Virtual Landmarking for Locality Aware Peer IDs

Alexander Allan, James Bradbury and Giuseppe Di Fatta
School of Systems Engineering
The University of Reading
Whiteknights, Reading, Berkshire, RG6 6AY, UK
A.J.M.Allan@student.reading.ac.uk, siu06jb@reading.ac.uk and G.DiFatta@reading.ac.uk

***Abstract*—In Peer-to-Peer (P2P) networks, it is often desirable to assign node IDs which preserve locality relationships in the underlying topology. Node locality can be embedded into node IDs by utilizing a one dimensional mapping by a Hilbert space filling curve on a vector of network distances from each node to a subset of reference landmark nodes within the network. However this approach is fundamentally limited because while robustness and accuracy might be expected to improve with the number of landmarks, the effectiveness of 1 dimensional Hilbert Curve mapping falls for the curse of dimensionality. This work proposes an approach to solve this issue using Landmark Multidimensional Scaling (LMDS) to reduce a large set of landmarks to a smaller set of virtual landmarks. This smaller set of landmarks has been postulated to represent the intrinsic dimensionality of the network space and therefore a space filling curve applied to these virtual landmarks is expected to produce a better mapping of the node ID space. The proposed approach, the Virtual Landmarks Hilbert Curve (VLHC), is particularly suitable for decentralised systems like P2P networks. In the experimental simulations the effectiveness of the methods is measured by means of the locality preservation derived from node IDs in terms of latency to nearest neighbours. A variety of realistic network topologies are simulated and this work provides strong evidence to suggest that VLHC performs better than either Hilbert Curves or LMDS use independently of each other.**

***Keywords*-Peer-to-Peer Networks; Hilbert Curve; Landmark Multidimensional Scaling; Virtual Landmarks; Network Coordinates**

I. Introduction

In Peer to Peer (P2P) networks it is useful in many circumstances for a node to be aware of its relative locality, the neighbourhood in which it resides. Latency optimizations that neighbourhood knowledge provides bring clear benefits to search and routing algorithms [1] [2] [3] [4]. In addition, locality information itself is key to many self organizing, cooperative and gossip based networks which represent an emerging paradigm in P2P technology [5] [6].

A node's calculation of its network locality is problematic as it is impractical for each node to apply a distance measure such as Round Trip Time (RTT) to all other nodes in the network.

A solution to this problem comes in the form of landmark clustering [3] used to create a scalar node ID space [7].

The assumption behind this technique is that RTT distances from any node to a predetermined set of landmark nodes (a subset of nodes which act as reference points) will be similar for other nodes in the immediate neighbourhood. The vector of distances to landmarks can be used in itself if the network protocols support a multidimensional index [8]. This work will be mainly focusing on the 1D scalar index that can be generated from this vector, noting that such an index can be more easily integrated into many existing peer-to-peer distributed hash table (DHT) systems. A scalar index provides an intuitive notion of locality for node naming schemes in all areas of P2P networks.

In order to create this scalar node ID from a distance vector, a dimensionality reduction algorithm is required. Previous work [9] has shown that in the context of landmark vector reduction, Hilbert Curves (HC), which are a type of space filling curve with good locality preserving properties [10] [11], outperform Principal Component Analysis (PCA) and Sammon Mapping [12] in terms of neighbourhood preservation. The Hilbert Curve can also be deployed in a distributed manner, requiring only a set of landmark vectors and a predefined HC granularity at each node to produce a homogeneous indexing for all nodes.

A problem identified with this method and with landmark indexing in general stems from the vulnerability and network traffic implication inherent in having a small number of landmark nodes upon which all other nodes depend to produce their node ID. Furthermore the less landmarks used, the greater the importance of landmark placement to the accuracy of the overall algorithm [13]. It is desirable therefore to increase the number of landmarks used as much as possible so as to negate the need for a heuristic landmark selection process and to decrease the vulnerability to single node failures.

However, Hilbert Curves are affected considerably by the curse of dimensionality [9], which causes accuracy to fall and computational load to increase with each additional dimension.

