

# An Interval Type-2 Fuzzy Neural Network for Cognitive Decisions

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**Abstract**—In an ambient assisted living environment, raw data can often be very noisy making is difficulty to interrupt by a decision and reasoning system. To help reduce the effects of noise, we propose a decision and reasoning system which combines an interval fuzzy system and a self-organising fuzzy neural network (SOFNN) is presented in this paper. The method exploits the use of a trained standard SOFNN structure from a fuzzy neural network to initialise the proposed approach. Simulation results show that the proposed structure is more suitable for uncertain situations demonstrating a high level of robustness.

**Keywords**- interval type-2 fuzzy system; self-organising fuzzy neural network; cognitive decisions; modelling capability; robustness.

## I. INTRODUCTION

Robotic UBIquitous COgnitive Network (RUBICON) [16] is an EU project that aims to create a self-sustaining, self-organizing, learning and goal-oriented robotic ecology, which consists of four layers: Learning Layer, Control Layer, Cognitive Layer and Communication Layer. The Cognitive Layer obtains events from the Learning Layer and generates goals for the Control Layer. The Cognitive Layer is described in detail in [1] and includes a cognitive reasoning and decision module.

The objective of the cognitive decision module is to generate decisions signal by integrating the status outputs determined by the cognitive reasoning module, taking into account current and historical information. These decision signals are then used to set the actual goals for the Control Layer so as to attempt to ensure that the RUBICON ecology behaves appropriately. Ultimately, the RUBICON system will be embedded in real environments where, potentially, the collected data will be noisy. To minimise the impact of noise and the inherent uncertainty of the generated status outputs, the decisions module is expected to show a high level of robustness.

A Self-Organising Fuzzy Neural Network (SOFNN) has a degree of robustness [2][3]. However, the membership functions of a SOFNN are type-1 fuzzy systems, which are presented as crisp numbers. This limits the ability of the network to model uncertainty. Type-2 fuzzy sets have been extended from type-1 fuzzy sets by Zadeh [4] and are attracting increasing attention. The advantage of type-2 fuzzy systems is that the membership functions can be presented as a fuzzy set, not simply as crisp numbers. This enhances the

ability of the network to handle the uncertainty in the rule base [5], which is vitally important for deployment in real applications, such as the RUBICON ecology. In this work, the combination of an interval type-2 fuzzy system and a SOFNN, denoted as SOFNN-IT2, has been proposed for the cognitive decision module.

Background information on the SOFNN and an interval type-2 fuzzy system are presented in Section II. Section III presents the proposed SOFNN-IT2 learning method. Simulation results in Section IV are presented to verify the proposed method in terms of its modelling capability and robustness. The work is summarised in Section V.

## II. BACKGROUNDS

### A. Overview of SOFNN

A Self-Organising Fuzzy Neural Network (SOFNN) [2] is a hybrid network which has the capability to model and forecast a complex nonlinear system. The SOFNN is a five-layer network, namely, the input layer, the Ellipsoidal Basis Function (EBF) layer, the normalised layer, the weighted layer, and the output layer, as shown in Figure 1.

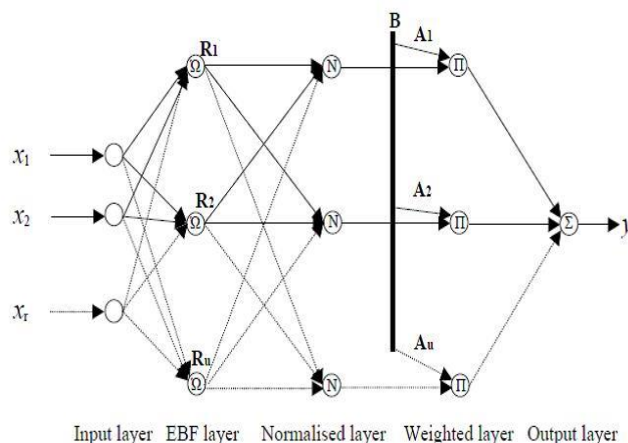


Figure 1. Structure of SOFNN.

