the deviation of the aggregated pattern of all appliances from a prespecified pattern is minimized. Secondly, we investigate how a desired pattern for the group can be specified based on global parameters and which patterns can be realized afterwards by the developed planning approach. In this way we get insight what kind of pattern may be offered on the market. The presented results show that we can find near optimal solutions using a column generation technique and that we can offer patterns with large variation on the market, as long as the running average does not deviate too much from the possible production.

Keywords - *column generation; scheduling*

I. INTRODUCTION

During the last years the electricity supply chain (the production, consumption, distribution, storage and load management of electricity) has changed. Important aspects in this change are the increase of the availability of renewable energy sources, the development of distributed electricity generators and the demand for more energy efficient appliances [1].

A specific type of upcoming distributed electricity generators is microCHP (Combined Heat and Power on a domestic scale). When running, a microCHP produces both heat and electricity for household usage at the kW level. The produced electricity can be delivered back to the electricity grid or consumed locally. The control of the microCHP is heat led, meaning that the heat demand of the building defines the possible production of heat and, simultaneously, the possible electricity output. Combined with a heat buffer, the production of heat and electricity can be decoupled to some extent and an operator has flexibility in the times that the microCHP is producing, which creates a certain degree of freedom in electricity production. This freedom of electricity production

may be used to increase stability in the grid, on a large scale it may allow to replace a conventional power plant, and more. To effect these possibilities the individual microCHPs need to be controlled, as standalone devices, but also in cooperation with other microCHPs or generators in a so-called Virtual Power Plant (VPP) (see for a German example [2]). For possible methodologies of the control of a VPP we refer to [3], [4] and [5].

The production of a VPP can be traded on different markets, from which the day-ahead electricity trading market is one option. To act on this market, the VPP has to provide an offer a day ahead and has to match this offer in real-time by the group of generators the next day. In this paper we investigate for a fleet of microCHPs how to 1) use a good day ahead planning to provide and afterwards match an offered production and 2) derive a simple guideline on how such an offer could look like.

The paper is organized as follows. In Section II, an introduction on the electricity trading market and on column generation is given. Section III gives a more detailed formulation of the considered problem, the way column generation is applied to it and a derivation of a lower bound on the quality of the achieved solutions. In Section IV, a scenario is proposed; results are given in Section V. The paper ends with some conclusions and future work.

II. RESEARCH FOCUS

The research in this paper focuses on the application of a column generation technique to find a planning for a group of microCHPs to reach a prespecified production pattern on the electricity trading market. To understand the idea of the planning problem, first a short introduction of the relevant electricity market is given. Then the general idea of the column generation approach is explained.

A. Electricity trading market

For electricity trading many different type of markets exist around the world, ranging from long term trading up to real-time balancing. For the setting considered in this paper, mainly the day ahead markets are of interest, i.e. the markets where electricity is traded for one day ahead, e.g., [6], [7]. In this market electricity is traded on an hourly basis, whereas on

intraday (balancing) markets trade is on a 15 minute basis [6]. Bids can be made for specific hours and on blocks of consecutive hours. The electricity is traded in multiples of 100 kWh. When the market is cleared, the blocks that were traded need to be generated on the next day as specified in the contracts. Penalty costs are incurred when the contracts cannot be supplied.

B. Column generation

In this paper we apply a general method called column generation to find a good solution for our specific optimization problem in reasonable computation time. The idea of column generation is to divide the given problem in two parts: a main problem and (a) sub problem(s). The main problem consists in principle of the original problem but reduced to a small part of the solution space, indicated by a set of solution vectors, the so-called 'columns'. Based on the solution for this (restricted) main problem, new columns are added to the solution space, as long as they improve the objective value of the main problem. In the sub problem(s) the new columns are found, by optimizing on the added value of the new columns, taking into account the current solution of the main problem.

Column generation is widely applied in practice. An example of the use of this technique is given in [8], where it is applied to solve the Cutting Stock Problem. It has also been used to solve the Vehicle Routing Problem, as in [9].