This work proposes a method to avoid the critical trade off in the number of landmark nodes. The notion of *virtual landmarks* is exploited to decouple the two issues. A large number L of actual landmark nodes are projected into an opportune small number K ($K < L$) of virtual landmarks which define a network coordinates space for a given topology. The HC is applied to the network coordinates space

 ISBN: 978-1-61208-173-1

which has lower dimensionality than the space defined by the actual landmarks. In particular, a decentralised version of the Landmark Multidimensional Scaling (LMDS) [14] algorithm is adopted to map the landmark vectors into virtual landmark vectors of lower dimensionality.

The proposed method, Virtual Landmarks Hilbert Curve (VLHC), is expected to be more accurate in preserving locality in the peer ID space than using either LMDS or Hilbert Curve alone.

In the experimental analysis realistic network topologies are used to compute and compare the accuracy of the methods in preserving nearest neighbour relations.

The remainder of the paper is organized in the following manner: Section II shows an overview of the various algorithms and techniques used, Section III describes the proposed method, Section IV describes the experimental methodology used for the comparison and benchmarking, Section V displays the experimental results and presents their analysis and, finally, Section VI provides some conclusive remarks and suggests future extensions to this work.

II. Overview of Techniques

A. Hilbert's Curve

The Hilbert Curve (HC) is a method of sequentially indexing points in space via a non intersecting line. The idea of space filling curves was first proposed theoretically by Giuseppe Peano in 1890 then geometrically by David Hilbert a year later. These curves allow points in an N dimensional space to be ordered on a 1D line in a manner which preserves locality relationships. Although a number of other space filling curves exist, such as Lebesque Curves, it has been shown that Hilbert Curves are among the best at preserving locality in terms of compact regions of space [15] [16].

B. Virtual Landmarking

Virtual landmarks in the context of network coordinates were proposed by Tang et al in 2003 [17]. The concept arose from analysis of the process in which locality information was embedded as sets of vectors to landmarks. This analysis showed that the intrinsic dimensionality of these embeddings in Internet-like networks was typically around 8 [17].

The concept of intrinsic dimensionality can be illustrated by imagining a mechanical arm with 5 joints. A data set might have the angle of each joint sampled at a certain time interval with the tip of the arm represented by a 5 dimensional coordinates. Since the mechanical arm exists in a 3D lab however, this position could be described by a 3 dimensional coordinate system with no reduction in accuracy. The embedding dimension of this data set would then be 5, with an intrinsic dimensionality of 3.

This work is based on the assumption that the intrinsic dimensionality of computer networks is relatively small. The approximate figure of 8 given by Tang et al. suggests that projecting a high dimensional coordinate space to a few dimensions will retain most of the locality information.

C. LMDS

Multidimensional Scaling (MDS) [18][19] is a family of dimensionality reduction methods. In classical MDS an $N \times N$ distance matrix is required, where N is the number of objects (nodes). The algorithm performs the eigendecomposition of the distance matrix and has a complexity of $O(N^3)$. In order to generate the landmark vectors each node would need to determine the communication latency to every other node in the topology. This approach clearly has practical limitations in a large-scale P2P network.

Landmark Multidimensional Scaling (LMDS) [20], [21] is a scalable MDS variant which does not require computationally expensive matrix calculations, nor the entire distance matrix. Landmark MDS instead performs a classical MDS on the subset of landmark nodes only, and computes embedding coordinates for the other nodes by using distance-based triangulation by means of the decomposed landmark matrix.

Landmark MDS was developed primarily as a technique to speed up the ISOMap procedure [20], and has been shown to be equivalent to the Nyström approximation of the eigenvectors and eigenvalues of a matrix [22], The method works by utilising properties of kernel matrices to calculate embedding coordinates based upon estimations of eigenvalues and eigenvectors.

Two distance matrices are calculated, landmark node to landmark node matrix A, and landmark node to non-landmark node matrix B. To distinguish Landmark MDS from the Nyström approximation, A and B must undergo double-centering, akin to the classical MDS procedure [22]. The Nyström approximation then calculates estimated embedding coordinates using values from the eigendecomposed A matrix and values from the B matrix only, therefore negating the need for the costly calculation of the $N \times N$ distance matrix as well as its eigendecomposition. LMDS has a complexity of $O(NLk + L^3)$, where L ($L \ll N$) is the number of landmarks and k ($k \leq L$) is the number of the largest positive eigenvalues used in the approximation.