In the EBF layer, each neuron is a T-norm of Gaussian fuzzy membership functions [2] belonging to the inputs of the network. Every Membership Function (MF), thus, has its own distinct centre and width, which means every neuron has both a centre vector and a width vector and the dimensions of these vectors are the same as the dimension

of the input vector. Figure 2 illustrates the internal structure of the  $j$ th neuron, where  $X = [x_1 \ x_2 \ \dots \ x_r]$  is the real valued input vector,  $C_j = [c_{1j} \ c_{2j} \ \dots \ c_{rj}]$  is the centre vector in the  $j$ th EBF neuron, and  $\Sigma_j = [\sigma_{1j} \ \sigma_{2j} \ \dots \ \sigma_{rj}]$  is the width vector in the  $j$ th neuron.

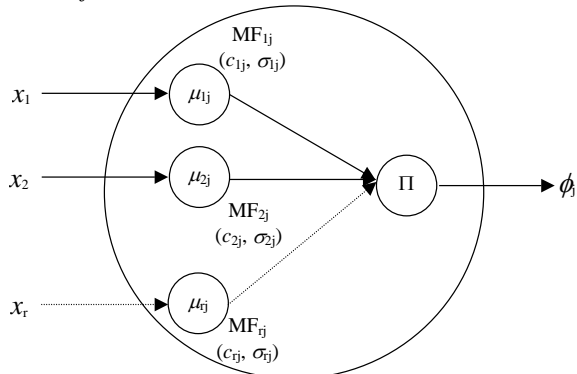


Figure 2. Structure of the  $j$ th neuron  $R_j$  with  $c_j$  and  $\sigma_j$  in EBF layer.

The SOFNN is thus constructed based on EBFs consisting of a centre vector and a width vector. The adding approach is based on the geometric growing criterion [6] and the  $\varepsilon$ -completeness of fuzzy rules [7]. The pruning method is based on the Optimal Brain Surgeon (OBS) method [8] and depends on the second order derivative of the objective function with respect to the parameters of each neuron, i.e., the Hessian matrix [2]. The Hessian matrix can be easily obtained as part of the proposed on-line parameter learning algorithm. Further information on this network is available from [2] and [3].

The SOFNN can be used for on-line learning and the adding and pruning strategies have the self-organising capability to produce a fuzzy neural network with a flexible structure that grows in order to minimise the training error. The SOFNN has demonstrated good performance in applications of function approximation, complex system identification, and time-series prediction [2] [3].

### B. Interval Type-2 Fuzzy System

An Interval Type-2 Fuzzy Logic System (IT2FLS) is shown in Figure 3. This is similar to a Type-1 FLS (T1FLS) containing a fuzzifier, rule base, fuzzy inference engine, and output processing. The main difference is that a type-2 FLS has a type-reducer in the output processing. The type-reducer has the ability to generate a type-1 fuzzy set from a type-2 fuzzy set. The defuzzifier then can defuzzify this type-1 fuzzy set to a crisp number. IT2FLSs have demonstrated better ability to handle uncertainties than their type-1 counterparts [9].

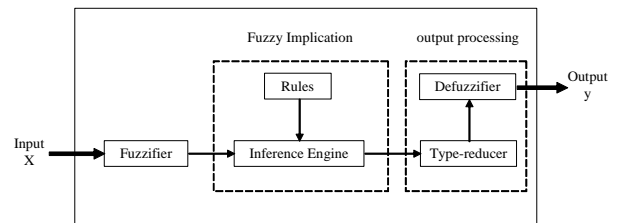


Figure 3. Interval type-2 fuzzy logic system (IT2FLS).

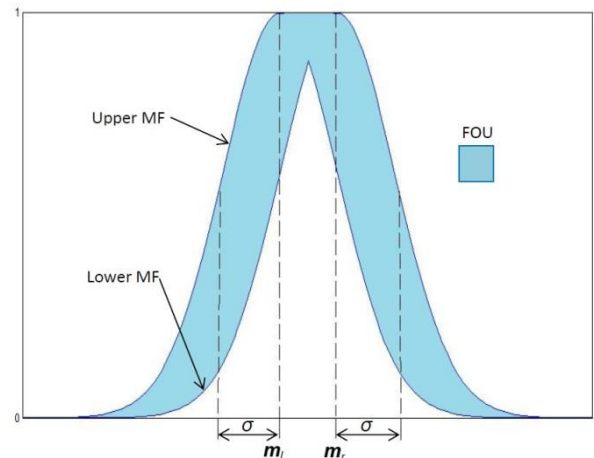


Figure 4. Interval type-2 Gaussian primary membership function.