III. APPROACH

The planning problem for a group of microCHPs on the one hand focuses on the individual control of the microCHPs, installed in (small) buildings, satisfying the local (household) constraints, but on the other hand aims at achieving an aggregated electricity output of the group which can be traded on a day ahead market and, thus, has to fulfill specified requirements. In general, this problem is NP-complete in the strong sense [10].

A concrete formulation of the problem is given in Section III-A. Next, a column generation technique is explained in Section III-B, which is a heuristic method that is used to solve the problem in reasonable computation time. Finally, in Section III-C lower bounds are derived for the problem, to analyze the quality of the developed heuristic method and to analyze the impact of choices for a specific offer on the electricity trading market.

A. Problem formulation

The main objective for a VPP, when acting on a market, is to maximize its profit (or revenue). For this, the VPP has to decide on which times of the day it wants to produce how much electricity. The overall amount of electricity a VPP consisting of a group of microCHPs can offer to the market is limited by the individual production capacities of the microCHPs. These can be calculated based on predictions of the local heat demand. However, for trading, not only the overall capacity of the VPP but its deviation over the time periods of a day needs to be specified to be able to act on

the day ahead market. For this, a planning of the production of all individual microCHP's is needed. This planning may be steered by bounds on the overall production profile, whereby these bounds result from possible ways to act on the market.

A feasible planning of a microCHP naturally should respect the individual heat demand within the building where the microCHP has been installed. In addition, the total electricity production should be within the given bounds resulting from possible actions on the market. The goal of the planning problem in this paper is to find such a feasible planning, in contrast to a planning with the objective of maximizing profit in the market.

Let microCHPs $i = 1, \dots, N$ be given and let the time horizon of 24 hours be discretized in time intervals $j = 1, \dots, N_T$. Furthermore, let P be the set of possible binary production patterns for the type of microCHP that is used. These patterns do not take into account heat demand requirements or total desired electricity production, but include the (technical) restrictions of the microCHP. Note that the set P can be extremely large.

In general, the offered bounds on the market (i.e., the desired production pattern) are represented by upper and lower bound vectors $P^{upper} = (P_1^{upper}, \dots, P_{N_T}^{upper})$ and $P^{lower} = (P_1^{lower}, \dots, P_{N_T}^{lower})$. These vectors specify per time period a lower and upper bound on the total production of the microCHPs that should be generated during that time period. A pattern $p \in P$ for a microCHP is defined as a binary vector $p = (p_1, \dots, p_{N_T})$ where $p_j \in \{0, 1\}$ specifies whether the microCHP is on or off. The problem is to pick exactly one pattern for each microCHP, such that the sum of all production patterns falls between the lower and upper bound of the desired production pattern in all time intervals. For this, let $F_i \subset P$ be a subset of all possible production patterns that takes the local heat demand into account of the building where microCHP i is installed, in case that a heat buffer is used. Thus, this set specifies the locally feasible patterns for microCHP i . The choice for using this set is explained in Section III-B, where we restrict ourselves to solving the problem by only using those production patterns that can supply the heat demand.

The planning problem for the VPP now can be formulated by the following Integer Linear Program (ILP):

$$\min \sum_{j=1}^{N_T} (sl_j + ex_j) \quad (1)$$

$$\sum_{i=1}^N \sum_{p \in F_i} p_j y_{ip} + sl_j \geq P_j^{lower} \quad j = 1, \dots, N_T \quad (2)$$

$$\sum_{i=1}^N \sum_{p \in F_i} p_j y_{ip} - ex_j \leq P_j^{upper} \quad j = 1, \dots, N_T \quad (3)$$

$$\sum_{p \in P} y_{ip} = 1 \quad i = 1, \dots, N \quad (4)$$

$$sl_j, ex_j \geq 0 \quad j = 1, \dots, N_T \quad (5)$$

$$y_{ip} \in \{0, 1\}, \quad (6)$$

where y_{ip} is a binary decision variable indicating whether pattern $p \in P$ is chosen for generator i (in this case $y_{ip} = 1$)

or not ($y_{ip} = 0$). In Equations (2) and (3) slack and excess variables sl_j and ex_j are introduced to calculate the deviation from the desired (and predefined) production pattern (P^{upper}, P^{lower}). The sum of slack and excess variables is minimized in Equation (1). Finally, Equation (4) requires that exactly one pattern is chosen for each generator.

