Game Refinement Theory and Multiplayer Games: Case Study Using UNO

Alfian Ramadhan and Nur Ulfa Maulidevi
School of Electrical Engineering and Informatics
Bandung Institute of Technology
Bandung, Indonesia
email: masphei@gmail.com, ulfa@stei.itb.ac.id

Hiroyuki Iida
School of Information Science
Japan Advanced Institute of Science and Technology
Nomi, Japan
Email: iida@jaist.ac.jp

***Abstract*—Game refinement theory has started to provide some interesting tools to measure sophistication of board games, sport games, and video games. In this paper, we apply game refinement theory to UNO® card game, from which we identify valuable aspects regarding multiplayer and incomplete information game. Specifically, we analyze game refinement value zone of UNO and reveal recommended number of players to play. Furthermore, we compare the measure of enjoyment between the players. Experiments have been conducted by developing various computer player types and simulating about 1.4 million UNO games. Results show that critical states of the game and number of card played are the important factors and confirm that UNO is best to play with 4, 5, or 6 players. Furthermore, another result shows that the second last and the last player get the most enjoyment out of the game.**

Keywords*—*UNO card game; game refinement theory; multi-player game; incomplete information game.

I. Introduction

Game refinement theory has been proposed earlier by Iida *et al.* [1] to determine level of sophistication of games. Some applications have already been done, such as in domain of board games [1], for Mah Jong [2], and sports games [3]. Although there are still many types of games to cover, this theory has already performed well, and generalized fundamental concept. By using sophistication measurement, many facts have been revealed regarding changes of attractiveness of games in decades. In fact, there are still some challenging research questions, especially in applying game refinement theory to multi-player and incomplete information game.

Multi-player game is one of important research themes in game domains. Many works in multi-player game regarding incomplete information aspects have been published, such as multi-player algorithms and approaches [4], comparison of algorithms [5], multi-player Go [6], decision algorithms [7], computing equilibria [8], and lower bounds [9]. Moreover, every kind of games is changing historically by years or decades, even multi-player game. For instance, game refinement theory in multi-person and incomplete information of Mah Jong has been proposed [2]. In fact, recent studies in game refinement theory still focused on several types of games. Hence, multi-player and incomplete information game research in broader types of games are still considered as challenging topics to explore using game refinement theory.

In this paper, we extend game refinement theory with the case study of UNO (UNO®, is a registered trademark of Mattel Corporation) which has been widely recognized as a popular card games. UNO is commonly known as fascinating games and many variants have been developed in many countries. By analyzing game refinement theory, we discover refinement value and sophistication value zone in UNO that are appropriated, as has been found for other refined games such as chess [1], Mah Jong [2], and soccer [3]. Contribution of this paper indicates a promising concept of game refinement theory to be applied in any games generally.

Basically, there are some interesting aspects of UNO, because it is categorized as multi-player game, regarding impact or feelings of each player during the game. Exploring recommended number of players to play UNO is challenging. Basically, there is a promising idea, proposed in sports games [3], of using game progress to measure difference of impact for each of player. Furthermore, determining players who enjoy the game the most seems essential to us. Later, we propose some measurement, called enjoyment measurement, to analyze the impact of the game on each player. Thus, we pack our main works on this different problems which are exploring refinement value and sophistication measurement zone in UNO, investigating recommended number of players to play, analyzing enjoyment measure which leads to find who are the players enjoying the most the game.

This paper is organized as follows. Section II introduces game refinement theory. Then, Section III discusses UNO card game, its various versions, our UNO program, and our analysis in applying game refinement theory. In Section IV and Section V, we present our experiments and discussions of our explorations and discoveries. Finally, Section VI concludes and describes some future works.

II. Game-Refinement Theory

In this section, we first show a basic idea of game-refinement theory, which has been cultivated in the domain of board games. Then, we present the idea to bridge the gap between the board games and sports games based on a model of game progress and game information progress. Moreover, we consider the game progress model of UNO game.

