Guided Local Search in High Performance Detectors for MIMO Systems

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***Abstract*—This work analyzes efficient non-spreading (a)synchronous MIMO detection topologies under realistic channels which results in high throughput and good performance $\times$ complexity trade-off. In this sense, we look for near-optimum efficient MIMO detections suitable for (un)coding schemes. Main system and channel parameters are analyzed, such as increasing number for transmitter and receiver antennas, number of iterations for convergence under AWGN and Rayleigh fading channels. Two heuristic local search MIMO detectors are compared with other near-optimum detectors, specifically SDR (semidefinite relaxation), expectation-maximization (EM), and linear multiuser detectors. Besides, the MIMO detectors performances under large MIMO systems (high number of transmitter and/or receiver antennas) are analyzed. The performance $\times$ complexity tradeoff results have indicated promising features for the guided local search (GLS) procedures in high capacity MIMO detectors.**

***Keywords*-MIMO system, heuristic detectors, semidefinite relaxation.**

I. Introduction

The capacity of a DS/CDMA system in multipath channel is limited mainly by the multiple access interference (MAI), self-interference (SI), near-far effect and fading. The Rake receiver explores the path diversity in order to reduce fading effect, but it is not able to mitigate the MAI [1], [2].

An alternative to solve this limitation is to apply the multiuser detection (MUD). The best performance is acquired by the optimum multiuser detection (OMUD), based on the log-likelihood function (LLF) [2]. However, this is achieved at costs of huge computational complexity which increases exponentially with the number of users. In the last decade, a variety of multiuser detectors with low complexity and sub-optimum performance were proposed, such as linear detectors, subtractive interference canceling [1], [2], semidefinite programming (SDP) approach [3]–[5] and heuristic methods [6]–[10].

In the near-optimal multiuser detection based on semidefinite relaxation (SDR-MuD), the optimal maximum likelihood (ML) detection problem is carried out by *relaxing* the associated combinatorial programming problem into an semidefinite programming (SDP) problem with both the objective function and the constraint functions being convex functions of continuous variables. SDR-MuD approach has been shown to yield near-optimal detection performance in detecting binary/quadrature phase shift keying (BPSK/QPSK) signals [3]. On the other hand, there are few works dealing with high-order modulation heuristic detectors (HeurD) for MIMO systems. Particle swarm optimization (PSO) approach for MIMO detection with 16- and 64-QAM was considered in [11], [12]. A 16-QAM local search (LS) and hybrid PSO heuristic multiuser detectors suitable for DS/CDMA systems under SISO multipath channels has been considered in [13].

This work proposes a framework analysis for near-optimum detection suitable for non-spreading high-order modulation MIMO systems based on heuristic guided local search (GLS) approach, comparing with others well established detectors methods in the literature. For several MIMO detectors, the performance$\times$complexity trade-off analysis is carried out, considering different systems and channels parameters in order to confirm the efficiency of the heuristic GLS-MIMO detectors approach. The non-spreading squared M-PSK MIMO system configurations in flat fading channels have been explored.

II. Non-Spreading MIMO System Model

Figure 1 illustrates different four configurations possibilities for the channel that can be treated with the system model described herein. In this work we have explored the configuration a) single-user MIMO (non-spreading systems) non-selective fading channels with M-PSK or squared M-QAM modulation formats. Next we describe the adopted system model.

Consider a generic MIMO system with K transmit antennas and N receive antennas, and not necessarily $K \leq N$, where K symbols are transmitted from K transmit antennas simultaneously. Let s_k be the symbol transmitted by the kth transmit antenna. Each transmitted symbol goes through the wireless channel to arrive at each of N receive antennas. Denote the path gain from transmit antenna k to receive antenna n by h_{nk}. Considering a baseband discrete-time model for a AWGN or flat fading MIMO channel, the signal received at nth antenna is given by

$$y_n = \sum_{k=1}^{K} h_{nk} s_k + \eta_n \tag{1}$$

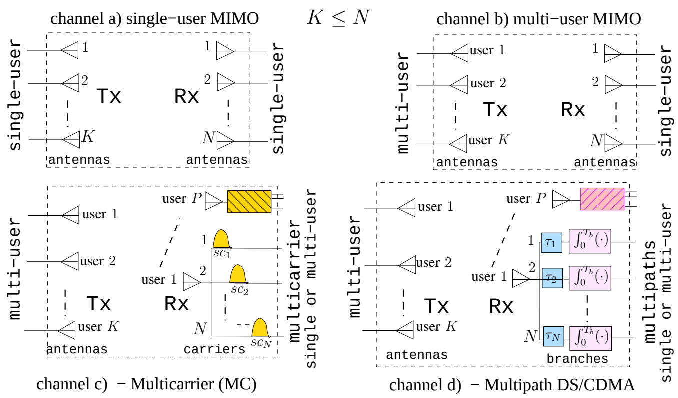

Figure 1. Channel configuration possibilities for the MIMO detection problem.

The $h_{nk}, \forall n \in \{1, 2, \ldots, N\}, \forall k \in \{1, 2, \ldots, K\}$ are assumed to be or i.i.d. complex Gaussian r.v's (fade amplitudes are Rayleigh distributed) with zero mean and $\mathbb{E}[(h_{nk}^I)^2] = \mathbb{E}[(h_{nk}^Q)^2] = 0.5$, where h_{nk}^I and h_{nk}^Q are the real and imaginary parts of h_{nk}, or the channel matrix is assumed unitary for the case of AWGN channel.

The noise sample at the nth receive antenna is assumed to be complex Gaussian with zero mean, and the samples $\{\eta_n\}, n = 1, \ldots, N$, are assumed to be independent with:

$$\mathbb{E}[\eta_n^2] = N_0 = \frac{K E_s}{\gamma}$$

where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna [14].