A feasible planning is achieved when the sum of slack and excess variables equals 0. If no feasible planning can be found, the objective value is a measure of the deviation from the desired production pattern.

B. Column generation

The problem formulated by equations (1)-(6) takes into account locally feasible production patterns from the sets F_i . These sets however are still very large and, therefore, it is already difficult to solve the problem for instances with 10 microCHPs if all possible patterns are added to the sets F_i [11]. For this reason a column generation technique is applied.

The column generation technique starts with a relatively small set of feasible patterns $S_i \subset F_i$ for each microCHP i . By looking at only a small set of patterns the problem can be solved relatively fast. However, this comes with a possible loss of patterns that are necessary for a high quality solution. The VPP might perform better, when some feasible production patterns from F_i not in S_i would be added to the set S_i . Unfortunately, we do not know on beforehand which patterns are useful in the final solution. Therefore it is the idea of the column generation technique to improve the current solution step by step, by searching for those patterns which promise to improve the current solution, and by adding these patterns to the (small) feasible pattern sets S_i of the corresponding microCHPs. We have chosen to expand the pattern set S_i by at most one pattern per iteration as the heuristic evolves.

The column generation technique uses a main problem and sub problems as indicated in Algorithm 1. The main problem is similar to equations (1)-(6), with the only difference that the set F_i is replaced by S_i :

$$\min \sum_{j=1}^{N_T} (sl_j + ex_j) \quad (7)$$

$$\sum_{i=1}^N \sum_{p \in S_i} p_j y_{ip} + sl_j \geq P_j^{lower} \quad j = 1, \dots, N_T \quad (8)$$

$$\sum_{i=1}^N \sum_{p \in S_i} p_j y_{ip} - ex_j \leq P_j^{upper} \quad j = 1, \dots, N_T \quad (9)$$

$$\sum_{p \in P} y_{ip} = 1 \quad i = 1, \dots, N \quad (10)$$

$$sl_j, ex_j \geq 0 \quad j = 1, \dots, N_T \quad (11)$$

$$y_{ip} \in \{0, 1\}. \quad (12)$$

The second phase of the column generation technique consists of creating new patterns that can be added to the current pattern sets S_i for each microCHP in the main problem. These new patterns should contribute to the existing sets in the sense that they must give possibilities to decrease the objective value in the first phase (i.e., the sum of slack and excess). A new

pattern p is only added to S_i if it 1) promises to improve the existing solution and 2) is a locally feasible pattern ($p \in F_i$).

Let λ_j represent the shadow prices for equations (8) and (9), obtained from the optimal dual solution of the linear relaxation of (7)-(12). A pattern p only may improve the existing relaxed solution if:

$$\sum_{j=1}^{N_T} \lambda_j p_j > 1. \quad (13)$$

However, this does not necessarily mean that this pattern can be automatically selected in the new solution of the main problem, since newly added patterns of other microCHPs (by solving these sub problems) can lead to different choices for this specific microCHP. As a consequence, the main problem has to be solved for the new sets S_i completely in each iteration.

The second requirement (p is locally feasible) has not been formalized yet. The reason for this is that the constraints that we use for forcing this requirement are only used in the sub problem of the column generation technique. For the main problem it suffices to know that the patterns have been checked for feasibility before; these feasible patterns are given input data for the main problem. The feasibility check is controlled by using two parameter sets, specifying in each interval j the minimum production the microCHP generator i should have generated ($MinOn_{ij}$) and the maximum production the generator could have generated ($MaxOn_{ij}$) up to and including the current interval. These parameters $MinOn_{i,j}$ and $MaxOn_{i,j}$ are fixed in such a way that they fulfill technical runtime/off-time constraints of the microCHP and the heat demand requirements of the building (for details see [10]). Startup and shutdown phases are neglected.