A. Basic idea of game-refinement theory

We give a short sketch of the basic idea of game refinement theory [3]. This section describes the idea based on Sutiono et al. [3] and additional knowledge from authors. The "game progress" is twofold. One is game speed or scoring rate, while another one is game information progress with focus on the game outcome. In sports games such as soccer and basketball, the scoring rate is calculated by two factors: (1) goal, i.e., total score and (2) time or steps to achieve the goal. Thus, the game speed is given by average number of successful shoots divided by average number of shoot attempts. For other score-limited sports games, such as Volleyball and Tennis in which the goal (i.e., score to win) is set in advance, the average number of total points per game may correspond to the steps to achieve the goal [10].

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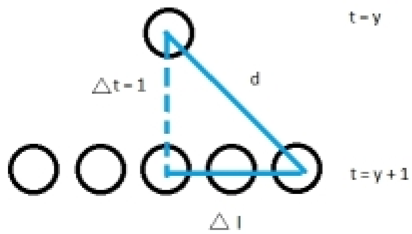

Figure 1. Illustration of one level of game tree.

Game information progress presents how certain is the result of the game in a certain time or steps. Let G and T be the average number of successful shoots and the average number of shoots per game, respectively. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq T$ and $0 \leq x(t) \leq G$, as shown in (1).

$$x(t) = \frac{G}{T}\, t \tag{1}$$

However, the game information progress given by (1) is usually unknown during the in-game period. Hence, the game information progress is reasonably assumed to be exponential. This is because the game outcome is uncertain until the very end of game in many games. Hence, a realistic model of game information progress is given by (2).

$$x(t) = G(\frac{t}{T})^n \tag{2}$$

Here, Sutiono et al. [3] described that n stands for a constant parameter which is given based on the perspective of an observer in the game considered. Then, acceleration of game information progress is obtained by deriving (2) twice. Solving it at $t = T$, the equation becomes

$$x''(T) = \frac{Gn(n-1)}{T^n} t^{n-2} = \frac{G}{T^2} n(n-1)$$

It is assumed in the current model that the game information progress in any type of games is happening in our minds. We do not know yet about the physics in our minds, but it is likely that the acceleration of information progress is related to the force in mind. Hence, it is reasonably expected that the larger the value $\frac{G}{T^2}$ is, the more the game becomes exciting due to the uncertainty of game outcome. Thus, we use its root square, $\frac{\sqrt{G}}{T}$, as a game refinement measure for the game considered.

B. Board games and sports games

Here, we show the idea to bridge the gap between board games and sports games by deriving a formula to calculate the game information progress of board games [3]. Let B and D be average branching factor (number of possible options) and game length (depth of whole game tree), respectively. One round in board games can be illustrated as decision tree. At each depth of the game tree, one will choose a move and the game will progress. One level of game tree is illustrated in Fig. 1. The distance d, which has been shown in Fig. 1, can be found by using simple Pythagoras theorem, thus resulting in $d = \sqrt{\Delta l^2 + 1}$.

Assuming that the approximate value of horizontal difference between nodes is $\frac{B}{2}$, then we can make a substitution and get $d = \sqrt{(\frac{B}{2})^2 + 1}$. The game progress for one game is the total level of game tree times d. For the meantime, we do not consider Δt^2 because the value ($\Delta t^2 = 1$) is assumed to be much smaller compared to B. The game length will be normalized by the average game length D, then the game progress $x(t)$ is given by $x(t) = \frac{t}{D} \cdot d = \frac{t}{D}\sqrt{(\frac{B}{2})^2} = \frac{Bt}{2D}$. Then, in general we have, $x(t) = c\frac{B}{D}t$, where c is a different constant which depends on the game considered. However, we manage to explain how to obtain the game information progress value itself. The game progress in the domain of board games forms a linear graph with the maximum value $x(t)$ of B. Assuming $c = 1$, then we have a realistic game progress model for board games, which is given by

$$x(t) = B(\frac{t}{D})^n. \tag{3}$$

Equation (3) shows that the game progress in board games corresponds to that of sports games as shown in (2).

