# Blocking Performance of Multi-Rate OCDMA Passive Optical Networks 

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#### Abstract

Optical Code Division Multiple Access (OCDMA) is one promising candidate for the provision of moderate security communications with large dedicated bandwidth to each end user. We present a new teletraffic model for the call-level performance of an OCDMA Passive Optical Network (PON) configuration that accommodates multiple serviceclasses. Parameters related to the local blocking, Multiple Access Interference (MAI) and user activity are incorporated to our analysis, which is based on a two-dimensional Markov chain. In this paper we focus on the derivation of recursive formulas for the efficient calculation of blocking probabilities. To evaluate the proposed model, the analytical results are compared with simulation results to reveal that the model's accuracy is quite satisfactory.


Keywords-PON; OCDMA; Multiple Access Interference; Loss; Blocking Probability; Recursive Formula; Markov chain.

## I. Introduction

The popularity of broadband applications, including bandwidth-hungry multimedia services and Internet Protocol Television (IPTV), has promoted the fiber-to-the-enduser as a feasible access networking technology. Among different optical access solutions, Passive Optical Networks (PONs) have received a tremendous attention from both industrial and academic communities, mainly due to the low operational cost, the enormous bandwidth offering and the absence of active components between the central office and the customers premises [1].

Over the years, several standards for PONs have evolved, in the form of the G. 983 ITU-T recommendations series, which include Asynchronous Transfer Mode PONs (ATMPONs), Ethernet PONs (EPONs) and Broadband PONs (BPONs). These architectures are based on a Time Division Multiple Access (TDMA) scheme and they typically use a 1550 nm wavelength for the downstream traffic and a 1310 nm for the upstream traffic [2]. While these TDMA-PONs employ two wavelengths for the upstream and downstream direction, respectively, the Wavelength Division Multiplexing (WDM)-PON utilizes multiple wavelengths, so that two wavelengths are allocated to each user for down/upstream
transmissions. A different approach for the provision of multiple access in PONs is the Optical Code Division Multiple Access (OCDMA). In contrast to the other multiple access schemes OCDMA can multiplex a number of channels on the same wavelength and on the same time-slot [3]. In addition, OCDMA offers full asynchronous transmission, soft capacity on demand, low latency access, simple network control and better security against unauthorized access [4].

In OCDMA, each communication channel is distinguished by a specific optical code. The encoding procedure involves the multiplication of each data bit by a code sequence either in the time domain [5], in the wavelength domain [6], or in a combination of both [7]. The decoder receives the sum of all encoded signals (from different transmitters) and recovers the data from a specific encoder, by using the same optical code. All the remaining signals appear as noise to the specific receiver; this noise is known as Multiple Access Interference (MAI) and is the key degrading factor of the networks performance.

To perform service differentiation in OCDMA networks, different solutions have been investigated. A simple approach is based on the utilization of multi-length codes [8]; however, under multi-length coding, short-length codes introduce significant interference over long-length codes, while high error probability emerges for high rate users. Optical fast-frequency hopping has been also proposed for multi-rate OCDMA networks [9]. This technique is based on multiple wavelengths; therefore it requires multi-wavelength transmitters with high sensitivity on power control. Another way to provide service differentiation is the assignment of several codes to each service-class. This procedure is known as the parallel mapping technique [10]. In this case the number of codes is proportional to the data rate of the assigned service-class.

The research activity on OCDMA networks is mainly focused on the performance of several OCDMA components; only a few analytical models have been presented in the literature involving the computation of blocking probabilities in OCDMA networks. Goldberg and Prucnal [11] provide
analytical models for the determination of blocking probabilities and for the teletraffic capacity in OCDMA networks. A similar study is performed in [12]. In both cases, a single service-class is considered, while the different sources of noise that are present in an OCDMA network are not taken into account. In [13] we provided a call-level analysis of hybrid WDM-OCDMA PONs, which is based on a similar teletraffic model for the call-level performance modeling of Wireless OCDMA (W-OCDMA) networks [14].