Summarizing, the sub problems of the column generation are given by the following ILP formulation for all microCHPs i :

$$\max \sum_{j=1}^{N_T} \lambda_j p_j \quad (14)$$

$$\sum_{k=1}^j p_k \leq MaxOn_{ij} \quad j = 1, \dots, N_T \quad (15)$$

$$\sum_{k=1}^j p_k \geq MinOn_{ij} \quad j = 1, \dots, N_T \quad (16)$$

$$p_j \in \{0, 1\} \quad j = 1, \dots, N_T, \quad (17)$$

where from all locally feasible patterns the one is chosen that maximizes the added value to the main problem. If constraint (13) is satisfied, the pattern p is added to the set S_i .

To summarize, the solution method is given in Algorithm 1. In each iteration, the main problem is solved first, after which for each microCHP the sub problem is solved and new feasible and improving patterns are added. If at least one sub problem leads to an improvement, the routine is repeated.

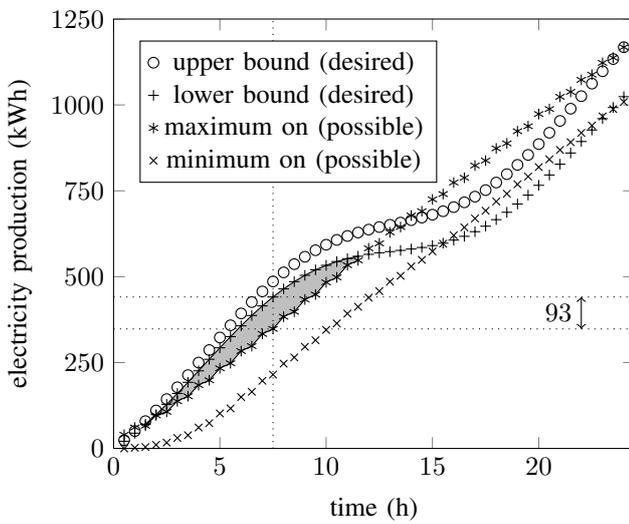
C. Lower bound of the objective

The lower and upper bounds P^{lower} and P^{upper} (representing the desired production pattern) and the possible production bounds $MaxOn_i$ and $MinOn_i$ form the basic input

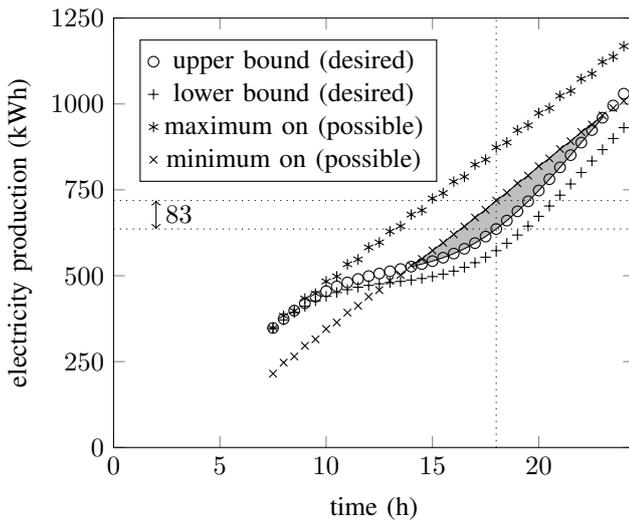
Algorithm 1 Column generation

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init  $S_i$  for all  $i$ ; finished  $\leftarrow$  false
while not finished do
    finished  $\leftarrow$  true
    solve main problem
    for all  $i$  do
         $p \leftarrow$  optimal solution of sub problem  $i$ 
        if  $\sum_{j=1}^{N_T} \lambda_j p_j > 1$  then
             $S_i \leftarrow S_i \cup p$ ;
            finished  $\leftarrow$  false
        end if
    end for
end while
    