To support the effectiveness of proposed game refinement measures, some data of games such as Chess and Go [1] from board games and two sports games [3] are compared. We show, in Table I, a comparison of game refinement measures for various type of games. From Table I, we see that sophisticated games have a common factor (i.e., same degree of acceleration value) to feel engagement or excitement regardless of different type of games.

TABLE I. MEASURES OF GAME REFINEMENT FOR BOARD GAMES AND SPORTS GAMES.

Game	*B or G*	*D or T*	$\frac{\sqrt{B}}{D}$ *or* $\frac{\sqrt{G}}{T}$
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073

III. UNO CARD GAME

UNO is one of the well known card games in the world and characterized as a multi-player, imperfect-information, and uncooperative combinatorial game [11]. In addition, a poll found on the website BoardGameGeek, a website specialised on board games and card games, shows that UNO is recommended to play with 2 to 10 players and best to play with 4, 5, or 6 players [12].

Research of UNO card game has attracted many people globally. Recent works have been performed from the viewpoint of a combinatorial algorithmic game theory [11], also in playful probing [13], and intelligent system [14]. Thus, UNO has been recognized not only in entertainment, but also in academic domain.

There are many variants of UNO with different rules, number of cards, or number of players which can be found in various countries in the world. Pagat [15] is a website which collects information of UNO variants including Hold'em UNO, Magic UNO, Speed UNO, Solitaire UNO, and so forth.

There is also a modified version of UNO rules which is played slightly different. The only one difference is that the game ends until there is only one player left who still holds cards in hand. This type of game is similar with DaiFuGo [16] card game from Japan. Moreover, the modified rules of UNO are used in our experiments to measure enjoyment value of each player.

A. Basic rules

UNO official rules can be found at official site [17] or [18]. There are 108 cards in total which are organized as follows: 19 Blue cards, 19 Green cards, 19 Red cards, 19 Yellow cards, 8 Draw Two, 8 Reverse, 8 Skip, 4 Wild, and 4 Wild Draw Four. Accordingly, Draw Two, Reverse, Skip, Wild, and Wild Draw Four cards are defined as Action cards which have effects as they are played in the game. Object of the game is to be the first player in games to get score 500 points. Specifically, only the winner gets score from a game by getting rid of all the cards in hands before other players, and this score is calculated from all of opponents' cards left.

Basically, UNO is easy to play. First of all, the game begins by deciding who among participants is to play first. In this part, every player picks a card and the first player is determined by the one who gets the highest number of numbered cards. Then, each player when beginning his turn firstly has to determine whether he wants to draw a card, or play a card in his hand. He can choose to play a card in his hand, otherwise he draws a card from deck and can play the drawn card if the card is possible to play. In official rules, the game ends until there is one player which has no cards left in his hands. However, we add some modifications in this study that the game ends until there is only one player holding cards in his hands. By doing so, there will be ranking from first player as the winner until the last player as the final loser.

There are several action cards which have to be understood before playing such as Draw Two, Skip, Reverse, Wild, and Wild Draw Four. Draw Two card forces the next player to draw two cards and skip his turn. Skip card means that next player misses his turn. Reverse card is used to invert turn direction. Wild card can be used to change color to play. Then, Wild Draw Four card is used to force next player to draw four cards, skip next player's turn, and change color in the game. These cards have their own effect and affect game play. Thus, these basic rules and action cards lead analysis of UNO in next sections.

B. Game refinement theory and game progress in UNO

The idea that had been the basis of previous works on sports games [3] is to find some critical enjoyment points in the game, and only measure those, assuming that they are key point and are the only point that we need to study. For example, in soccer, this critical action are the shoots. A game of soccer is actually more than a succession of shoot, but we can restrict our study to only those for two reasons: the first is all the other actions during the game take place only to decide which side will be able to try a shoot, and how probably it will be a success, and the second is that shoots are the moment the spectator can enjoy the most, because it is the most intense action.