In [15] we proposed a call-level performance analysis of OCDMA PONs, where multiple service-classes of infinite traffic source population are accommodated. The shared medium is modeled by a two dimensional Markov chain. Based on it, we provided an approximate recursive formula for the calculation of blocking probabilities in the PON. The analysis takes into account the user activity, by incorporating different service times for active and passive (silent) periods. In this paper, we provide the proof of the recursive formula that describes the distribution of the occupied bandwidth in the PON that we presented in [15]. We present the necessary assumptions that have to be taken into account in order to derive the approximate recursive formula. The capacity of the PON is defined by the total interference caused by both active and passive users. An arriving call may be blocked if the resulting interference exceeds a predefined maximum threshold. This case defines the Hard Blocking Probability (HBP). A call may also be blocked in any other system state, due to the existence of different forms of additive noise (thermal noise, shot noise, beat noise). The latter case is expressed by the Local Blocking Probability (LBP). The accuracy of the proposed algorithm is evaluated through simulation and is found to be quite satisfactory.

This paper is organized as follows. Section II includes the system model. In Section III, we derive recursive formulas for the blocking probabilities calculation. Section IV is the evaluation section. We conclude in Section V.

## II. System Model

We consider the OCDMA PON of Fig. 1. A number of Optical Network Units (ONUs) located at the users' premises are connected to a centralized Optical Line Terminal (OLT) through a Passive Optical Splitter/Combiner (POSC ). The PO-SC is responsible for the broadcasting of traffic from the OLT to the ONUs (downstream direction) and for the grouping of data from the ONUs and the transmission of the collected data to the OLT through one fiber (upstream direction). We study the upstream direction; however the following analysis can be applied to the downstream direction. Users that are connected to an ONU switch between active and passive (silent) periods. The PON supports $K$ serviceclasses. The service-differentiation in this OCDMA system is realized using the parallel mapping technique. When a single codeword is applied to an active call, the interference that this call causes to the receiver is denoted by $I_{\text {unit }}$.


Figure 1. A basic configuration of an OCDMA PON

Since different service-classes require different data-rates, a number of codewords is assigned to each service-class; therefore the interference $I_{k}$ that an active service-class $k$ call causes to the receiver is proportional to $I_{\text {unit }}$. We define the bandwidth requirement $b_{k}$, of service-class $k$, as the number of codewords assigned to the specific service-class, which is equal to the ratio of $I_{k}$ to $I_{u n i t}$, therefore:

$$
\begin{equation*}
b_{k}=\frac{I_{k}}{I_{u n i t}} \tag{1}
\end{equation*}
$$

Calls that are accepted for service start an active period and may constantly remain in the active state for the entire duration of the call, or alternate between active and passive states. Throughout an active state, the traffic source sends bursts, while during a passive state no transmission of data occurs. When a call is transferred from the active state to the passive state, the bandwidth (which is expressed by the interference that this call produces to the receiver) held by the call in the active state is released and this bandwidth becomes available to new arriving calls. When a call attempts to become active again, it re-produces the same amount of interference (as in the previous active state); if the total interference at the receiver does not exceed a maximum value, a new active period begins; if not, burst blocking occurs and the call remains in the passive state. At the end of the active period the total interference at the receiver is reduced by $I_{k}$ and the call either jumps to the passive state with probability $v_{k}$, or departs from the system with probability $1-v_{k}$. At the end of the passive period the call returns to an active state only if the call will not be blocked due to the presence of the additive noise. Furthermore, calls that belong to service-class $k$ arrive to an ONU according to the Poisson process; the total arrival rate from all ONUs is denoted $\lambda_{k}$. The service time of serviceclass $k$ calls in state $i$, ( $i=1$ indicates the active state, $i=2$ the passive state) is exponentially distributed with mean $\mu_{i k}^{-1}$.