The received signals are collected from all receive antennas, so (1) can be re writing in a vectorial form as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{2}$$

where $\mathbf{y} = [y_1\, y_2\, \ldots\, y_N]^T$ is the received signal vector, $\mathbf{s} = [s_1\, s_2\, \ldots\, s_K]^T$ is the transmitted symbol vector, the $N \times K$ channel matrix $\mathbf{H}$, with channel coefficients h_{nk}, and $\mathbf{n} = [\eta_1\, \eta_2\, \ldots\, \eta_N]^T$ is the noise vector. In a first analysis, $\mathbf{H}$ is assumed to be known perfectly at the receiver, and afterward errors in channel estimation matrix at received can be introduced. At the transmitter, the channel matrices are assumed completely unknown.

III. Optimum Detection

The optimal maximum likelihood (ML) detector estimates the symbols for all K users by choosing the symbol combination associated with the minimal distance metric among all possible symbol combinations in the $M = 2^m$ constellation points. So, ML detection in a memoryless non-spreading MIMO Gaussian channels ($K \times N$) can be formulated as:

$$\begin{aligned} \min_{\mathbf{s}} \quad & \|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2 \\ \text{s.t.} \quad & \{s_k\} \in \mathcal{A}, \quad k = 1, \ldots, K \end{aligned} \tag{3}$$

In order to avoid handle complex-valued variables, the separable squared QAM or $M-$PSK constellation is adopted. Hence, (3) can be redefined as the following decoupled optimization problem:

$$\begin{aligned} \min_{\mathbf{r}} \quad & \|\mathbf{z} - \mathbf{M}\mathbf{r}\|_2^2 \\ \text{s.t.} \quad & r_i \in \mathcal{C} \subset \mathbb{Z}, \quad i = 1, \ldots, 2K \end{aligned} \tag{4}$$

with definitions:

$$\mathbf{z} := \begin{bmatrix} \mathrm{Re}\{\mathbf{y}\} \\ \mathrm{Im}\{\mathbf{y}\} \end{bmatrix} \in \mathbb{R}^{2N\times 1}; \quad \mathbf{r} := \begin{bmatrix} \mathrm{Re}\{\mathbf{s}\} \\ \mathrm{Im}\{\mathbf{s}\} \end{bmatrix} \in \mathbb{R}^{2K\times 1} \tag{5}$$

$$\mathbf{M} := \begin{bmatrix} \mathrm{Re}\{\mathbf{H}\} & -\mathrm{Im}\{\mathbf{H}\} \\ \mathrm{Im}\{\mathbf{H}\} & \mathrm{Re}\{\mathbf{H}\} \end{bmatrix} \in \mathbb{R}^{2N\times 2K}; \tag{6}$$

Clearly, (4) is a quadratic optimization problem with discrete variables in the set $\mathcal{A}$ and can be expressed as:

$$\begin{aligned} \min_{\mathbf{r}} \quad & \mathbf{r}^T\mathbf{Q}\mathbf{r} + \mathbf{q}^T\mathbf{r} \\ \text{s.t.} \quad & r_i \in \mathcal{C} \subset \mathbb{Z}, \quad i = 1, \ldots, 2K \end{aligned} \tag{7}$$

where $\mathbf{Q} = \mathbf{M}^T\mathbf{M}$, $\mathbf{q} = -2\mathbf{H}^T\mathbf{z}$, and $\mathbf{r} = [r_1^I, r_2^I, ..., r_K^I, r_1^Q, r_2^Q, ..., r_K^Q]^T$, with r_k^I and r_k^Q the in-phase and quadrature component, respectively, for the kth user evaluated symbol. Note that the solution $\mathbf{r}^*$ in (7) represents the estimation symbol for all K users, simply by composing the in-phase and quadrature components as: $r_k^* = r_k^{I^*} + jr_k^{Q^*}$. Note that if $K > N$, $\mathbf{Q}$ could become singular for some channel realization, once that $\mathbf{Q}$ is merely positive semidefinite. This difficulty can be removed by adding $\epsilon\,\mathbf{I}$ with small $\epsilon > 0$ to $\mathbf{Q}$.

The vector $\mathbf{r}$ in (7) is a discrete set with size dependence of M and K and can be solved directly using $m-$dimensional ($m = \log_2 M$) search method. Therefore, the associated combinatorial problem in an exhaustive search fashion has an exponential computational complexity that becomes prohibitive even for a moderate product $M\,K$ This ML detection problem can be solved efficiently by expanding the discrete feasible set into a continuous and convex feasible region [15]. Hence, manipulations, simplifications and relaxation over (4) or (7) is explored in the next section.

IV. Sub-Optimal MIMO Detectors

Based on the recently proposed non-spreading MIMO detectors suitable either for coding as uncoding MIMO schemes, herein we investigate near-optimum detectors MIMO detectors under same system framework used in [16], but aiming to improve the performance $\times$ complexity trade-off. The goal is to obtain a structure which high overall throughput with good performance and relatively simple detection (or even decoding) using low complexity detectors topologies. Under uncoded-MIMO systems context, we have proposed LS, Hyb-opt LS and PSO (named GLS-MIMO) heuristic detectors. Hence, the performance $\times$ complexity trade-off of the proposed HeurD, specially GLS-MIMO detectors, are compared with SDR, EM, linear parallel interference cancellation (PIC) and minimum mean squared error (MMSE) MIMO detectors. Below we discuss relaxations, simplifications, heuristic and classical criteria such as expectation-maximization and minimum mean squared error approaches suitable to non-spreading MIMO detectors.

A. Semidefinite Programming Relaxations (SDR)

In a brute-force fashion, the conventional ML detector requires to examine all symbol combinations, i.e., 2^{mK} possibilities, or M^{2K} for the equivalent decoupled optimization problem (4). Hence, the difficulty in adopting the OMUD is its high computational complexity, which is proportional to $\mathcal{O}(M^K)$. Therefore, when K or/and M increase, the computational complexity increases rapidly and this option becomes impractical. SDR and/or heuristic approaches are alternatives to deal with this problem, reducing complexity substantially, avoiding this huge complexity at an affordable performance loss in relation to optimum performance.