In game refinement theory, branching factors and game length are the main factors to determine game information outcome [1]. Iida *et al.* proposed average number of possibilities and turns to apply game-refinement theory in board games [2]. Furthermore, Sutiono *et al.* proposed some relationships between game-refinement theory and game progress concluding that number of goals and shoots are factors to measure sophistication of games, as well as game information outcome [3]. Thus, each game may have its own measurement to be identified as game refinement value regarding game characteristics.

UNO card game is different from sports and board games. Although there are some similarities between board games and card games such as turns and type of actions, different rules or characteristics of games can result contrast impact to players in terms of game refinement theory. For instance, different versions of Mah Jong game in history affect its attractiveness [2]. Furthermore, broad and deep analysis of UNO are required since it is characterized as a multi-player, imperfect-information, and uncooperative combinatorial game [11]. Thus, there are some different considerations to identify main factors of game refinement theory in UNO.

In this study, we highlight multi-player and imperfect-information characteristics as main aspects. First of all, multi-player games characteristic is simply identified by the number of players. Basically, each player is supposed to perform any actions which affect the game in any conditions. For instance, a player may play any action cards to attack other players or skip his turn to give other player's chance. Consequently, each player has contributions to increase or decrease attractiveness or flow of the game. Meanwhile, treating imperfect-information game is more challenging than perfect-information game because of hidden information. However, there is global information which is visible to all of players in game and simply measured as global variables which is state of the game such as number of cards or number of remained cards in deck.

Because of the imperfect information nature, it is much harder to measure the progression of the game. The end of the game may not be expected by every player. We will only look at information that are shared by every player for our measurement, and because the game ends when a player had no more cards in his hand, game progress can be measured by looking at the number of remaining cards in hand for each player.

The solution we propose here is to consider the times a player say "UNO", when he has only one card left in hand, which we name critical state of the game. This is a viable option because it is emphasised by the game itself as a big point of interest of the progression of a player toward victory. When a player says "UNO", he is more likely to become the target of every punishing card or strategy from other players (for example playing yellow if it is known that he had no yellow card in hand). Also, when considering the ratio of "UNO" over the total number of card played, the value obtained is in relation with the balance of the game. The most card you need to play in average before having only one card left is a measure of how slow the game is.

In experiment, we use both values in average of each player because UNO is a multi-player game. Thus, determining sophistication of the game is simplified by using average number of UNO times and average number of played cards for each per player.

$$x(t) = U(\frac{t}{P})^n \tag{4}$$

According to (3), let U to replace B as average number of saying UNO, and P to replace D as average number of played cards for each player independently. By using our analysis and referring to game refinement theory, game information outcome of UNO is defined in (4). Then, acceleration of game information outcome value in UNO is shown in (5). Finally, the sophistication measurement can be obtained using $\frac{\sqrt{U}}{P}$.

$$x''(t) = \frac{Un(n-1)}{P^n} t^{n-2} \tag{5}$$

When $t = P$, the equation becomes

$$x''(P) = \frac{U}{P^2} n(n-1)$$

On the other hand, there is another analysis regarding excitement for each player which argues that the first player, the winner, does not feel more excitement than other late player. Likewise, there is some different feeling when comparing multi-player game with two-player game due to the number of players. Although the first player is lured by prize and high score, as he enjoys his win, but it stops his play. But players still in the game can continue to enjoy it. Our model does not include feeling about winning, and only focus on enjoying the content of the game, not the side effects like winning or losing. Our analysis argues that the second and the last player in the game are the player who enjoy the most and feel the most excitement in the game.

Basically, measuring enjoyment feeling is more likely to outlook the game in overall. This can be done by using overall number of critical states of the game and contributions of the game as well as generalizing our analysis in measuring sophistication value of UNO. In other words, overall critical state of the game is reflected by total number of UNO times in the game. Moreover, instead of having overall number of played cards, we can change it by number of rounds in the game. Thus, according to the sophistication measurement formula, the enjoyment measure can be similarly defined using $\frac{\sqrt{U}}{P}$ with U as overall UNO times and P as number of rounds, in order to specifically investigate the excitement of each player.