According to the principle of the CDMA technology, a call should be blocked if it increases the noise of all inservice calls above a predefined level, given that a call is noise for all other calls. This noise is known as MAI. We distinguish the MAI from other forms of noise, the shot noise, the thermal noise and the fiber-link noise. The thermal noise is generally modeled as Gauss distribution ( $0, t h$ ), and the fiber-link noise is modeled as Gauss distribution ( 0 , $\sigma_{f b}$ ) [16]. The shot noise is modeled as a Poisson process where its expectation and variance are both denoted by $p$ [16]. According to the central limit theorem, we can assume that the additive shot noise is modeled as Gauss distribution ( $\mu_{N}, \sigma_{N}$ ), considering that the number of users in the PON is relatively large. Therefore, the interference $I_{N}$ caused by the the three types of noise is modeled as a Gaussian distribution with mean $\mu_{N}=p$ and variance $\sigma_{N}=\sqrt{\sigma_{t h}^{2}+\sigma_{f b}^{2}+p^{2}}$.

The Call Admission Control (CAC) in the OCDMA PON under consideration is performed by measuring the total interference at the receiver. When a new call arrives (which automatically enters an active state), the CAC checks if two conditions are valid. The first condition (condition A) estimates the total received interference and if it exceeds a maximum value $I_{\max }$, the call is blocked and lost. Therefore, condition A is expressed by the following relation:

$$
\begin{equation*}
\sum_{k=1}^{K}\left(n_{k}^{1} I_{k}\right)+H_{k} H_{N}>I_{\max } \Leftrightarrow \frac{I_{N}}{I_{\max }}>1-\sum_{k=1}^{K}\left(n_{k}^{1} \frac{I_{k}}{I_{\max }}\right)-\frac{I_{k}}{I_{\max }} \tag{2}
\end{equation*}
$$

where $n_{k}^{1}$ represent the number of the service-class $k$ calls in the active system. The same condition is used at the receiver, when a passive call jumps to an active state. Based on (2), we define the LBP $l b_{k}\left(n_{k}^{1}\right)$ that a service-class $k$ call is blocked due to the presence of the additive noise, when the number of active calls is $n_{k}^{1}$ :

$$
\begin{equation*}
l b_{k}\left(n_{k}^{1}\right)=P\left(\frac{I_{N}}{I_{\max }}>1-\sum_{k=1}^{K}\left(n_{k}^{1} \frac{I_{k}}{I_{\max }}\right)-\frac{I_{k}}{I_{\max }}\right) \tag{3}
\end{equation*}
$$

or

$$
\begin{equation*}
1-l b_{k}\left(n_{k}^{1}\right)=P\left(\frac{I_{N}}{I_{\max }} \leq 1-\sum_{k=1}^{K}\left(n_{k}^{1} \frac{I_{k}}{I_{\max }}\right)-\frac{I_{k}}{I_{\max }}\right) \tag{4}
\end{equation*}
$$

Since the total additive noise $I_{N}$ follows a Gaussian distribution $\left(\mu_{N}, \sigma_{N}\right)$, the variable $I_{N} / I_{\max }$, which is used for the LBP calculation also follows a Gaussian distribution ( $\mu_{N} / I_{\max }, \sigma_{N} / I_{\max }$ ) Therefore the right-hand side of (4), which is the Cumulative Distribution Function (CDF) of $I_{N} / I_{\max }$, is denoted by $F_{n}(x)=P\left(I_{N} / I_{\max } \leq x\right)$ and is given by:

$$
\begin{equation*}
F_{n}(x)=\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x-\left(\mu_{N} / I_{\max }\right)}{\left(\sigma_{N} / I_{\max }\right) \sqrt{2}}\right)\right) \tag{5}
\end{equation*}
$$

where $\operatorname{erf}(\bullet)$ is the well-known error function. Using (2) and (4) we can calculate the LBP, $l b_{k}\left(n_{k}^{1}\right)$ by means of the
substitution $x=1-\sum_{k=1}^{K}\left(n_{k}^{1} \frac{I_{k}}{I_{\max }}\right)-\frac{I_{k}}{I_{\max }}$ :

$$
l b_{k}(x)= \begin{cases}1-F_{n}(x), & x \geq 0  \tag{6}\\ 1, & x<0\end{cases}
$$

## III. The Distribution Of The Number Of Active And Passive Calls

The following analysis is inspired by the multi-rate ONOFF model for the call-level performance of a single link, presented in [17], [18], which considers discrete state space. The discretization of the total interference $I_{\max }$ is performed by using the interference caused by a single-codeword call:

$$
\begin{equation*}
C_{1}=\left|\frac{I_{\max }}{I_{\text {unit }}}\right| \tag{7}
\end{equation*}
$$