1) Relaxation for Decoupled ML Uniform QAM MIMO Detection Problem: Utilizing upper and lower bounds on the symbol energy in the relaxation step, a high-order QAM SDR MIMO detector with complexity that is independent of the constellation order for uniform QAM was proposed [17].

Under the hypothesis that $\mathcal{A}$ is an square alphabet and symmetric about the origin[1], the decoupled optimization problem posed by (4) can equivalently be rewritten as [17]:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \mathrm{Trace}(\mathbf{QX}) && (8)\\ \text{s.t.} \quad & \mathbf{X} \geq \mathbf{0}; \quad \mathrm{rank}(\mathbf{X}) = 1; \\ & \mathbf{X}_{2K+1,2K+1} = 1 \quad \mathbf{X}_{i,i} \in \mathcal{C}^2, \; i = 1, \ldots, 2K; \end{aligned}$$

$$\text{with:} \quad \mathbf{x} := \begin{bmatrix} \mathbf{r}^T \\ t \end{bmatrix} \in \mathbb{R}^{2K+1}; \quad t \in \{\pm 1\} \qquad (9)$$

$$\mathbf{Q} := \begin{bmatrix} \mathbf{M}^T\mathbf{M} & -\mathbf{M}^T\mathbf{z} \\ -\mathbf{z}^T\mathbf{M} & 0 \end{bmatrix}; \quad \text{and} \quad \mathbf{X} := \mathbf{x}\mathbf{x}^T \qquad (10)$$

Since the optimization problem (8)–(10) has nonconvex constraints: a) rank constraint, $\mathrm{rank}(\mathbf{X}) = 1$; b) squared finite alphabet constraints; $\mathbf{X}_{i,i} \in \mathcal{C}^2$, $i = 1, \ldots, 2K$, than, dropping the rank-one constraint a), and relaxing the constraints b) to the convex half-space (lower and upper) constraints:

$$\mathtt{L} := \min_{a \in \mathcal{C}} a^2 \leq \mathbf{X}_{i,i} \leq \max_{a \in \mathcal{C}} a^2 =: \mathtt{U}, \quad i = 1, \ldots, 2K,$$

we finally obtain the the SDR detector for non-spreading MIMO system:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \mathrm{Trace}\left(\begin{bmatrix} \mathbf{M}^T\mathbf{M} & -\mathbf{M}^T\mathbf{z} \\ -\mathbf{z}^T\mathbf{M} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{r}^T \\ t \end{bmatrix} \begin{bmatrix} \mathbf{r}^T & t \end{bmatrix} \right) \\ \text{s.t.} \quad & \mathbf{X} \geq \mathbf{0}; \quad \mathtt{L} \leq \mathbf{X}_{i,i} \leq \mathtt{U} \;\; i = 1, \ldots, 2K; \\ & \mathbf{X}_{2K+1,\,2K+1} = 1 && (11) \end{aligned}$$

[1]Always valid for the QAM constellations.

with $\mathbf{x} \in \mathbb{R}^{2K+1}$, $t \in \{\pm 1\}$. As suggest in [17], the relaxed problem in (11) can be solved using any of the available modern SDP solvers, based on interior point (IP) methods, such as SeDuMi [18].

After this step, an approximate solution to the original problem can be generated using Gaussian randomization:

- drawing random vectors $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{X}_{\text{opt}})$, where $\mathbf{X}_{\text{opt}}$ denotes the solution of (11),
- quantizing each element of $\mathbf{x}$ to the nearest point in $\mathcal{C}$: $\check{\mathbf{x}} = \mathrm{quantize}(\mathbf{x})$,
- reconstructing $\mathbf{s}$ from the quantized $\mathbf{x}$, i.e., $\mathbf{s}_i \leftarrow \check{\mathbf{x}}_i$, with e.g. $\mathbf{s}_i \in \{\pm 1, \pm 3\}$ for $16-$QAM.
- and finally picking the $\widehat{\mathbf{s}}$ that yields the smallest cost in original minimization problem (4).

Other strategies for approximating $\mathbf{s}$ in a squared-QAM NO-MUD problem can be obtained using a simple quantization or eigenvalue decomposition, as well as a simple randomization procedures [19], [20].

Previous results using different SDR higher-order QAM MIMO near-optimum detection strategies [17], [20]–[26]. In this sense, an efficient SDR detector was proposed in [26]. The focus is to achieve near-optimal BER performance with worst-case polynomial complexity. This is made by combining an optimized dual-scaling IP method for the relaxed SDP with a truncated version of the Sphere Decoder (SD) [27] and a dimension reduction strategy.

2) SD $\times$ SDR Detectors: The SD demonstrates impressive low running time for small systems operating in the high SNR regime; however for large systems or under low SNR regions, the running time grows exponentially. The core of the SD is based on exhaustive search. This feature is responsible for the increasing complexity under low SNR and/or huge number of users or antennas in MIMO systems.

On the other hand, the SDR detector is by nature insensitive to SNR changing, and its running time scales gradually with problem size. The insensitivity to SNR is a major ally in the low SNR regime, where the ML detection problem shows great difficulty. However, in the high SNR regime, the SDR algorithm fails to take advantage of the low noise property of the channel, when the SD takes advantage. The Sphere Decoding algorithm with adjustable radius search serves as a fast heuristic test of low noise channel realizations. The maximum number of sphere expansions is selected to ensure that complexity of the truncated Sphere Decoder does not dominate complexity of the dual-scaling algorithm.

It is worth to note that unlike the excellent performance obtained with BPSK modulation, the performance of SDR detector under higher-order QAM modulation formats is still considerably worse than achievable with the ML detector. This observation motivate us to propose heuristic alternatives for low-order modulation as well as high order squared-QAM formats.