Figure 4 illustrates the Footprint of Uncertainty (FOU) of an interval type-2 Gaussian primary Membership Function (MF). This MF can be represented by the two bounding MFs: upper MF and lower MF. For the  $i$ th type-2 fuzzy rule  $R_i$ , the  $j$ th input variable  $x_j$  has the interval type-2 fuzzy set  $\tilde{A}_j^i$  which has a Gaussian primary MF with the standard deviation  $\sigma_j^i$  and the uncertain mean  $m_j^i$  within the range  $[m_{jl}^i, m_{jr}^i]$ , i.e.,

$$\mu_{\tilde{A}_j^i} = \exp\left[-\frac{(x_j - m_j^i)^2}{2\sigma_j^{i2}}\right] \equiv N(m_j^i, \sigma_j^i, x_j), \quad (1)$$

$$m \in [m_{jl}^i, m_{jr}^i], \quad \mu_{\tilde{A}_j^i} \in [\underline{\mu}_{\tilde{A}_j^i}, \bar{\mu}_{\tilde{A}_j^i}]$$

The FOU of this MF for the input  $x_j$  can be represented as an upper MF

$$\bar{\mu}_{\tilde{A}_j^i} = \begin{cases} N(m_{jl}^i, \sigma_j^i; x_j), & x_j < m_{jl}^i \\ 1, & m_{jl}^i \leq x_j \leq m_{jr}^i \\ N(m_{jr}^i, \sigma_j^i; x_j), & x_j > m_{jr}^i \end{cases} \quad (2)$$

and a lower MF

$$\underline{\mu}_{\tilde{A}_j^i} = \begin{cases} N(m_{jr}^i, \sigma_j^i; x_j), & x_j \leq \frac{m_{jl}^i + m_{jr}^i}{2} \\ N(m_{jl}^i, \sigma_j^i; x_j), & x_j > \frac{m_{jl}^i + m_{jr}^i}{2} \end{cases} \quad (3)$$

For a type-2 Takagi-Sugeno (TS) model [10][11], the type-2 fuzzy rule  $R_i$  can be represented as

$$R_i: \text{IF } x_1 \text{ is } N([m_{1l}^i, m_{1r}^i], \sigma_1^i) \text{ and } x_2 \text{ is } N([m_{2l}^i, m_{2r}^i], \sigma_2^i), \quad (4)$$

$$\text{THEN } y^i = [a_{0l}^i, a_{0r}^i] + [a_{1l}^i, a_{1r}^i]x_1 + [a_{2l}^i, a_{2r}^i]x_2$$

where  $y^i$  is the output of the  $i$ th rule and  $[a_{kl}^i, a_{kr}^i]$  is the interval set of the parameter for TS model.

The type-reducer [12][13] reduces the outputs of the rules to the type-1 output of the system as an interval-valued fuzzy set  $[y_l, y_r]$ . This type-1 interval-valued fuzzy set can be defuzzified as

$$y = \frac{y_l + y_r}{2}. \quad (5)$$

### III. SOFNN-IT2 LEARNING

An algorithm combining SOFNN and IT2FLS has been developed for the cognitive decision module to attain a high level of robustness. The strategy for development exploits the following steps:

1. A trained SOFNN structure, which is a type-1 fuzzy structure, is obtained following learning.
2. This type-1 fuzzy structure is then initialised as a type-2 fuzzy structure.
3. The initialised type-2 structure is trained off-line, based on gradient descent and Kalman filter algorithms. The obtained type-2 fuzzy neural structure can be represented as a set of type-2 fuzzy rules similar to (4).
4. The final output of the system can be generated after information has passed through type-reduction and defuzzification.

The method to obtain a SOFNN structure has been described in [2] and [3]. Details on attaining the final output of the system by type-reduction and defuzzification can be found in [12] and [13]. Steps 2 and 3 are outlined in the following sections.