D. Landmark Selection

The landmark selection problem has itself been the subject of much work as for small numbers of landmarks it can have a large impact on any triangulation based methods [23][13][24]. Various heuristics have been proposed which attempt to select landmarks spread throughout the network with a uniform distribution.

However, Tang et al (2004) [13] find that as the number of landmarks in a network surpass 20, most landmark selection techniques (with the exception of a computationally infeasible greedy approach) are no better than random selection because a uniform distribution inevitably emerges from any random selection method given sufficient points.

ISBN: 978-1-61208-173-1

This work adopts a random selection of a large number of landmarks, thus avoiding the complexity of implementing a distributed non random selection heuristic.

III. Virtual Landmarks Hilbert Curve (VLHC)

This work proposes a novel approach for 1 dimensional index generation by combining the notion of virtual network landmarks with the two techniques HC and LMDS. In a two-stage process at each node, LMDS is applied to the local RTT vector v^L obtained from a large set of L landmarks. LMDS is used to project the high-dimensional RTT vector to a lower K-dimensional space, where $K < L$. The corresponding low-dimensional vector v^K is referred to as the *virtual landmark vector*.

In the second stage the local virtual landmark vector is converted into a scalar index by means of a Hilbert Curve.

A traditional approach would apply HC to the landmark vectors in L dimensions. In the proposed approach HC is applied to the virtual landmark vectors in K dimensions. The LMDS dimensionality reduction is adopted to overcome the restriction of the Hilbert Curve that performs badly in high dimensional spaces. This combination is also likely to be more effective than using the LMDS to project directly into the 1D index space, as Hilbert Curves have more favourable locality preserving properties.

A. Decentralised Algorithm

A distributed approach for conducting the LMDS at each node would be as follows.

In the initialisation step the landmark nodes ping each other in order to create a matrix of latencies (RTT) between them. The landmark nodes perform an all-to-all broadcast operation to propagate their local latency vector and to generate the $L \times L$ matrix. Each landmark node then uses this matrix to perform classic MDS locally and independently. All landmark nodes will compute the eigenvalue decomposition of the matrix. The landmark nodes generate an identical reference set of K[1] eigenvalues and vectors.

When a node joins the network, it pings the landmarks to create a local landmark vector. The node also requests and receives the reference set of eigenvalues and eigenvectors from any one of the landmarks. From the reference set and its local RTT vector, the node can calculate its own approximate position in the lower dimensional MDS projection using the Nyström approximation [22]. The coordinates of this approximate position is then taken as the node's *virtual landmark vector* to which the Hilbert Curve is applied to produce the 1D node index (peer ID).

The approach is scalable as nodes have to communicate with landmark nodes only. The number and choice of actual landmarks is not critical as already discussed.

[1]The actual number of virtual landmarks corresponds to the maximum between the parameter K and the number of positive eigenvalues.

The VLHC technique can be applied in P2P networks without the need for global communication and synchronisation. This can be done in a fully decentralised approach as outlined in the following steps.

Using a Gossip-based protocol nodes can randomly select the landmarks by exchanging sorted lists of IP addresses and choosing the top L after a suitable convergence period.

The elected landmark nodes collect RTTs to each other and create the matrix needed for LMDS. The landmark nodes distribute the matrix throughout the network at a certain rate to prevent a traffic bottleneck at the landmarks.

When each node receives the matrix, it can then calculate its virtual landmarks vector and from this its node ID.

IV. Comparative Analysis

The comparative analysis of the different methods is based on the average latency to the 30 Nearest Neighbour (NN) nodes. Two methods provided the baseline and the optimal value of the performance index. The ideal performance was computed by searching the 30 actual NNs in the topology for each node. The random selection of 30 nodes provided a baseline value of the worst performance. The performance indices from several random selections of landmark sets were averaged.

In the experimental evaluation four methods were tested to carry out a comparative analysis:

- 1-dimensional Landmark Multidimensional Scaling (1D LMDS),
- Hilbert Curve applied to the landmark vectors (Hilbert),
- the proposed Virtual Landmarks Hilbert Curve (VLHC) and
- 8-dimensional LMDS Network Coordinates (8D Network Coordinates).