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B. Guided Local Search Heuristic MIMO Detectors

Other approach to reach near-optimum performance consists in to apply heuristic procedures over (4). Comparisons among heuristic techniques performed for the MUD problem with BPSK modulation were carried out in [13], [28] and show that they are able to achieve performances close to the OMUD with low complexity. A local search $M-$QAM SISO MuD based on the BPSK s-LS-MUD has been analyzed in [28]. In a hybrid heuristic detection, the conventional Rake, or MMSE, or EM receiver output is adopted as the initial solution. Next, all unitary Hamming distance (from the initial solution) are evaluated individually through the equivalent quadratic minimization problem, eq. (7). The third step consists of switch the search to the simplified k-opt local search multiuser detector (s-LS-MUD), with $k = 1, 2$ [28], or even adopting PSO, genetic algorithm (GA), simulating annealing (SA), or other heuristic approach.

1) 1-LS MIMO Detector: The MIMO detector is based on the guided local search (GLS-MIMO detector) of all candidates with unitary Hamming distance regarding to the current solution.

In an exhaustive search approach, there are M branches originating from each node in any tree search algorithm. So, for BPSK modulation ($M = 2$) always there are two branches, while in 16-QAM there are 16 branches originating from each node, regarding to a symbol of kth user or Tx antenna in a MIMO context. Therefore, the complexity is exponential with the number of users: M^K candidates exist.

On the other hand, in a k-opt local search ($k-$LS) approach with binary modulation, at each iteration, the fitness function (LLF) is evaluated only $\begin{pmatrix} K \\ k \end{pmatrix}$ times. Under strong interference environment, 1-LS MIMO degradation can be significant. However, the performance degradation can be mitigated combining k-opt and p-opt LS, with $p > 1$ and $p \neq k$, with marginal complexity increasing regarding to single k-opt LS. In general, the simplest strategy in k-p-opt LS consists in swapping the local search from k-opt to p-opt and vice-versa along the iterations every time there are no improvement in the fitness function value, eq. (4) or (7). Indeed, the performance-complexity of 1-2-opt LS (named ”Hyb. 1-2-LS”) MIMO detector are evaluated in Section V. The local search setting is swapped (1-opt to 2-opt) after 3 iterations with no fitness values improvement and a total number of iterations is set to $It = 3K$.

C. Iterative Expectation-Maximization (EM) Detector

In order to compare the performances and complexities of the proposed GLS-MuD for MIMO channels, in this section an extension of the iterative expectation-maximization multiuser detector (EM-MuD) under BPSK DS/CDMA systems [29], [30] is provided. In [29], a EM-MuD for BPSK synchronous DS/CDMA under SISO AWGN channels was proposed, while in [30] an extension for MIMO flat-fading channels has been proposed.

In [30], the EM algorithm is applied to the maximum likelihood detection of BPSK synchronous DS/CDMA under MIMO (layered space-time codes) flat fading channels systems. The single data stream in the input is demultiplexed into K substreams, and each substream is modulated independently; then transmitted over a rich-scattering wireless channel to N received antennas The conditional log likelihood function (LLF) of a single layer is iteratively treated, rather than maximizing the intractable likelihood function of all layers. Computer simulations demonstrate some improvement of the EM-MIMO detection scheme with BPSK modulation in relation to the original V-BLAST scheme.

Herein, we analyze the performance-complexity of the BPSK EM-MIMO detector [30] under different scenarios. Since the symbols can only take the values $\{\pm 1\}$, the iterative decisions for the kth substream of a synchronous EM detector take the form:

$$\widehat{s}_k^{n+1} = \text{sign}\left\{\text{Re}\left[\widehat{s}_k^n \mathbf{h}_k^H \mathbf{h}_k + \beta_k \mathbf{h}_k^H \left(\mathbf{y} - \mathbf{H}\widehat{\mathbf{s}}^n\right)\right]\right\} \quad (12)$$

where $\widehat{\mathbf{s}}^n = [\widehat{s}_1^n \, \widehat{s}_2^n \, \dots \, \widehat{s}_K^n]^T$ is the symbol vector estimative at the nth iteration, and β_k's are arbitrary real valued scalars satisfying $\sum_{k=1}^K \beta_k = 1, \;\; \beta_k \geq 0$.

For QPSK modulation, the kth symbol estimation at nth iteration above is given by $s_k^{n+1} = \text{csign}\{\cdot\}$, with the same argument of (12), where the complex decisor $\text{csign}\{a + jb\} = \text{csign}\{a\} + j\text{csign}\{b\}$.

Eq. (12) provides an iterative method to detect the symbols of all substream (or users). An appropriate initial value for symbol estimate is given by the output of the conventional maximum ratio combining (MRC) receiver $\mathbf{s}^0 = \mathbf{s}_{\text{mrc}}$. On the other hand, parameter β_k has a critical role in the EM-based algorithm convergence. By setting $\beta_k = 0$, eq. (12) loses its iteration capability and reduces to the MRC receiver:

$$\widehat{s}_k = \text{sign}\left\{\text{Re}\left[\mathbf{h}_k^H \mathbf{h}_k \widehat{s}_k\right]\right\} = \text{sign}\left\{\text{Re}\left[\mathbf{h}_k^H \mathbf{y}\right]\right\}. \quad (13)$$

Assuming $\beta_k = 1$, eq. (12) becomes a linear PIC detector:

$$\widehat{s}_k^{n+1} = \text{sign}\left\{\text{Re}\left[\widehat{s}_k^n \mathbf{h}_k^H \mathbf{h}_k + \mathbf{h}_k^H \left(\mathbf{y} - \mathbf{H}\widehat{\mathbf{s}}^n\right)\right]\right\} \quad (14)$$

In [31], the β_k's values were found experimentally, and found $\beta_k = 0.8$ for the best performance in a system with one and two receive antenna scenarios. In our simulation results, for K and N in the range of $[5; 20]$ antennas, the best $\beta_k = 0.8$ value was confirmed.