#### A. Initialise Type-1 Structure to Type-2 Structure

The structure of the SOFNN can be represented as a set of type-1 fuzzy rules, for example

$$R_i: \text{IF } x_1 \text{ is } N(m_1^i, \sigma_1^i) \text{ and } x_2 \text{ is } N(m_2^i, \sigma_2^i), \quad (6)$$

$$\text{THEN } y^i = a_0^i + a_1^i x_1 + a_2^i x_2.$$

Firstly, we initialise the centres of the Gaussian membership function in the IF-part as

$$[m_k^i - \delta m_{0k}^i, m_k^i + \delta m_{0k}^i],$$

$$\text{i.e., } m_{kl}^i = m_k^i - \delta m_{0k}^i, \quad m_{kr}^i = m_k^i + \delta m_{0k}^i, \quad (7)$$

$$k = 1, 2, \dots$$

and then initialise the parameters in THEN-part as

$$[a_n^i, \delta s_{0n}^i], \text{ i.e., } a_{nl}^i = a_n^i - \delta s_{0n}^i, \quad a_{nr}^i = a_n^i + \delta s_{0n}^i, \quad (8)$$

$$n = 0, 1, 2, \dots$$

where  $\delta m_{0k}^i$  and  $\delta s_{0n}^i$  are predefined values. This is the initial type-2 structure of the system, similar to (4). Thus, the parameters  $m_{kl}^i$ ,  $m_{kr}^i$ ,  $\sigma_k^i$ ,  $a_n^i$  and  $\delta s_n^i$  should be adapted during the learning process.

#### B. Proposed Training Algorithm

The proposed training algorithm is based on the gradient descent and Kalman filtering algorithms.

Considering the objective function

$$E = \frac{1}{2} [y(t) - y_d(t)]^2 \quad (9)$$

where  $y(t)$  and  $y_d(t)$  are the real and desired outputs of the system, respectively. Parameters of the IF-part are tuned by the gradient descent algorithm as:

$$m_{kl}^i(t+1) = m_{kl}^i(t) - \eta \frac{\partial E}{\partial m_{kl}^i} \quad (10)$$

$$m_{kr}^i(t+1) = m_{kr}^i(t) - \eta \frac{\partial E}{\partial m_{kr}^i} \quad (11)$$

$$\sigma_k^i(t+1) = \sigma_k^i(t) - \eta \frac{\partial E}{\partial \sigma_k^i} \quad (12)$$

where  $\eta$  is a learning rate.

For training purposes, the output of the system can be described in matrix form as

$$y = \mathbf{W}_2 \Psi. \quad (13)$$

Similar to [2][14][15],  $\mathbf{W}_2$  is relevant to parameters of the THEN-part and  $\Psi$  is a matrix obtained by parameters of the IF-part and input data. Parameters of the THEN-part  $\mathbf{W}_2$  can be updated by executing the Kalman filtering algorithm

$$\mathbf{W}_2(t+1) = \mathbf{W}_2(t) + Q(t+1)\Psi(t+1)[y_d(t+1) - \mathbf{W}_2(t)\Psi(t+1)] \quad (14)$$

$$Q(t+1) = Q(t) - \frac{Q(t)\Psi(t+1)\Psi^T(t+1)Q(t)}{1 + \Psi^T(t+1)Q(t)\Psi(t+1)}. \quad (15)$$

The proposed training algorithm, combining the gradient descent and Kalman filtering algorithms is performed in one iteration for each incoming training data and repeated for incremental offline learning. The trained type-2 fuzzy neural network is thus obtained. This network is employed as the RUBICON cognitive decision module to generate the decision signal to set the actual goals for Control Layer.

### IV. SIMULATION

For the purposes of validating the approach, a synthesised dataset consisting of 4500 samples was used as input to the reasoning module [1], which generates the data needed for the decision module. These data describe typical events in a domestic environment, one of the application areas of the RUBICON project's ecology. Figure 5 shows the inputs to the decision module are status outputs from the reasoning module [1], and the outputs of the decision

module (the SOFNN-IT2) are decision signals to set actual goals. The decision module receives 10 status outputs from the reasoning module plus an additional 10 one-step-back status outputs to make a combined set of 20 inputs. These status outputs are then categorised into 1 of 7 goals by the decision module. The status outputs and goal labels are listed in Table I and Table II, respectively.