The 8D LMDS generates a network coordinates scheme which is suitable to assess the information loss associated with the virtual landmarks. Ideally a loss-less latency vector projection from the L-dimensional space to the K-dimensional space would produce a performance index comparable to the ideal NN method. It should be noticed that this method does not produce a scalar index which can be used as peer ID and it serves as reference only.

In each test L landmark nodes were chosen randomly and RTTs between them were used to create the matrix for the LMDS calculations with the Nyström approximation. RTTs from each node i to the landmark nodes were determined, v_i^L, and were input into the LMDS algorithm to create a vector of distances to K virtual landmarks v_i^K.

The Douge Moore's C implementation [25] of a recursion free algorithm of the Hilbert's space filling curve [26] was adopted. It was applied to v_i^K to produce the 1D index $H(v_i^K)$. The same algorithm was also applied to the vector v_i^L to produce the 1D index $H(v_i^L)$ for the classical Hilbert Curve method.

 ISBN: 978-1-61208-173-1

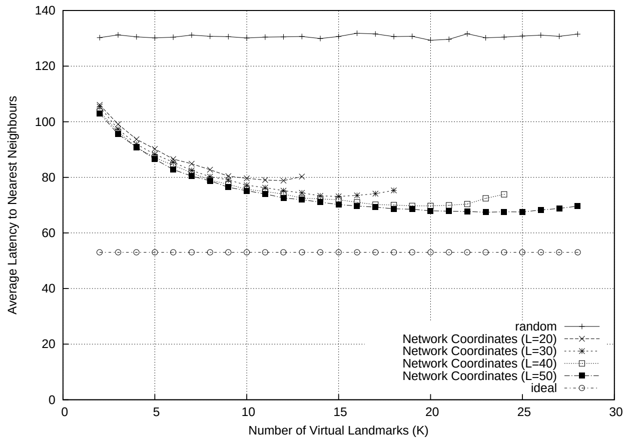


Figure 1. Average latency to nearest neighbours vs. the number of virtual landmarks: reference methods only

LMDS with $K = 1$ was applied to vectors v_i^L to produce a 1D index $L(v_i^L)$. Each of the three methods generates a different set of peer IDs, respectively, $\{H(v_i^K)\}$, $\{H(v_i^L)\}$ and $\{L(v_i^L)\}$.

Each generated index set was used to search the set of 30 nearest neighbours nodes for each node. Multiple sets of random nodes were generated and the exact NNs in the topology were computed using the Floyd Warshals algorithm [27]. The set of NNs in the 8D network coordinates scheme was determined by means of the Euclidean distance.

A set of NNs was used to compute an accuracy index. The distances between every node and its NNs was used to produce an average NN latency for each method.

The accuracy index is a normalized average NN latency ($\in [0, 1]$) which is designed to be relevant across heterogeneous topologies. An accuracy index of 1 is assigned to the ideal average NN latency; 0 is assigned to the worst average NN latency average associated with the random method. The average NN latency of each method and on each topology was normalized between these two indices. Using this accuracy measure results close to 1 indicate the method was almost as good as was possible and results which were closer to 0 imply that the method was almost as bad as a purely random selection of neighbours.

V. Simulations and Results

We have tested the algorithms in two different types of network topologies:

- 10 Internet-like topologies with 1000 nodes generated by the topology generator BRITE [28] with a Waxman model [29] to simulate a flat level Autonomous System;
- 10 2D mesh topologies with dimensions 40×25 (1000 nodes).

In the first test the average NN latency of the three reference methods (ideal, random and Network Coordinates) was compared to verify the intrinsic dimensionality of the Internet-like topologies. In Figure 1 the average NN latency for these three methods is shown for a varying number of virtual landmarks K. The number of actual landmarks L is fixed to 20, 30, 40 and 50. For the Network Coordinates method and for a given value L, the test was executed till the eigendecomposition would return enough positive eigenvalues to generate the desired number K of virtual landmarks. For $K < 10$ the average NN latency of the Network Coordinates scheme clearly improves as K increases. For $K > 10$ it does not improve further or it worsens. The test was run on a single Internet-like topology; similar results were obtained for the other topologies. This experiment shows that a choice of 8 virtual landmarks is a good trade off, confirming the intrinsic dimensionality of the Internet-like topologies as determined in [17]. In all other simulations we have fixed the number K of virtual landmarks to 8.