C. UNO program

We have created a program which is developed in Java to run our simulation of UNO. The program works as automatic simulation playing UNO and records each player's activity. Information of player's activities is collected during the game including the number of turns, played cards, UNO times, and so forth. Furthermore, the program has been published as an open source which can be found on Github [19].

Process of each player who is having a turn is illustrated in Fig. 2. Square and diamond shape stands for process and decision, respectively. From the beginning until the end of turn, a player is given some several processes and decisions including decision of drawing a card, process of picking a card, and so forth. In general, there are several basic actions called Module Actions which require an action of each player regarding their strategy and mind. These actions are drawing a card, playing drawn card, playing a card, yelling UNO, and choosing a color. Consequently, by having well defined and separated actions, the program becomes well structured and modularized, especially in building computer players.

Basically, our simulation is played fully by computer players which are inspired from multi-player algorithms and approaches [4]. However, our implementation has been performed in simpler ways. We have created four different profiles as computer players: Amateur, Offensive, Defensive, and

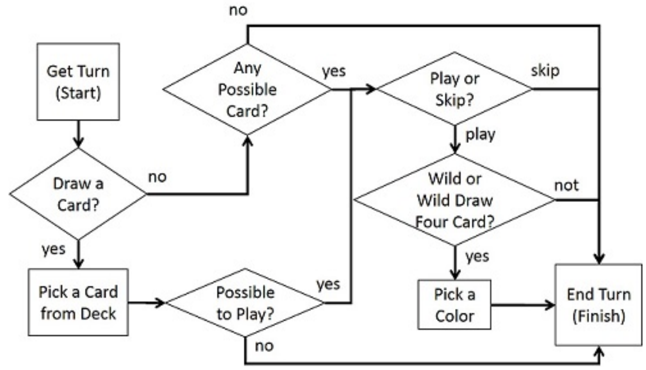

Figure 2. Flowchart of Player Turn Process.

Observer. These types of players are identified because they are most reasonable strategies and easy to understand.

First of all, Amateur is the easiest player to imagine. Amateur is more likely to be analogous to weak human players who still lack of experiences. In general, Amateur does not have good strategy to play, so that he does not consider any actions or situations. Specifically, Amateur plays all of Module Actions randomly and recklessly. Thus, action of Amateur is not specified as an important algorithm.

Algorithm III.1: OFFENSIVE($possible\_cards$)

procedure OFFENSIVE_ACTION(P)
$C \leftarrow \{\}$ $//$ $candidate\ stack$
$C \leftarrow TOP(P)$ $//$ $P\ possible\ cards\ as\ stack$
for each $p \in \mathcal{P}$
do { **if** $(stronger\ (p,\ TOP(C)))$
then { $//\ p\ is\ stronger$; $//\ push\ p\ to\ top\ of\ stack\ C$; $PUSH(C\ p)$ }

return (C) $//$ $return\ cards\ in\ offensive\ order$

Figure 3. Offensive step.

Secondly, Offensive roles as an active player and is more likely to be analogous to ambitious players. In general, Offensive always plays offensively, so that he takes the most strongest and offensive actions, especially to attack other players as shown in Fig. 3. Ranking of card's strength from the strongest to the weakest is given as follows, Draw Four, Draw Two, Skip, Reverse, and Wild card.

Meanwhile, Defensive player acts as a passive player. In contrast, Defensive is more likely to be the opposite of Offensive because he always plays defensively, so that he mainly chooses harmless actions as shown in Fig. 4. Ranking of cards is prioritized in inverted order with Offensive algorithm.

Finally, Observer player chooses his strategy by considering opponent actions. Basically, he plays cards which other players do not have the color or number by remembering others' missing cards from recent turns, especially when they draw a card as shown in Fig. 5.

Algorithm III.2: DEFENSIVE($possible\_cards$)

```
procedure DEFENSIVE_ACTION(P)
  C ← {} // candidate stack
  C ← TOP(P) // P possible cards as stack
  for each p ∈ 𝒫
    do { if (weaker (p, TOP(C)))
           then { // p is weaker
                  // push p to top of stack C
                  PUSH(C, p)

  return (C) // return cards in defensive order
```

Figure 4. Defensive step.