D. Linear MMSE and Pseudo-Inverse $\mathbf{H}$ MIMO Detectors

The well known linear MMSE and channel pseudo-inverse (Pinv-H) based multiuser sub-optimal detectors are easily represented for $M-$PSK MIMO detection, respectively, as

$$\widehat{s}_k = \text{sign}\left\{\text{Re}\left[\mathbf{h}_k^H \left(\mathbf{h}_k^H \mathbf{h}_k + \sigma_k \mathbf{I}_N\right)^\dagger \mathbf{y}\right]\right\}, \;\; \text{(MMSE)} \quad (15)$$

and $$\widehat{\mathbf{s}} = \text{sign}\left\{\text{Re}\left[\mathbf{H}^{\dagger}\mathbf{y}\right]\right\}, \qquad \text{(Pinv-H)} \qquad (16)$$

where $(\cdot)^{\dagger}$ represents the pseudo-inverse operator.

Numerical results for the analyzed and proposed NO-MUD MIMO detectors are discussed in the next section.

V. NUMERICAL RESULTS

The performance of MIMO detectors were obtained by Monte-Carlo simulations, considering both AWGN and NLOS flat Rayleigh fading channels; the transmitted and received antennas were grouped into two categories: determined ($K \leq N$) and undetermined ($K > N$) MIMO channels. Figure 2 shows typical statistics for flat Rayleigh channel coefficients deployed in simulations. In order to facilitate the performance-throughput comparison analysis among the several MIMO detectors, low order modulation (BPSK) was assumed.

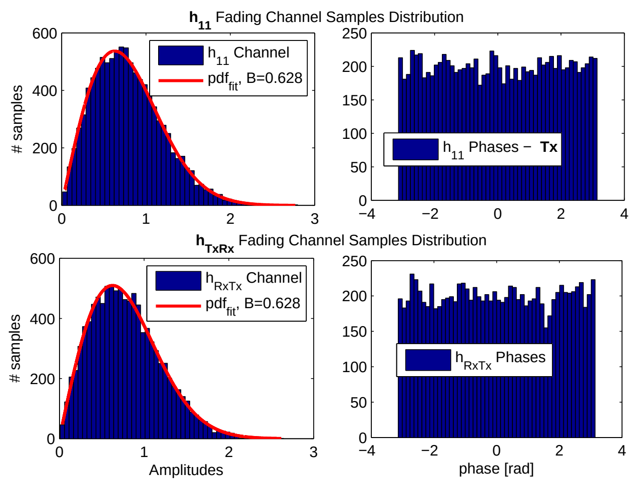

Figure 2. Typical statistics for the $h_{1,1}$ and $h_{10,10}$ Rayleigh channel coefficients.

A. Performance under AWGN Channels

Figures 3 presents the MIMO detectors performance tendency under AWGN channels when number of transmitted antennas increases from $K = 5$ to 10, and to 20, while the number of receive antennas is held, $N = 10$. The GLS-MIMO detector (1-LS) advantage increases when the channel approaches to the determined limit condition, i.e. $K = N$ antennas. For the undetermined MIMO channel condition, Fig. 3.c indicates that a single guided local search (1-LS) is not enough to deal with the interference generated under degraded spatial eigen-mode ($K >> N$). Alternativaly, a low complexity GLS-MIMO is evaluated in the next subsection, namely hybrid 1-shift-2-LS.

Note that although the SDR approach result in high performance, the complexity is quite high when compared to the GLS-MIMO detectors, mainly when $K \geq N$, as discussed in Section V-C.

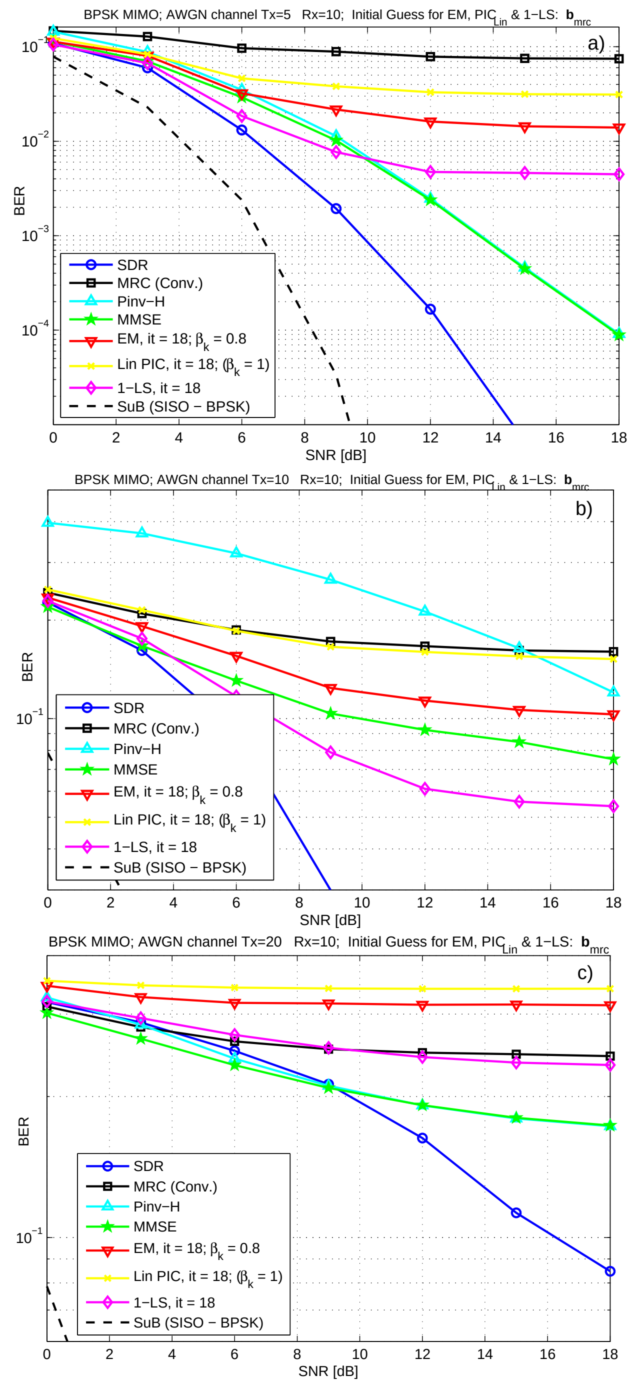

Figure 3. MIMO detectors performance for AWGN channels: a) $K = 5$ $N = 10$; b) $K = 10$ $N = 10$; c) $K = 20$ and $N = 10$ (undetermined).