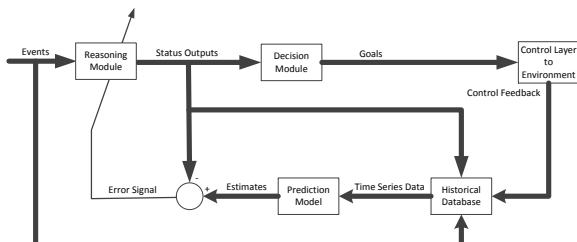


Figure 5. Block diagram of Cognitive Layer.

TABLE I. LIST OF STATUS OUTPUTS

ID	Status Outputs
1	User Exercising
2	User Relaxing
3	User in Kitchen
4	Phone Ringing Confirmed
5	Visitor at Door
6	User Cook Activity
7	Fire Alert
8	Burglary Alert
9	Dripping Alert
10	Cleaning Situation

TABLE II. LIST OF GOALS

ID	Goals
1	Bring Drink for User
2	Set Bath for User
3	Bring Phone for User
4	Attend Door
5	Attend Drip
6	Suspend Clean
7	Attend Fire

A. Testing of Modelling Capability

The first 3900 points of the data set were used as the training data of the decision module, and the remaining 600 points as the testing data. Using the algorithm outlined in the previous section a type-2 structure, SOFNN-IT2, is then obtained. The results of training and testing of the SOFNN-IT2 are shown in Table III, where RMSE is the Root Mean Square Error between the output of the network and the

desired decision signal of the goal (i.e., target) and CD is the percentage of correct decision in terms of goals against desired goal data. Figures 6 to 8 give results of the decision signal of the goal, "Bring Drink for User", and type-2 membership functions (MFs) of selected inputs.

TABLE III. RESULTS OF SOFNN-IT2

ID	Goals	Number of Rules	Result	SOFNN-IT2	
				Training	Testing
1	Bring Drink for User	4	RMSE	0.1373	0.1195
			CD	94.92	97.17
2	Set Bath for User	3	RMSE	0.1636	0.1326
			CD	92.36	94.17
3	Bring Phone for User	33	RMSE	0.1553	0.1297
			CD	93.79	96.33
4	Attend Door	11	RMSE	0.1223	0.0856
			CD	96.56	99.00
5	Attend Drip	9	RMSE	0.1425	0.1319
			CD	93.03	94.67
6	Suspend Clean	3	RMSE	0.0945	0.1115
			CD	96.15	94.33
7	Attend Fire	3	RMSE	0.1080	0.1093
			CD	97.21	97.17

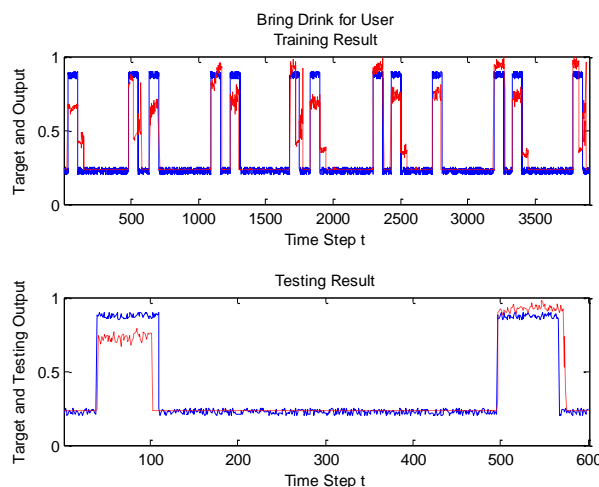


Figure 6. Results of Bring Drink for User.

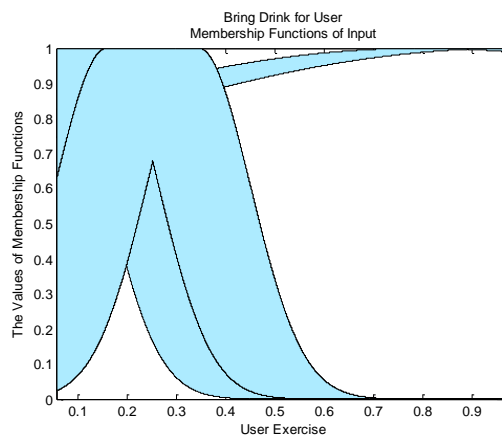


Figure 7. MFs of current input User Exercising.