Algorithm III.3: OBSERVER($possible, forbidden$)

```
procedure OBSERVER_ACTION(P, F)
  C ← [ ] // list of candidates

  // copy P into C
  for each p ∈ 𝒫
    do {INSERT (C, p)

  for each f ∈ ℱ
    do { for each p ∈ 𝒫
           do { if match (f, p)
                  then {VOTE_DOWN(C, p)

  SORT(C) // sort candidates in vote order

  return (C) // return cards in vote order
```

Figure 5. Observer step.

IV. EXPERIMENTAL DESIGN AND RESULT

In this paper, we conduct experiments by simulating our UNO computer program to obtain refinement value and sophistication measurement. Then, we quickly compare a particular result with the real UNO games played with human players. Another experiment is done by modifying rule of the game to identify enjoyment value using UNO computer program.

A. Game Refinement Experiment

First of all, we collected data from 1,432,089 game simulations run by several type of players described previously. Composition of player types in each of the game is randomly organized. Measures of game refinement in simulations of UNO is illustrated in Table II. The measures are applied for 10 different number of players playing in the game from 3 to 12 players. Each number of players gives a different value of three variables which are average UNO times per player U, average played cards per player P, and division between square root of U and P as game information outcome.

According to Table II, U, P, and $\frac{\sqrt{U}}{P}$ are decreasing from 3 to 12 players. That is, chance to have UNO is decreasing when the number of players is increasing. In addition, each player also has less number of played cards in the game with more players. Furthermore, third variable called sophistication

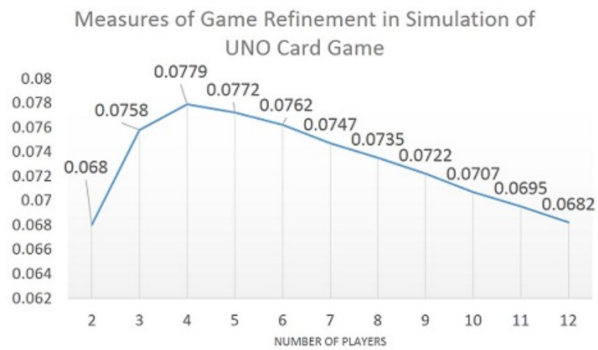

Figure 6. Graph of Game Refinement Measurement UNO Simulation.

measurement is scored 0.0682 as minimum value and 0.0758 as maximum value. Specifically, sophistication measurement reaches below 0.07 in 11 and 12 players. Understandable visualization can be seen from Fig. 6, which shows visualization of Table II.

TABLE II. MEASURES OF GAME REFINEMENT IN SIMULATION OF UNO CARD GAME.

Player	*U*	*P*	$\frac{\sqrt{U}}{P}$
3	1.260	14.802	0.0758
4	0.976	12.684	0.0779
5	0.813	11.679	0.0772
6	0.702	10.994	0.0762
7	0.617	10.511	0.0747
8	0.554	10.129	0.0735
9	0.506	9.856	0.0722
10	0.460	9.594	0.0707
11	0.427	9.404	0.0695
12	0.398	9.246	0.0682

On the other hand, although our experiments using simulation show fascinating results, statistical data which was obtained from real games shows slight difference. By conducting 19 real UNO games in four human players, we have U at 1.118 and P at 10.947, so that the $\frac{\sqrt{U}}{P}$ becomes 0.095. This phenomenon is interesting to be explained in discussion section later on.

B. Enjoyment Value Experiment

We conduct another experiment by using UNO modified rules, so that the game continues until only one player had cards in hand. The results shown in Table III and Table IV illustrate measures of game refinement in simulation of UNO with modified rules. The measurement is performed for 10 different number of players from 3 to 12 players. Ranking are grouped by number of players in the game, and the enjoyment value is expressed in function of the rank of the player.

According to Table III and Table IV, score of each player is decreasing from first rank until last rank in the game, so that the first rank gets the highest score and the last rank gets the second lowest score. Specifically, the lowest and highest score in each different player of the game is from about 0.074 to 0.076 and 0.11 to 0.22, respectively. Basically, only the second and last player from all of different number of players perform score from about 0.071 to 0.076.

