Reliability Measure for a System Operating under Random Environment

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***Abstract*—In this paper, we consider a system operating in a random external shock process. The underlying system performance is modelled by a quality (output) function which is decreasing due to degradation. Shocks affect the failure rate of the system directly and, at the same time, they additionally decrease the quality function. Expectations (unconditional and conditional on survival) and variability of this time-dependent quality function are analyzed.**

Keywords-quality characteristic; random environment; shock process; intensity process; variability measure.

I. INTRODUCTION

The performance of various engineering systems is often characterized not only by reliability characteristics, but also by characteristics of performance (output). For instance, a quality function for production systems can be described by the production rate, i.e., the number of items produced in a unit interval of time. For navigation systems, this quality is characterized by the accuracy of navigation parameters such as heading, altitude and longitude. It is well understood that most engineering systems are deteriorating in some stochastic sense and deterioration affects not only reliability indices but also the quality of performance [1][2]. [1] and [2] have considered mostly deterministic quality function. However, the quality or performance of a system should depend on random operational environment. In this regard, in this paper, we will consider stochastic quality functions.

In this paper, we study the reliability measure for a system operating in a random environment. The random environment is modeled by a process of external shocks. We suggest a novel approach in shocks modeling when shocks have a double effect, i.e., they act directly on the failure rate (more precisely, on the corresponding failure rate process) that characterizes the time to failure of a system and, at the same time, on the quality function as well. For example, for a network system, if a shock (e.g., external attack) occurs, the susceptibility to a failure of the network increases and, at the same time, the performance of the network decreases. To account for this complex influence and to obtain explicit expressions for characteristics of interest, we derive the necessary conditional and unconditional average characteristics under the assumption of the Non-homogeneous Poisson Process (NHPP) of shocks. Specifically, we obtain the expectation and the variance of the quality function of a system on condition that a system is operable at a given instant of time and without this condition.

In Section 2, we introduce the model studied in this paper. Furthermore, the unconditional and conditional expected quality functions are derived. In Section 3, the unconditional and conditional variability measures are obtained. Finally, in Section 4, we provide a brief conclusion.

II. EXPEXTED QUALITY OF THE SYSTEM

Assume that a non-repairable system is operating in a random environment modeled by the NHPP of shocks $\{N(t), t \ge 0\}$ with the rate of occurrence $\lambda(t)$, where $N(t)$ is the number of shocks by time *t*. Define its lifetime by the following conditional failure rate (intensity process) [3]

$$\lambda_t = r_0(t) + \eta N(t), \tag{1}$$

where $r_0(t)$ is the baseline failure rate of a system that is operating in the absence of shocks and $\eta > 0$ is a constant jump in the failure rate on occurrence of each shock. Thus, each shock increases λ_t in each realization of this stochastic process by the same deterministic value.

Let $Q(t)$ be a deterministic quality or performance function of an operating system, which is monotonically decreasing [1]. Moreover, assume also that the quality or performance is decreasing on each shock. To account for this effect of the shock process in a consistent way, we assume that the quality at time t under a shock process is given by the following stochastic process

$$\tilde{Q}(t) = Q(t) \prod_{i=1}^{N(t)} \exp\{-\psi(T_i)\}, \tag{2}$$

where $\psi(t) > 0$ is a deterministic function and $0 \le T_1 \le T_2 \le \ldots$ are the sequential arrival times of shocks in the NHPP.

Let $I(t)$ denote the corresponding indicator of the system state (1 if the system is operating at time *t* and 0 if it is in the state of failure). Our first measure of interest is

$$Q_E(t) = E[\tilde{Q}(t) \cdot I(t)], \tag{3}$$

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which is the expectation of the quality function of a system at time t (assuming that the quality is 0 when a system is in the state of failure). Note that when $\tilde{Q}(t) \equiv 1$, for all $t \geq 0$, $Q_E(t)$ in (3) is the usual 'reliability function'.