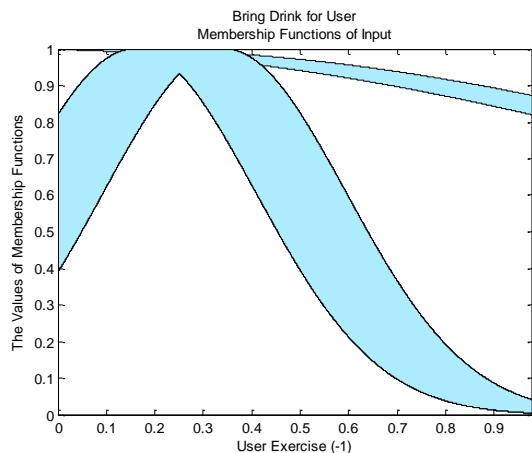


Figure 8. MFs of one-step-back input User Exercising.

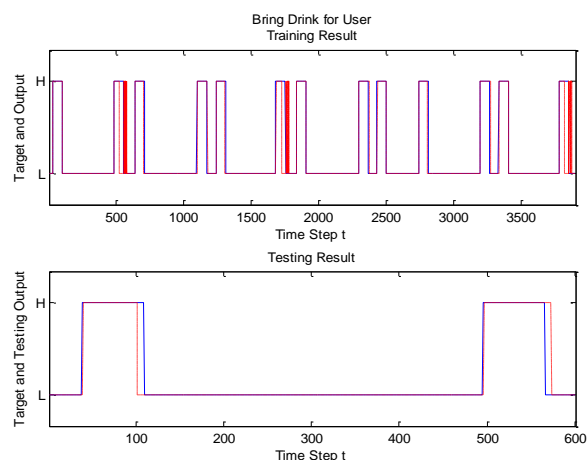


Figure 9. Results of Bring Drink for User's decision goals.

In Figure 6, the red line represents outputs of the decision model and the blue line represents targets. As an example, Figure 9 shows the results for the goal "Bring Drink for User". In Figure 9, the red line represents the goals obtained by the decision module and the blue line represents desired goals. In Figure 9, H is the high-level signal which triggers the actual goal, L is the low-level signal. Any decision signal with a confidence greater than 0.5 is defined as the high-level goal data; otherwise it is defined as the low-level goal data. These results prove that the obtained type-2 system, SOFNN-IT2, has the capability to model the complex input-output relationship to achieve high accuracy decisions.

**B. Testing of Robustness**

To investigate the robustness of the type-2 network SOFNN-IT2, its type-1 counterpart network SOFNN is used for comparison. Inputs for the obtained SOFNN and SOFNN-IT2 structures are a combination of current and one-step-back historical data from the reasoning module. The 20 inputs are combined with white-noise to assess the robustness of both networks. White-noise is chosen from the different standard deviations listed in Table IV. Again, the

RMSE is between the output with noise inputs and the output without noise inputs. Table IV provides the results for both the SOFNN and SOFNN-IT2 for the goal "Bring Drink for User".

TABLE IV. RESULTS OF SOFNN AND SOFNN-IT2 FOR BRING DRINK FOR USER

No.	White Noise (SD)	Result	SOFNN	SOFNN-IT2
1	$\sigma=0.00001$	RMSE	1.16E-05	3.66E-06
		CD%	98.84	95.24
2	$\sigma=0.0001$	RMSE	0.0001	3.48E-05
		CD%	98.84	95.24
3	$\sigma=0.001$	RMSE	0.0012	0.0004
		CD%	98.84	95.24
4	$\sigma=0.01$	RMSE	0.0116	0.0038
		CD%	98.84	95.29
5	$\sigma=0.1$	RMSE	0.124	0.0731
		CD%	96.29	93.62
6	$\sigma=0.15$	RMSE	0.1569	0.1525
		CD%	70.38	86.27
7	$\sigma=0.2$	RMSE	0.177	0.2214
		CD%	71.67	79.96
8	$\sigma=0.25$	RMSE	0.1914	0.2512
		CD%	73	77.71
9	$\sigma=0.3$	RMSE	0.2034	0.2676
		CD%	72.18	76.67

Figure 10 and Figure 11 plot the trends of RMSE and CD% against increased noise on the inputs for the goal "Bring Drink for User".