There are some essential information extracted from statistics regarding different type of computer players. Other statistics from our simulation of UNO with modified rules in overall

TABLE III. ENJOYMENT MEASURE OF UNO SIMULATION WITH MODIFIED RULES 3-7 PLAYERS.

Rank	3P	4P	5P	6P	7P
1	0.110	0.135	0.152	0.166	0.177
2	0.071	0.106	0.130	0.148	0.164
3	0.074	0.072	0.103	0.126	0.144
4		0.074	0.072	0.102	0.123
5			0.074	0.073	0.101
6				0.075	0.074
7					0.075

TABLE IV. ENJOYMENT MEASURE OF UNO SIMULATION WITH MODIFIED RULES 8-12 PLAYERS.

Rank	8P	9P	10P	11P	12P
1	0.188	0.197	0.205	0.213	0.220
2	0.178	0.191	0.202	0.212	0.222
3	0.160	0.174	0.187	0.199	0.210
4	0.141	0.156	0.170	0.183	0.195
5	0.121	0.138	0.153	0.167	0.180
6	0.100	0.119	0.135	0.150	0.164
7	0.074	0.098	0.117	0.134	0.148
8	0.075	0.074	0.098	0.116	0.132
9		0.075	0.074	0.097	0.115
10			0.076	0.075	0.097
11				0.076	0.075
12					0.076

is collected in Table V. Statistics are given as a percentage and score from all of different type of players, which are Amateur, Defensive, Offensive and Observer. Furthermore, according to Table V, the lowest and highest percentage of winning rate are performed by Amateur at 10.04% and Defensive at 30.5%. In addition, Amateur and Defensive also perform the lowest score at 0.0862 and the highest score at 0.1337 of enjoyment measure, respectively. On the other hand, Amateur reaches the highest percentage of being second last player at 38.42% and being last player at 57.82%.

TABLE V. STATISTICS OF UNO.

Level	Amateur	Defensive	Offensive	Observer
Winning rate	10.04%	30.50%	29.62%	29.84%
Being second last player	38.42%	21.55%	21.21%	18.82%
Being last player	57.82%	13.35%	13.84%	14.99%
enjoyment measure	0.0862	0.1337	0.1323	0.1246

Enjoyment measure of 8-Player UNO is represented in Fig. 7 ordered by ranking. First rank player gets the highest score at 0.188, but the second last player gets the lowest score at 0.074. In general, the point is decreasing from the first rank until the second last player. Meanwhile, the second and the last player perform score 0.074 and 0.075, respectively.

V. DISCUSSION

In this section, there are discussions regarding refinement value of UNO and enjoyment measurement in the game according to the experimental results. First investigation focuses on the difference of refinement value between UNO computer program and the real game with human players. Then, the second issue points the enjoyment value of the game which is specifically related with the second and the last player in the game.

First of all, by observing comparison of game refinement measurement between computer simulation and human UNO games, there is difference about 0.02. The difference is shown by 4-Player human game and 4-Player UNO game simulation

Figure 7. Enjoyment Measure of 8-Player UNO.

in Fig. 6. There are several possible issues to be drawn regarding this difference such as computer player quality, playing against human in reality, and method of gathering data.

Basically, our implementation of UNO players, which are Amateur, Offensive, Defensive, and Observer may not fully represent real human players' abilities. They are only very simple models, and could be improved to be closer to human real strategies. Real human players' abilities may vary broader and deeper in terms of skill compared to our implementation. Moreover, playing UNO against human players is more likely to be uncontrollable, so that the game can be various depending on various human skills. For example, human social nature may cause players to not be equally considered by each other. Besides that, another problem may be drawn by error in recording data in human game. For instance, gathering data in real human game may be less accurate because of various game flow speed. There are many factors affecting the game flow such as luck, number of turns, strategies and so forth. However, our implementation has reflected only a few of the various player types in the world. Although the difference appears, our implementation is fair enough since the difference is not significant. Furthermore, the number of data obtained from human real games is less than 20, which is very small compared to our computer simulation with only slight difference of result. Thus, the statistics gathered from computer simulation is still acceptable to be analyzed.