When a call is at the passive state, it is assumed that it produces a fictitious interference of a fictitious system, with a discrete capacity $C_{2}$ : This passive system is used to prevent new calls to enter the system when a large number of calls are at the passive state. In order to employ the analysis presented in [17], we use the following notations:

1) the total interference of the in-service active calls represents the occupied bandwidth of the active system, and is denoted by $j_{1}$.
2) the total (fictitious) interference of the in-service passive calls represents the occupied bandwidth of the passive system, and is denoted by $j_{2}$.
Based on the analysis presented in the previous section, a new call will be accepted for service if the call's interference (which is expressed by the bandwidth requirement $c_{k}$ together with the interference caused by all the in-service active calls, (which is expressed by the parameter $j_{1}$ ), will not exceed $I_{\max }$. Moreover, in order to avert the acceptance of new calls when a large number of calls are at the passive state, the interference of the new call together with the total interference of all in-service active calls and the fictitious interference of all in-service passive calls should not exceed the fictitious capacity (which is expressed by the discrete value $C_{2}$ ). Based on this analysis, a new service-class $k$ call will be accepted for service in the system, if it satisfies both the following constraints:

$$
\begin{equation*}
j_{1}+b_{k} \leq C_{1} \text { and } j_{1}+j_{2}+b_{k} \leq C_{2} \tag{8}
\end{equation*}
$$

If we denote by $\Omega$ the set of the permissible states, then the distribution $\vec{j}=\left(j_{1}, j_{2}\right)$, denoted as $q(\vec{j})$ can be calculated by the proposed two-dimensional approximate recursive formula:

$$
\begin{equation*}
\sum_{i=1}^{2} \sum_{k=1}^{K} b_{i, k, s} p_{i k}(\vec{j}) q\left(\vec{j}-B_{i, k}\right)=j_{s} q(\vec{j}) \tag{9}
\end{equation*}
$$

where

$$
\begin{equation*}
\vec{j} \in \Omega \Leftrightarrow\left\{\left(j_{1} \leq C_{1} \cap\left(\sum_{s=1}^{2} j_{s} \leq C_{2}\right)\right)\right\} \tag{10}
\end{equation*}
$$

The parameter $s$ refers to the systems ( $s=1$ indicates the active system, $s=2$ the passive system), while $i$ refers to the states ( $i=1$ specifies the active state, $i=2$ specifies the passive state). Also,

$$
b_{i, k, s}= \begin{cases}b_{k}, & \text { if } s=i  \tag{11}\\ 0, & \text { if } s \neq i\end{cases}
$$

and $B_{i, k}=\left(b_{i, k, 1}, b_{i, k, 2}\right)$ is the $i, k$ row of the $(2 K \times 2)$ matrix $B$, with elements $b_{i, k, s}$. Also, $p_{i k}(\vec{j})$ is the utilization of the $i$-th system by service-class $k$ :

$$
p_{i, k}(\vec{j})= \begin{cases}\frac{\lambda_{k}\left[1-l b\left(j_{1}-b_{k}\right)\right]}{\left(1-v_{k}\right) \mu_{1 k}} & \text { for } i=1  \tag{12}\\ \frac{\lambda_{k} \sigma_{k}}{\left(1-v_{k}\right) \mu_{2 k}} & \text { for } i=2\end{cases}
$$

Moreover, $j_{s}$ is the occupied capacity of the system:

$$
\begin{equation*}
j_{s}=\sum_{i=1}^{2} \sum_{k=1}^{K} n_{k}^{i} b_{i, k, s} \tag{13}
\end{equation*}
$$