```



(a) The total desired production and the total possible production result in a first phase lower bound



(b) The second phase of the lower bound calculation and the resulting lower bound improvement

Fig. 1. The calculation of the lower bound of the group planning problem

parameters of a problem instance. To derive a lower bound z_{LB} for the objective of the main problem, we concentrate on these parameters. Since we have a minimization problem and the sum of slack and excess variables cannot be negative, the theoretical lower bound z_{LB} is at least 0.

The calculation of the lower bound works in phases. In each phase a minimal guaranteed mismatch (slack or excess) z_{LB}^{extra} is found and added to the current lower bound.

In the first phase, the additional value of the lower bound z_{LB}^{extra} equals:

$$z_{LB}^{extra} = \max_j \begin{cases} \sum_{k=1}^j P_k^{lower} - \sum_i MaxOn_{ij} \\ \sum_i MinOn_{i,j} - \sum_{k=1}^j P_k^{upper} \\ 0. \end{cases} \quad (18)$$

This value equals the maximum deviation of the aggregated possible production from the aggregated desired production pattern. An example of this phase is shown in Figure 1(a), where the aggregated minimal mismatch per time interval is given by the gray area. In this example, the maximum difference between the maximal possible production and the minimal desired production is found at time 7.5, with a value of 93. So, in this example, the lower bound has now improved from 0 to 93.

The first value of j for which z_{LB}^{extra} is found is the starting point r for the calculation of the next phase. This starting point is important in two ways. First, the mismatch in previous intervals cannot be undone, if we only look at intervals $j > r$. Secondly, the starting point r offers a natural reset point; the sum of desired maximum (minimum) production upto and including interval r can be replaced by the maximum (minimum) possible production upto and including interval r . Other reset values either are not allowed (these total productions are not possible at r) or would increase the value of z_{LB}^{extra} . Considering the second option, these values do not represent the lower bound, since the chosen reset values are reachable (at least in the lower bound calculation) and give a smaller mismatch. So, $\sum_{k=1}^j P_k^{lower}$ is replaced by $\sum_i MinOn_{i,r} + \sum_{k=r+1}^j P_k^{lower}$ and $\sum_{k=1}^j P_k^{upper}$ by $\sum_i MaxOn_{i,r} + \sum_{k=r+1}^j P_k^{upper}$, and we look for mismatch in the future (time intervals $j > r$):

$$z_{LB}^{extra} = \max_{j>r} \begin{cases} (\sum_i MinOn_{i,r} + \sum_{k=r+1}^j P_k^{lower}) - \sum_i MaxOn_{ij} \\ \sum_i MinOn_{ij} - (\sum_i MaxOn_{i,r} + \sum_{k=r+1}^j P_k^{upper}) \\ 0. \end{cases} \quad (19)$$

In the example, the second phase calculation is shown in Figure 1(b), where an additional lower bound z_{LB}^{extra} of 83 is found. The lower bound is now: $z_{LB} = 93 + 83 = 176$.

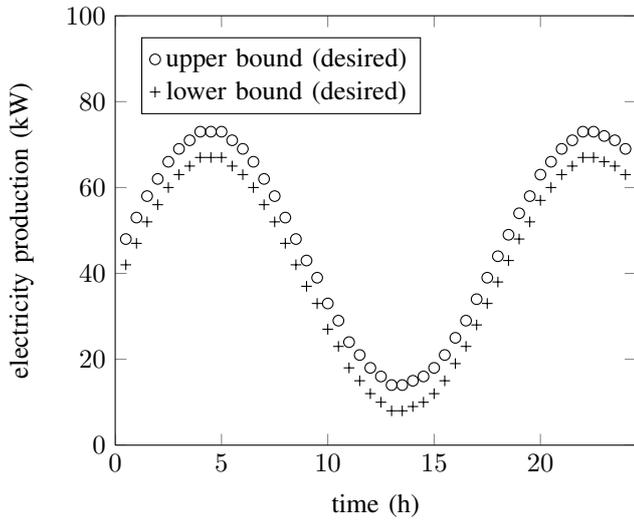


Fig. 2. An example of a desired production pattern; a sine with amplitude 30 and period 18

IV. SCENARIO