B. Performance under Flat Rayleigh Channels

Figures 4 and 5 illustrate the BER degradation reduction obtained with 1-2-opt LS ("Hyb. 1-2-LS" in legend) under BPSK modulation and flat Rayleigh channels. The swapping procedure between the two guided LS algorithms ($1 \rightleftarrows 2$-opt LS) occurs after 3 iterations with no fitness values improvement; the total number of iterations was set to

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$It = 3K$. Figure 5 indicates the performance degradation *versus* SNR for the Conventional (MRC), Linear PIC, 1-opt LS, 1-2-opt LS, and SDR detectors under the increasing number of transmitted antennas (from $K = N = 3$ to $K = N = 12$).

The same MIMO detectors performance tendency in Figure 4 was observed with 16-QAM modulation. For the sake of space limitation, the MIMO detectors performance under higher $M-$QAM modulation orders were not shown herein.

It is worth to note that when the number of transmit and/or receive antennas increase, characterizing large MIMO systems, i.e., high number of K and/or N antennas (tens to hundreds) [32], Figs. 4.b and 5.c indicates a relative improvement performance of GLS MIMO detectors (1-LS and Shift 1-2-LS) regarding to the linear strategies (MMSE and Pinv-H) for low SNR. Additionally, the computational cost/complexity to obtain the channel matrix inverse is high in comparison with heuristic strategies, indicating a relative gain for heuristic approaches in these scenarios.

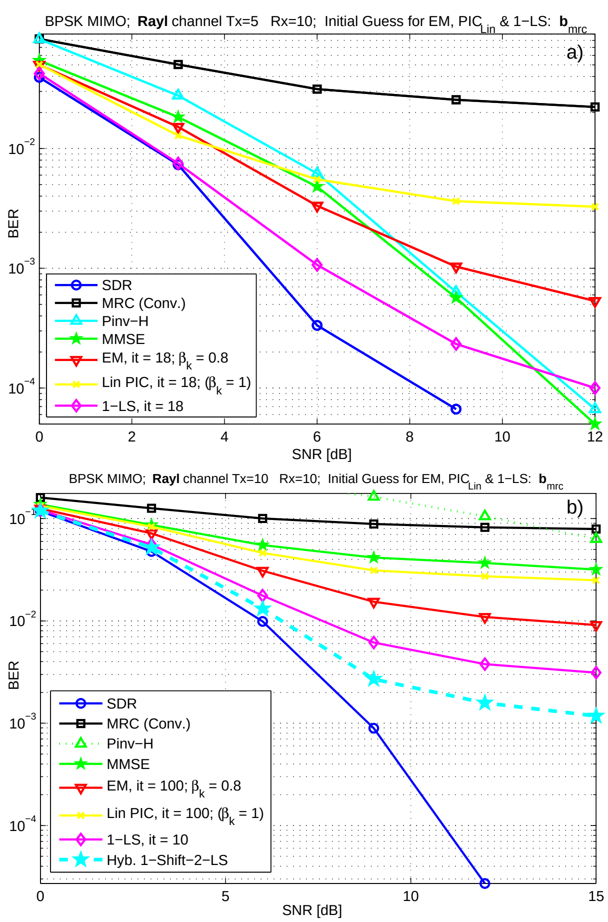

Figure 4. Performance results with $N = 10$, non-selective Rayleigh channels. a) $K = 5$; b) $K = 10$. $1 \rightleftarrows 2$-opt LS occurs after 3 iterations with no improvement; $It = 3K$.

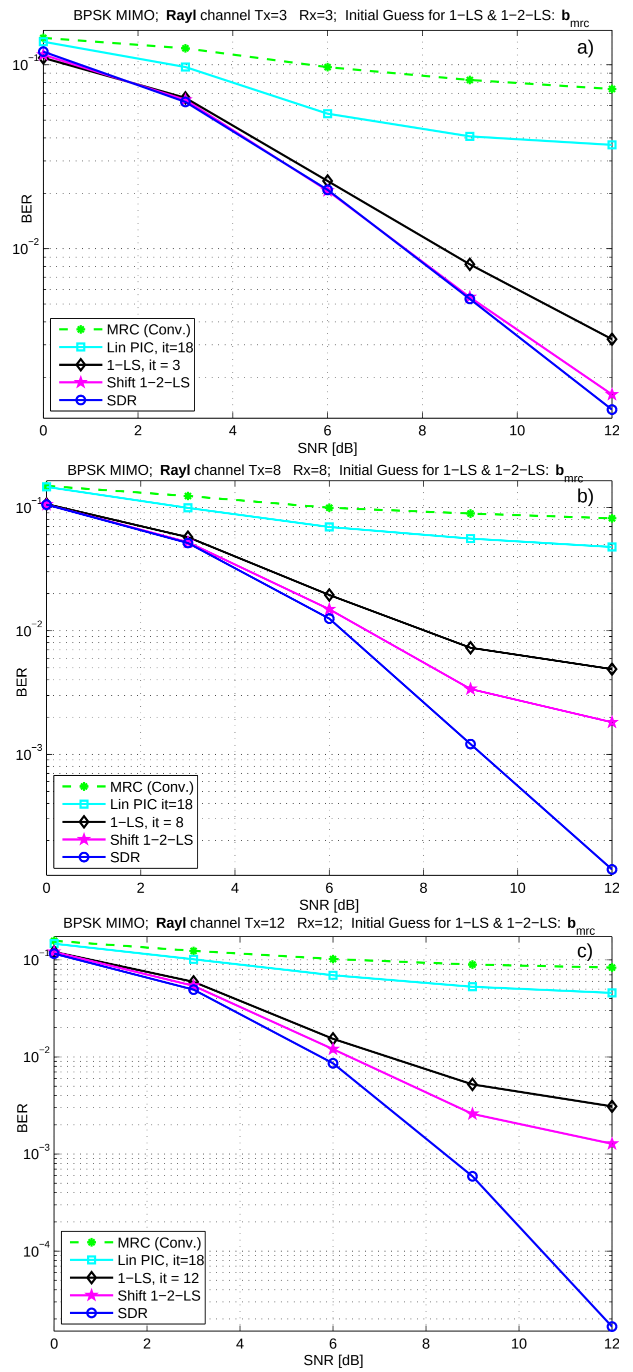

Figure 5. Performance for 1-LS and 1-2-LS with $K = N$ antennas under flat Rayleigh channel: a) $K = N = 3$; b) $K = N = 8$; c) $K = N = 12$. $1 \rightleftarrows 2$-opt LS occurs after 3 iterations with no improvement; $It = 3K$.