Proof: In order to derive the recursive formula of (9) we introduce the following notation:

$$
\begin{align*}
& \vec{n}=\left(n^{1}, n^{2}\right), n_{k}^{i}=\left(n_{1}^{i}, n_{2}^{i}, \ldots, n_{K}^{i}\right), \\
& n_{k+}^{i}=\left(n_{1}^{i}, \ldots, n_{k}^{i}+1, \ldots, n_{K}^{i}\right), \\
& n_{k-}^{i}=\left(n_{1}^{i}, \ldots, n_{k}^{i}-1, \ldots, n_{K}^{i}\right),  \tag{14}\\
& \vec{n}_{k+}^{1}=\left(n_{k+}^{1}, n^{2}\right), \vec{n}_{k+}^{2}=\left(n^{1}, n_{k+}^{2}\right), \\
& \vec{n}_{k-}^{1}=\left(n_{k-}^{1}, n^{2}\right), \vec{n}_{k-}^{2}=\left(n^{1}, n_{k-}^{2}\right)
\end{align*}
$$

Having determined the steady state of the system $\vec{n}=$ $\left(n^{1}, n^{2}\right)$, we proceed to the depiction of the transitions from and to state $\vec{n}$, as it is shown in Fig. 2. The horizontal axis of the state transition diagram of Fig. 2 reflects the arrivals on new calls and the termination of calls. More specifically, when the system is at state (A) it will jump to state (B) with a rate $\lambda_{k}$, when a new service-class $k$ call arrives at the system. This rate is multiplied by the probability $1-l b\left(n_{k}^{1}-1\right)$ that this call will not be blocked due to the presence of the additive noise. Similarly, we define the rate from state (C) to state (A). From state (B) the system will jump to state (A) $\mu_{1, k}\left(n_{k}^{1}+1\right)\left(1-v_{k}\right)$ times per unit time, since one of the $n_{k}^{1}+1$ active calls of service-class $k$ (in state (B)) will depart from the system with probability $\left(1-v_{k}\right)$. The transition from state (A) to state (C) is defined in a similar way.

The vertical axis of the state transition diagram of Fig. 2 defines the transition from the active state to the passive state and vice versa. In particular, when the system is at state (A) it will jump to state (D) $\mu_{2, k} n_{k}^{2}\left[1-l b_{k}\left(n_{k}^{1}\right)\right]$ times per unit time. In this case a transition from the passive state to the active state occurs; this transition will be blocked only due to the presence of the additive noise, which is expressed by the LBP. The reverse transition (from state (D) to state (A)) occurs when one of $n_{k}^{1}+1$ active calls jumps to the passive state with probability $v_{k}$. Similarly, we can define the transitions between states (E) and (A).

Let $P(\vec{n})$ be the probability of the steady state of the state transition diagram. Assuming that local balance exists


Figure 2. State transition diagram of the OCDMA system with active and passive users.
between two subsequent states, we derive the local balance equations:

$$
\begin{align*}
& P(\vec{n}) \mu_{i k} n_{k}^{1} v_{k}=P\left(\vec{n}_{k-+}\right) \mu_{2 k}\left(n_{k}^{2}+1\right)\left[1-l b\left(n_{k}^{1}-1\right)\right] \\
& P(\vec{n}) \lambda_{k}\left[1-l b\left(n_{k}^{1}\right)\right]=P\left(\vec{n}_{k+}^{1}\right) \mu_{1 k}\left(n_{k}^{1}+1\right)\left(1-v_{k}\right) \\
& P(\vec{n}) \mu_{2 k} n_{k}^{2}\left[1-l b\left(n_{k}^{1}\right)\right]=P\left(\vec{n}_{k+-}\right) \mu_{1 k}\left(n_{k}^{1}+1\right) v_{k}  \tag{15}\\
& P(\vec{n}) \mu_{1 k} n_{k}^{1}\left(1-v_{k}\right)=P\left(\vec{n}_{k-}^{1}\right) \lambda_{k}\left[1-l b\left(n_{k}^{1}-1\right)\right]
\end{align*}
$$

We assume that the system of (15) has a Product Form Solution (PFS):

$$
\begin{equation*}
P(\stackrel{\rightharpoonup}{n})=\frac{1}{G} \prod_{i=1}^{2} \prod_{k=1}^{K} \frac{p_{i k}^{n_{k}^{1}}\left(n_{k}\right)}{n_{k}^{1}!} \tag{16}
\end{equation*}
$$

where $G$ is a normalization constant and $p_{i k}\left(n_{k}\right)$ is given by

$$
p_{i, k}\left(n_{k}\right)= \begin{cases}\frac{\lambda_{k}\left[1-l b_{k}\left(n_{k}^{1}-1\right)\right]}{\left(1-v_{k}\right) \mu_{1 k}} & \text { for } i=1  \tag{17}\\ \frac{\lambda_{k} \sigma_{k}}{\left(1-v_{k}\right) \mu_{2 k}} & \text { for } i=2\end{cases}
$$