In this section, some computational results achieved with the presented approach are given. For this, a scenario is set up to answer two kinds of problems. First, the defined instances should provide a framework to test the quality of the column generation technique. Secondly, the instances should give an indication of the type of pattern that qualifies for being offered on the electricity trading market.

To support both questions, we focus on variation in the offered/desired patterns and keep the possible total production constant. The variation is created by adding a sine function to a constant profile, where we vary both amplitude and period of the sine function, i.e. the sine function characterizes the variation of the profile.

The instances consist of a group of 100 microCHPs and a discretization of the one day time horizon into 48 time intervals. Although this group size is too small to be able to act on the electricity market at the moment (microCHPs generates at the 1 kW level), this size already gives a good indication of the possibilities of the planning method. Decisions are made on an half an hour basis. The discretization in half hour periods is more fine grained than required, since the day ahead electricity market works on an hourly basis, however, for this granulation the planning problem gets more realistic (and more difficult). If needed, the production patterns can be simply converted to hourly blocks.

The maximum and minimum possible production $MaxOn_i$ and $MinOn_i$ are chosen to differ per microCHP, i.e. the heat demand differs per building. The aggregated values of all microCHPs are already given in Figure 1.

The initial patterns in the sets S_i are derived from the parameters $MaxOn_i$ and $MinOn_i$. Each microCHP sub problem starts with two patterns, one resulting from the earliest possible time intervals that the microCHP can be switched on, and one resulting from the latest possible time intervals that the microCHP has to be switched on.

Upper and lower bounds of the desired production are

defined as follows. The upper bound P^{upper} is derived from the highest integer value of μ^{upper} for which a given amplitude amp and period per result in a total desired production that is still feasible, when only looking at the total possible production:

$$\max \mu^{upper} \quad (20)$$

$$\sum_j P_j^{upper} \leq \sum_i MaxOn_{i,N_T} \quad (21)$$

$$P_j^{upper} = rnd(amp \times \sin(f(per) \times j)) + \mu^{upper} \forall j, \quad (22)$$

where $f(per)$ is the frequency corresponding to the given period per and $rnd()$ is a rounding function. Likewise, the lower bound P^{lower} results from the lowest sine curve fitting in the possible minimum production:

$$\min \mu^{lower} \quad (23)$$

$$\sum_j P_j^{lower} \geq \sum_i MinOn_{i,N_T} \quad (24)$$

$$P_j^{lower} = rnd(amp \times \sin(f(per) \times j)) + \mu^{lower} \forall j. \quad (25)$$

Looking at the final time interval in Figure 1(a), we can conclude that the lower and upper bound of the example fit within the possible total production domain. Figure 2 gives the corresponding resulting individual values.

Based on the above, an instance is defined as a pair $I(amp, per)$ and a solution can be characterized by a tuple $(I(amp, per), z_{LB}, z_{found})$. For this paper, we choose $amp \in \{0, 1, \dots, 40\}$ and $per \in \{2, 3, \dots, 24\}$.

V. RESULTS