C. Complexity

Table I shows the complexity equations representing the number of complex multiplication/division and addition/subtraction operations for each analyzed MIMO detector. The four basic operations were considered with the same computational complexity. The complexity analysis was expressed by one iteration per substream/symbol (or antenna). The pseudo-inverse operator complexity was calculated with the Golub-Reinsch SVD [33].

Table I
NUMBER OF OPERATIONS COMPLEXITY FOR THE MIMO DETECTORS

Detector	Eq.	Complexity
SDR	(11)	$16K^2N + 17KN + 4K^2 - 2N + 14K + 2$
MRC	(13)	$2N - 1$
Pinv-$\mathbf{H}$	(16)	$11K^3 + 9K^2N + 5KN^2 - N$
MMSE	(15)	$24N^3 + N^2 + 5N - 2$
EM	(12)	$2NK + 6N - 1$
Lin PIC	(14)	$2NK + 5N - 1$
1-LS	Ref. [28]	$8NK + N - 2K - 1$

Table II indicates the number of operations for each MIMO detector considering the antennas scenarios discussed before. Hence, in AWGN channel the operation are considered real values, while under flat Rayleigh channels, the operations are assumed complex values. In all cases it was assumed $N = 10$ received antennas. The complexity for the 1-LS and Shift 1-2-LS is almost the same, been omitted the Shift 1-2-LS complexity. One can see the MIMO detector with less number of operation for all $K = 5, 10$ or 20 transmitted antennas is the MRC, but with the worst performance. While the larger complexity is achieved by the MMSE, followed by the EM (with $It = 100$), and Pinv-$\mathbf{H}$, respectively. Note that under $K = 10$ transmitted antennas scenario, the Pinv-$\mathbf{H}$, MMSE and Lin-PIC with $It = 100$ detectors present approximately the same complexity of EM MIMO detector with $It = 100$.

Table II
NUMBER OF OPERATIONS COMPLEXITY FOR THREE MIMO CHANNELS SCENARIOS. $N = 10$ RX ANTENNAS.

Detector (@$N = 10$)	Eq.	$K = 5$	$K = 10$	$K = 20$
SDR	(11)	5002	18218	69262
MRC	(13)	19	19	19
Pinv-$\mathbf{H}$	(16)	6115	24990	133990
MMSE	(15)	24148	24148	24148
EM $It = 18$	(12)	2862	4662	8262
EM $It = 100$	(12)	15900	25900	45900
Lin PIC $It = 18$	(14)	2682	4482	8982
Lin PIC $It = 100$	(14)	14900	24900	49900
1-LS $It = 18$	Ref. [28]	7002	14202	28242
1-LS $It = 10$	Ref. [28]	3890	7890	15690

Analyzing the performance-complexity trade-off provided by Figs. 4, 5 and Table II one can see that the best choice is the GLS-MIMO Detectors (1-LS and Shift 1-2-LS).

VI. CONCLUSIONS

The proposed simple guided local search MIMO detectors (1-LS and Shift 1-2-LS) have been shown attractive option regarding to the linear strategies (MMSE and Pinv-H), Expectation-Maximization approach, and conventional MIMO receivers (V-Blast and MRC topologies) as well, due to either high computational complexity in obtaining the channel matrix inverse (in case of linear strategies) or the very poor performance although lower complexity of MRC and EM strategies. On the other hand the SDR performance is always better than GLS-MIMO detectors, but the computational demand is much more intensive than heuristic approaches.

Finally, under large MIMO systems scenarios (tens to hundreds K and N), the GLS MIMO detectors presents a relative improvement performance under the conventional, linear, EM and inverted channel matrix approaches.

REFERENCES

[1] S. Moshavi, “Multi-user detection for ds-cdma communications,” *IEEE Communication Magazine*, vol. 34, pp. 132–136, Oct. 1996.

[2] S. Verdú, *Multiuser Detection*. New York: Cambridge University Press, 1998.

[3] P. H. Tan and L. K. Rasmussen, “The application of semidefinite programming for detection in cdma,” *IEEE Journal on Selected Areas in Communication*, vol. 19, no. 8, pp. 1442–1449, Aug. 2001.

[4] W.-K. Ma, T. N. Davidson, K. M. Wong, Z.-Q. Luo, and P. C. Ching, “Quasi-maximum-likelihood multiuser detection using semi-definite relaxation with applications to synchronous cdma,” *IEEE Trans. on Signal Processing*, vol. 50, no. 4, pp. 912–922, Apr. 2002.

[5] X. M. Wang, W.-S. Lu, and A. Antoniou, “A near-optimal multiuserdetector for ds-cdma using semidefinite programming relaxation,” *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2446–2450, Sept. 2003.

[6] C. Ergün and K. Hacioglu, “Multiuser detection using a genetic algorithm in cdma communications systems,” *IEEE Transactions on Communications*, vol. 48, pp. 1374–1382, 2000.

[7] H. S. Lim and B. Venkatesh, “An efficient local search heuristics for asynchronous multiuser detection,” *IEEE Communications Letters*, vol. 7, no. 6, pp. 299–301, June 2003.

[8] F. Ciriaco, T. Abrão, and P. J. E. Jeszensky, “Ds/cdma multiuser detection with evolutionary algorithms,” *Journal Of Universal Computer Science*, vol. 12, no. 4, pp. 450–480, 2006.