In order for (16) to satisfy all equations of the system of (15) using (17), we assume that $1-l b\left(n_{k}^{1}\right) \approx 1-l b\left(n_{k}^{1}-\right.$ 1), i.e. the acceptance of one additional call in active state does not affect the LBP. This is the first assumption that we take into account in order to derive (9). By using (16), the probability $P\left(\vec{n}_{k-}^{i}\right)$ can be expressed by:

$$
\begin{equation*}
n_{k}^{i} P(\vec{n})=p_{i k}\left(n_{k-}\right) P\left(\vec{n}_{k-}^{i}\right) \tag{18}
\end{equation*}
$$

The probability of $q(\vec{j})$ is given by:

$$
\begin{equation*}
q(\vec{j})=P(\vec{j}=\vec{n} \cdot B)=\sum_{\vec{n} \in \Omega_{\vec{j}}} P(\vec{n}) \tag{19}
\end{equation*}
$$

where $\Omega_{\vec{j}}=\left\{\vec{n} \in \Omega_{\vec{j}}: \vec{n} B=\vec{j}, \mathrm{n}_{k}^{i} \geq 0, k=1, \ldots, K\right\}$. By multiplying both sides of (19) with $b_{i, k, s}$, and summing over $k=1, \ldots, K$ and $i=1,2$, we have:

$$
\begin{equation*}
P(\vec{n}) \sum_{i=1}^{2} \sum_{k=1}^{K} b_{i, k, s} n_{k}^{i}=\sum_{i=1}^{2} \sum_{k=1}^{K} b_{i, k, s} p_{i k}\left(n_{k-}\right) P\left(\vec{n}_{k-}^{i}\right) \tag{20}
\end{equation*}
$$

By using (13) and summing both sides of (20) over the set of all states of $\Omega_{\vec{j}}$, we have:

$$
\begin{equation*}
j_{s} \sum_{\vec{n} \in \Omega_{\vec{j}}} P(\vec{n})=\sum_{i=1}^{2} \sum_{k=1}^{K} b_{i, k, s} p_{i k} \sum_{\vec{n} \in \Omega_{\vec{\jmath}}} p_{i k}\left(n_{k-}^{i}\right) P\left(\vec{n}_{k-}^{i}\right) \tag{21}
\end{equation*}
$$

The second assumption that we consider is that:

$$
\begin{equation*}
\sum_{\vec{n} \in \Omega_{\vec{j}}} p_{i k}\left(n_{k}^{i}\right) P\left(\vec{n}_{k-}^{i}\right) \approx p_{i k}\left(\hat{n}_{k}^{i}\right) \sum_{\vec{n} \in \Omega_{\vec{j}}} P\left(\vec{n}_{k-}^{i}\right) \tag{22}
\end{equation*}
$$

Based on the fact that $(\vec{n} B=\vec{j}) \Rightarrow\left(n_{k-}^{i} B=j-\mathrm{B}_{i, k}\right)$ (17) is equal to (12) and (19) can be rewritten as:

$$
\begin{equation*}
q\left(\vec{j}-B_{i, k}\right)=\sum_{\vec{n} \in \Omega_{\vec{j}}} P\left(n_{k-}^{i}\right) \tag{23}
\end{equation*}
$$