Result 1. The expected quality function $Q_E(t)$ is given by

$$Q_E(t) = Q(t)\exp\left\{-\int_0^t r_0(u)du\right\}\exp\left\{-\int_0^t \lambda(u)du\right\} \cdot \times \exp\left\{\int_0^t \exp\{-\eta t - \psi(x) + \eta x\}\lambda(x)dx\right\}.$$

Proof. It can be shown that the joint distribution of $(T_1, T_2, ..., T_{N(t)}, N(t))$ is given by

$$\left(\prod_{i=1}^{n}\lambda(t_i)\right)\exp\left\{-\int_0^t \lambda(u)du\right\},\ 0 \leq t_1 \leq t_2 \leq ... \leq t_n \leq t, n = 0,1,2,...,$$

and taking expectation of $[\tilde{Q}(t) \cdot I(t)]$ with respect to this distribution yields the desired result.

In many instances and especially when considering characteristics of quality in a population of systems, it could be more interesting and practically sound to obtain the expected quality for systems that are 'operating at time t'. Hence, our second measure of interest is the following conditional expectation:

$$Q_{ES}(t) = E[\tilde{Q}(t) \mid T > t], \tag{4}$$

where T is the system lifetime and "S" in $Q_{ES}(t)$ stands for "survived".

Result 2. The conditional expected quality function $Q_{ES}(t)$ is given by

$$Q_{ES}(t) = Q(t) \times \exp\left\{\int_0^t \exp\{-\eta t - \psi(x) + \eta x\}\lambda(x)dx - \int_0^t \exp\{-\eta(t-x)\}\lambda(x)dx\right\}.$$

Proof. It is similar to the proof of Result 1.

III. VARIABILITY IN QUALITY OF THE SYSTEM

Note that the quality of a system $\tilde{Q}(t) = Q(t)\prod_{i=1}^{N(t)} \exp\{-\psi(T_i)\}$ and the conditional quality of a system $(\tilde{Q}(t) \mid T > t)$ are stochastic processes. In the previous section, we have considered expectations of these quality measures as important reliability characteristics of a system. In this section, we will discuss the time-dependent variability of the quality, which can be represented by the variance or the conditional variance at each time instant. Thus, we now define the following measures for variability of quality.

$$VQ_E(t) = Var[\tilde{Q}(t)I(t)],$$

and

$$VQ_{ES}(t) = Var[\tilde{Q}(t) \mid T > t].$$

These measures are obtained in the following result.

Result 3. The variability measures $VQ_E(t)$ and $VQ_{ES}(t)$ are given by

$$\begin{aligned} VQ_E(t) &= Q(t)^2 \exp\left\{-\int_0^t r_0(u)du\right\}\exp\left\{-\int_0^t \lambda(u)du\right\} \\ &\times \exp\left\{\int_0^t \exp\{-\eta t - 2\psi(x) + \eta x\}\lambda(x)dx\right\} \\ &- Q(t)^2 \exp\left\{-2\int_0^t r_0(u)du\right\}\exp\left\{-2\int_0^t \lambda(u)du\right\} \\ &\times \exp\left\{2\int_0^t \exp\{-\eta t - \psi(x) + \eta x\}\lambda(x)dx\right\}, \end{aligned}$$

and

$$\begin{aligned} VQ_{ES}(t) &= Q(t)^2 \\ &\times \exp\left\{\int_0^t \exp\{-\eta t - 2\psi(x) + \eta x\}\lambda(x)dx - \int_0^t \exp\{-\eta(t-x)\}\lambda(x)dx\right\} \\ &- Q(t)^2 \exp\left\{2\int_0^t \exp\{-\eta t - \psi(x) + \eta x\}\lambda(x)dx - 2\int_0^t \exp\{-\eta(t-x)\}\lambda(x)dx\right\}, \end{aligned}$$

respectively.

Proof. It is similar to the proof of Result 1.

Note that $VQ_E(t)$ represents the unconditional variation, whereas $VQ_{ES}(t)$ provides the conditional variation.

IV. CONCLUSION

In this paper, we have considered a system operating under a random Poisson shock process. Each shock affects the failure rate of the system and the quality of the system simultaneously. Under the suggested model, the unconditional and conditional expected quality functions have been derived. Furthermore, the unconditional and conditional variability measures have also been obtained. This paper extends the previous works [1][2] by considering stochastic quality functions, which is practically meaningful generalization.

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