By accepting data of our simulation, we can analyze from Fig. 6 that UNO shows refinement value from 0.0682 to 0.0758 for different number of players. In fact, there is an interesting fact that the value reaches lower than 0.07 for 11 or 12 players. By referring to game refinement theory in board games [2] and sports games [3], we can say that UNO is sophisticated enough to play from 3 to 10 players since the refinement value ranges in between 0.07 and 0.08, which are identified as the reasonable values. Moreover, although our experiment did not cover 2-player game, we have successfully confirmed that recommended number to play UNO ranges between 2 and 10 players, which is what people voted on BoardGameGeek. Our measures shows that UNO is a sophisticated game to play, like chess, Mah Jong and soccer. Furthermore, we suggest that the most sophisticated game to play UNO is not more than 10 players.

In addition, according to Fig. 6, there is a peak area which is considered as the three highest refinement value in UNO, which are performed by 4, 5 and 6 players. By referring to game refinement value, this can be inferred that these three cases are special numbers of players which is recommended to enjoy the most the game of UNO, because it has the best sophistication for these values. This finding is relevant to the BoardGameGeek site, where people have voted optimal numbers to play UNO, as well as the same result in our experiment.

Moreover, according to our last experiment in Fig. 7, there is an another important result showing about enjoyment value in the game. Although the last player performs higher score than the second last player, their values are in the range of 0.07 to 0.08. According to related works, by using board and sports game's sophistication value which lies between 0.07 and 0.08, our enjoyment measure shows that late players feel more excitement comparing to players who left the game earlier, especially the first player. Hence, we have confirmed that the second and the last player enjoy the most in the game. Finally, these results endorse ideas to support the concept of game refinement theory to be applied generally.

VI. Conclusion and Future Work

Game refinement theory has been applied to measure sophistication of games in board games, video games, and sports. In this paper, we have extended this theory to card games and presented that UNO can be analyzed using this measurement. Specifically, we can prove the recommended number of players to play UNO and show some enjoyment measure to determine which player enjoy the most in the game.

In UNO card game, individual critical situation and individual contributions are the most considered values to be used in sophistication measurement in game refinement theory. In addition, the theory considers UNO card game as a sophisticated game which is consistent with the popularity of the game. Furthermore, recommended number of players to play UNO has been proven from 3 to 10 players and the best number to play lies from 4 to 6 players as mentioned on BoardGameGeek. Thus, game refinement theory is well applied in UNO card game using the individual critical situation and contribution.

Determining which player who feels the most engaging game is a challenging question to be figured out, especially in multi-player games. Directly, enjoyment measure can be simplified using critical states of the game and game length in modified UNO card game. Specifically, this variables show some facts that the second last and the last player enjoy playing the most in the game. Consequently, the critical state of the game and length of the game perform the most important role in identifying the enjoyment measure of player in the game.

Thus, a good deal of efforts have been done to analyze UNO card game using game refinement theory approach. This paper has successfully proposed attributes which can be used as sophistication and enjoyment measure in UNO which are individual critical situation, contributions, overall critical states of the game, and game length. In general, critical state of the game is reasonably a main factor in game refinement theory, especially in multi-player games in order to discover sophistication or enjoyment evaluation of games.

This research can be continued better by exploring external validation to discover fundamental formulas in game refinement theory. Besides, it is possible to capture whole picture of game in general by inspecting carefully all of the applied concepts so far and identifying global concept of game. Moreover, further works may consider other interesting aspects such as cooperation and non-cooperation in multi-player and incomplete information games using game refinement theory. In addition, improving computer player UNO in terms of quantity and quality may also be interesting, especially in developing a framework of more or less sophisticated multi-player game. Moreover, there are other challenging aspects to apply game refinement theory in multi-player and incomplete information games such as player modelling, social behavior, economy, and game sustainability.

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