By using the assumption of (22) and substitute (19) and (23) to (22), we derive the recursive formula of (9). The LBP is a function of the total interference of the in-service active calls $j_{1}$, i.e. $l b\left(n_{k}^{1}\right)=l b\left(j_{1}\right)$, since
$x=1-\sum_{k=1}^{K}\left(n_{k}^{1} \frac{I_{k}}{I_{\max }}\right)-\frac{I_{k}}{I_{\max }}=1-\sum_{k=1}^{K}\left(n_{k}^{1} \frac{b_{k}}{C_{1}}\right)-\frac{b_{k}}{C_{1}}=1-\frac{j_{1}}{C_{1}}-\frac{b_{k}}{C_{1}}$
CBP is calculated by combining LBP and HBP as follows:

$$
\begin{equation*}
P b_{k}=\sum_{\vec{j} \in \Omega-\Omega_{h}} l b_{k}\left(j_{1}\right) q(\vec{j})+\sum_{\vec{j} \in \Omega_{h}} G^{-1} q(\vec{j}) \tag{25}
\end{equation*}
$$

where $\Omega_{h}=\left\{\vec{j} \mid\left[\left(b_{i, k, 1}+j_{1}\right)>C_{1}\right] \cup\left[\left(b_{i, k, 2}+j_{1}+j_{2}\right)>C_{2}\right]\right\}$. The first summation of the right part of (25) refers to the probability that a new call could be blocked at any system state due to the presence of the additive noise. The second summation signifies the HBP, which is derived by summing the probabilities of all the blocking states that are defined by (8). Note that the bounds of the first summation in (25) are accidentally different than those of the corresponding equation in [16] due to a misprint in (10) of [16].

## IV. Evaluation

In this section we evaluate the proposed anaytical model through simulation. To this end we simulate the OCDMA PON of Fig. 1 by using the Simscript II. 5 [19] simulation tool. The simulation results are mean values from 6 runs with confidence interval of $95 \%$. The resulting reliability ranges of the simulation measurements are small and therefore we present only mean results. We consider that the OCDMA PON supports $K=2$ service-classes. The traffic parameters $\left(I_{k}, \mu_{1 k}, \mu_{2 k}, \sigma_{k}\right)$ of each service-class are ( $5,1,1.2,0.9$ ) and $(1,0.7,1,1)$, where $I_{k}$ is expressed in $\mu \mathrm{W}$ and $\mu_{i k}$ in $\mathrm{sec}^{-1}$. The threshold of the total interference at the receiver is $60 \mu \mathrm{~W}$, while the fictitious interference that describes the passive system is $70 \mu \mathrm{~W}$. The total additive noise follows a Gaussian distribution $(1,0.1) \mu \mathrm{W}$. The interference that a single-codeworded call is assumed to be $0.05 \mu \mathrm{~W}$. In Fig.


Figure 3. Analytical and simulation CBP results vs. offered traffic load of each service-class.

3 we comparatively present analytical and simulation CBP results vs. the arrival rate, which is assumed that is the same for both service-classes. The results of Fig. 3 show that the accuracy of the proposed analysis is quite satisfactory. Small declinations between analytical and simulation results are due to the assumptions that were taken into account in order to prove (9).

In order to demonstrate the effect of the additive noise to the CBP, in Fig. 4 we present analytical CBP results of the first service-class versus the arrival rate, for different noise distributions. As the results of Fig. 4 reveal, the increment of the additive noise results in the increase of the CBP.

We also study the effect of the fictitious capacity of the passive system to the CBP. To this end, we present analytical CBP results of both service-classes vs. different values of the fictitious capacity. the arrival rate of both service-classes is 0.3 calls $/ \mathrm{sec}$. The increment of $C^{*}$ results in lower CBP, since the passive system can accommodate more calls, and therefore less interference is present in the active system. However, this increment results in a higher probability that a call is blocked at its transition from passive to active state.

## V. Conclusion

We propose a new multi-rate loss model for the calculation of blocking probabilities in an OCDMA-PON. Our analysis takes into account the user activity and different service times for active and passive periods. We provide and prove an approximate recurrent formula for the efficient calculation of the CBP, which is a function of the LBP, and of the HBP. The accuracy of the proposed analysis is quite satisfactory, as it was verified by simulations.As a future work we will incorporate a finite population of traffic sources in the CBP calculation, while we will study the case where the receiver has an interference cancellation capability.


Figure 4. Analytical CBP results of the first service-class for different additive noise distributions.


Figure 5. Analytical and simulation CBP results vs. the fictitious capacity.

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