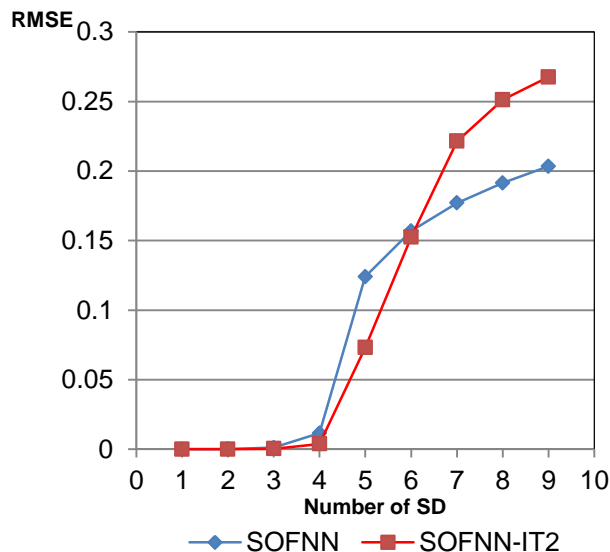


Figure 10. Results of Bring Drink for User's decision goals.

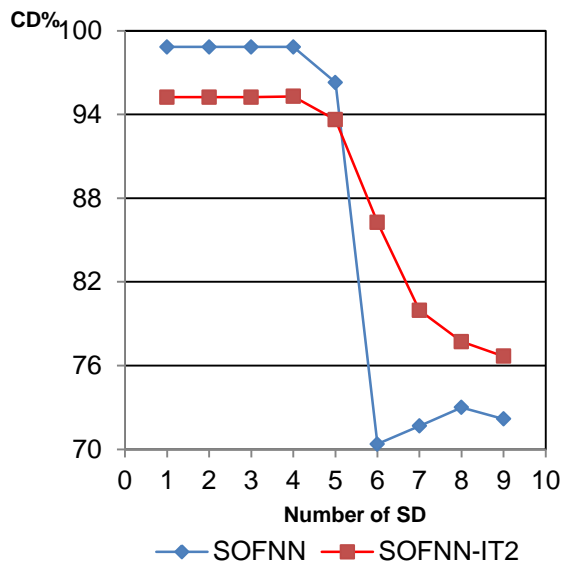


Figure 11. Results of Bring Drink for User's decision goals.

It is observed that the RMSEs of SOFNN and SOFNN-IT2 decrease as the standard deviation of the white-noise is decreased. This means that the SOFNN and SOFNN-IT2 both have a degree of robustness. For white-noise with  $\sigma \leq 0.1$ , the RMSEs of the SOFNN-IT2 for all outputs are smaller than those of the SOFNN and the CDs are similar. This illustrates that the SOFNN-IT2 is more robust to noise than the SOFNN. Furthermore, for increased white-noise, the CDs of the SOFNN-IT2 are better than those of the SOFNN, though the RMSEs of the SOFNN-IT2 are slightly larger than those of the SOFNN. These results show that, compared with its type-1 counterpart structure, the type-2 structure is more suitable for addressing uncertain situations with a high level of robustness.

V. CONCLUSION

A type-2 fuzzy neural network SOFNN-IT2, based on a SOFNN and interval type-2 fuzzy reasoning, has been proposed in this paper. The obtained type-1 SOFNN structure is firstly initialised to a type-2 SOFNN-IT2 structure and then the final type-2 SOFNN-IT2 structure is generated by the proposed training algorithm. Extensive testing of the cognitive decisions module using this SOFNN-IT2 algorithm has demonstrated that the approach is highly robust to noise and that performance is improved (compared to the traditional SOFNN approach) when considerable noise or uncertainty is present in the inputs. This is a very important attribute for RUBICON as it will be deployed in a real world environment, and thus, subject to uncertainty and noisy inputs.

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