[9] L. D. Oliveira, F. Ciriaco, T. Abrão, and P. J. E. Jeszensky, “Particle swarm and quantum particle swarm optimization applied to ds/cdma multiuser detection in flat rayleigh channels,” in *ISSSTA’06 - IEEE International Symposium on Spread Spectrum Techniques and Applications*, Manaus, Brazil, 2006, pp. 133–137.

[10] ——, “Simplified local search algorithm for multiuser detection in multipath rayleigh channels,” in *The 16th IST Mobile and Wireless Communications Summit*, Budapest, Hungary, July 2007, p. 5 pp.

[11] A. Khan, S. Bashir, M. Naeem, and S. Shah, “Heuristics assisted detection in high speed wireless communication systems,” in *IEEE Multitopic Conference*, Dec. 2006, pp. 1–5.

[12] H. Zhao, H. Long, and W. Wang, "Pso selection of surviving nodes in qrm detection for mimo systems," in *GLOBECOM - IEEE Global Telecommunications Conference*, Nov. 2006, pp. 1–5.

[13] T. Abrão, L. D. de Oliveira, B. A. Angélico, and P. Jeszensky, *PSO Assisted Multiuser Detection for DS-CDMA Communication Systems*, ser. Particle Swarm Optimization: Theory, Techniques and Applications. Nova Science Publishers, Sept 2010, vol. 1, pp. 247–278.

[14] H. Jafarkhani, *Space-Time Coding: Theory and Practice*. Cambridge University Press, 2005.

[15] H. Peng, L. Rasmussen, and T. Lim, "Constrained maximum-likelihood detection in cdma," *IEEE Transactions on Communications*, vol. 49, no. 1, pp. 142–152, Jan. 2002.

[16] N. S. J. Pau, D. P. Taylor, and P. A. Martin, "Robust high throughput space time block codes using parallel interference cancellation," *IEEE Transactions on Wireless Communications*, vol. 7, no. 5, pp. 1603–1613, May 2008.

[17] N. Sidiropoulos and Z.-Q. Luo, "A semidefinite relaxation approach to mimo detection for high-order qam constellations," *IEEE Signal Processing Letters*, vol. 13, no. 9, pp. 525–528, Sept. 2006.

[18] J. F. Sturm, "Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones," *Sofware Methods Optimization*, vol. 11-12, pp. 625–653, 1999, available online: http://sedumi.mcmaster.ca.

[19] W. K. Ma, P. C. Ching, and Z. Ding, "Semidefinite relaxation based multiuser detection for m-ary psk multiuser systems," *IEEE Transactions on Signal Processing*, vol. 52, no. 10, pp. 2862–2872, Oct. 2004.

[20] A. Wiesel, Y. C. Eldar, and S. S. (Shitz), "Semidefinite relaxation for detection of 16-qam signaling in mimo channels," *IEEE Signal Processing Letters*, vol. 12, no. 9, pp. 653–656, Sept. 2005.

[21] X. Wang and Z. Mao, "Multiuser detection for mc-cdma system with m-qam using semidefinite programming relaxation," in *PACRIM - IEEE Pacific Rim Conference on Communications, Computers and signal Processing*, Aug 2005, pp. 530–533.

[22] Y. Yang, C. Zhao, P. Zhou, and W. Xu, "Mimo detection of 16-qam signaling based on semidefinite relaxation," *IEEE Signal Processing Letters*, vol. 14, no. 11, pp. 797 – 800, Nov 2007.

[23] Z. Mao, X. Wang, and X. Wang, "Semidefinite programming relaxation approach for multiuser detection of qam signals," *IEEE Transactions on Wireless Communications*, vol. 6, no. 12, pp. 4275 – 4279, Dec. 2007.

[24] Y. Zhang, W.-S. Lu, and T. Gulliver, "Integer qp relaxation-based algorithms for intercarrier-interference reduction in ofdm systems," *Can. J. Elect. Comput. Eng*, vol. 32, no. 4, pp. 199–205, Fall 2007.

[25] S. Park, H. Zhang, H. Hongchao, D. Han, J. Kim, E. S. Kang, and W. W. Hager, "A fast suboptimal algorithm for detection of 16-qam signaling in mimo channels," in *MILCOM - IEEE Military Communications Conference*, Oct. 2007, pp. 1–7.

[26] M. Kisialiou and Z.-Q. Luo, "Efficient implementation of a quasi-maximum-likelihood detector based on semi-definite relaxation," in *ICASSP'07 - IEEE International Conference on Acoustics, Speech and Signal Processing*, vol. 4, April 2007, pp. 1329–1332.

[27] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," *Mathematics of Computation*, vol. 44, pp. 463–471, 1985.

[28] L. de Oliveira, F.Ciriaco, T. Abrao, and P. Jeszensky, "Local search multiuser detection," *AEÜ International Journal of Electronics and Communications*, vol. 63, pp. 259–270, 2009.

[29] M. Borran and M. Nasiri-Kenari, "An efficient detection technique for synchronous cdma communication systems based on the expectation maximization algorithm," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 5, pp. 1663–1668, Sept. 2000.

[30] K. R. Rad and M. Nasiri-Kenari, "Iterative detection for v-blast mimo communication systems based on expectation maximisation algorithm," *Electronics Letters*, vol. 40, no. 11, pp. 684–685, 27 May 2004.

[31] S. Iraji and J. Lilleberg, "Interference cancellation for space-time block-coded mc-cdma systems over multipath fading channels," in *VTC 2003 - IEEE 58th Vehicular Technology Conference*, vol. 2, Oct. 2003, pp. 1104–1108.

[32] K. V. Vardhan, S. K. Mohammed, A. Chockalingam, and B. S. Rajan, "A low-complexity detector for large mimo systems and multicarrier cdma systems," *IEEE Journal on Selected Areas in Communication*, vol. 26, no. 3, pp. 473–485, April 2008.

[33] G. H. Golub and C. F. V. Loan, *Matrix Computations*, 3rd ed. The Johns Hopkins University Press, 1996.