Figure 3 shows the lower bound and the best found solutions for the defined instances. The lower bound is plotted as a surface plot, whereas the found solutions are given as dots in the figure. For the example used throughout this paper, the found optimization value is 176.5, whereas the lower bound is 176, close to the lower bound.

In general, the column generation technique finds solutions, which are close to the derived lower bounds of the objective value. This means that when the total production capacity of the microCHPs is known and the variations on the average production per time interval is not to extreme, the column generation technique can find actual assignments of individual patterns to match the pattern. However, for small amplitudes the difference between the lower bound and the found value can be relatively large. The inability to find better patterns for these instances is probably due to the simple choice for the initial patterns in the sets S_i .

Regarding the quality of the eventual production pattern, Figure 3 shows that the period has the most influence on the deviation from the desired pattern. For small periods, the mismatch stays relatively low, even for large values of the amplitude. This indicates that we may use large variation in our market offer, as long as the sum of positive and negative deviations from the possible production is close to zero.

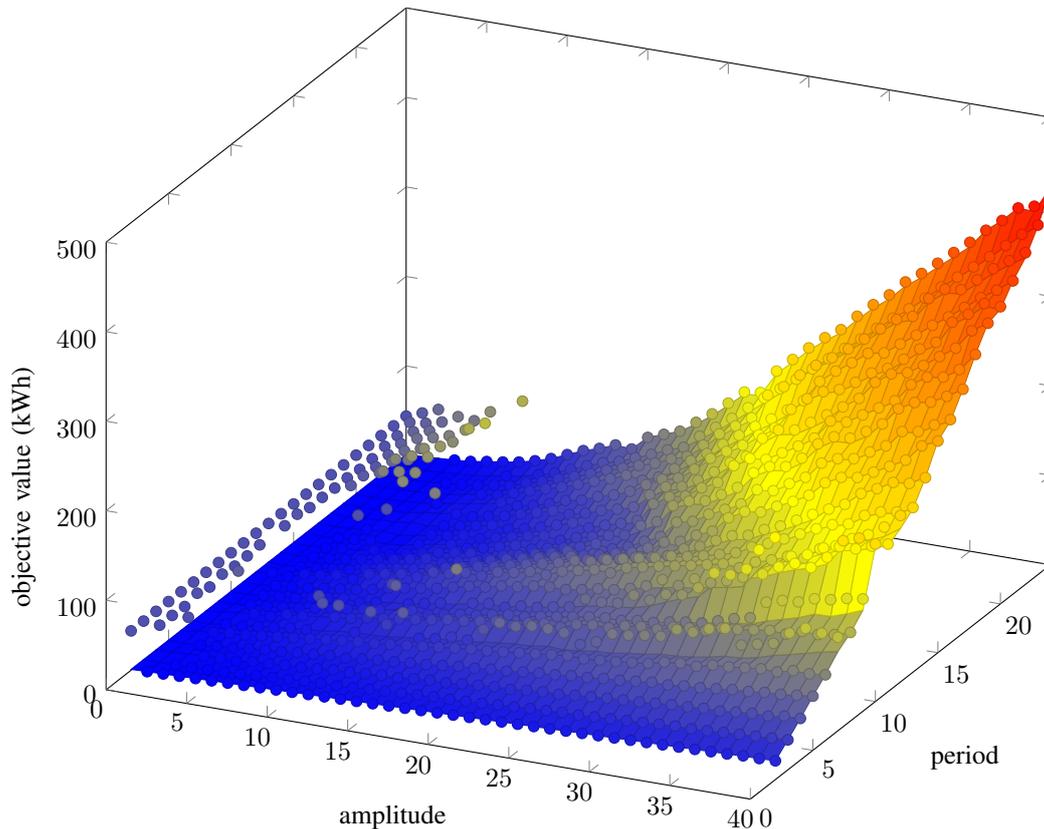


Fig. 3. Calculated lower bounds and solutions derived from the column generation technique, for sines with varying amplitude and period

VI. CONCLUDING REMARKS AND RECOMMENDATIONS

In this paper, we have presented an application of the column generation technique to the planning problem of a group of microCHPs. This technique finds near optimal solutions for a large fraction of the defined instances. It is possible to specify in advance a desired output pattern, which is calculated based only on global parameters, and afterwards create individual patterns which combine to an output of the whole group, which is close to the desired output. This enables the group to act on the electricity trading market by only using global information.

As a guideline to what kind of production pattern the group should offer on the market, we can state the following: large variations may occur, as long as they are 'corrected' in close-by intervals. In other words, we can offer patterns with large variation on the market, as long as the running average does not deviate too much from the possible production.

Future work - In future work, the influence of the initial pattern set on the resulting quality of the overall solution should be investigated. We believe that by a better choice the results also for small amplitudes improve.

In the current implementation of the column generation technique we neglected startup and shutdown times, opposite to the work in [11]. It is of interest to see the impact of adding these requirements, which results in a more accurate modeling of the electricity production.

Finally, to be able to trade on a real market, the group size has to increase. It has to be validated if the proposed planning

method also works for these larger groups.

VII. ACKNOWLEDGEMENTS

This research is conducted within the SFEER project (07937) supported by STW, Essent and GasTerra.

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