All methods were tested on each network topology multiple times. For each topology, the resulting performance indices were averaged over 20 tests with different random selections of landmarks. For each topology, the number of landmarks was increased from $L = 3$ to $L = 49$ and for every L value the normalised accuracy index was computed as the average over 20 tests with different random selections of landmarks.

Figures 2(a) and 2(b) show the accuracy of the methods when varying the number of the actual landmark nodes.

The curve for the Hilbert method is truncated at $L = 20$. Within Douge Moore's C implementation of the Hilbert Curve, the number of input bits (which defines the granularity of the curve) multiplied by the number of input dimensions cannot exceed the value of 8 times an unsigned 'long long' data type (which is 64 bytes in the used CPU architecture). A curve with 3-byte granularity was used to give sufficient precision, so when the input dimensionality increased beyond 21, the curve would no longer compute as $22 \times 3 > 64$.

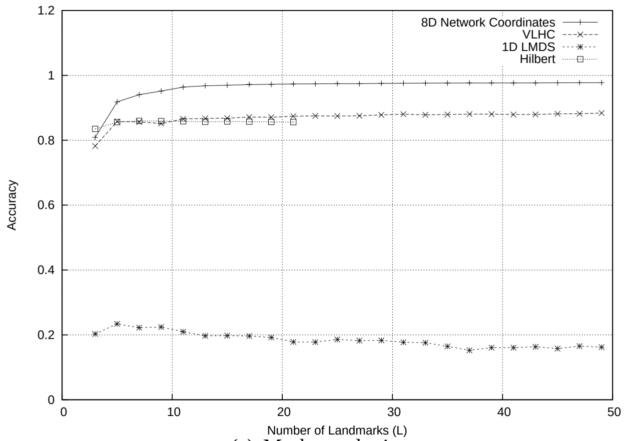
A. Discussion

The VLHC technique performs better than either the 1D LMDS or stand alone Hilbert curve on both mesh and Internet like topologies. Its accuracy plateaus at around 0.87 on the mesh topology, and around 0.41 on the Internet like topology. It achieves this after 18 or more landmarks are used, only improving marginally after this point as more landmarks are added.

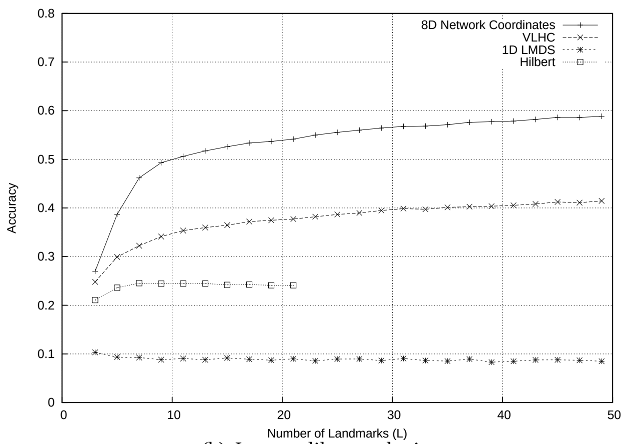
On the mesh topologies, where the intrinsic dimensionality is 2, the standard Hilbert Curve computed over all landmarks was comparable to VLHC. However, implementation issues did effect its actual scalability in terms of the number of landmarks.

On the Internet like topologies, the standard Hilbert Curve achieved a maximum accuracy of around 0.24 after 6 landmarks but could only maintain this until the implementation failed at 22 landmarks. On this type of topologies, where the

 ISBN: 978-1-61208-173-1



(a) Mesh topologies



(b) Internet-like topologies

Figure 2. Accuracy for mesh (a) and Internet-like topologies (b)

intrinsic dimensionality is about 8, it performed constantly worse than the VLHC.

On the mesh topologies the 1D index produced by the LMDS was the worst performer with a peak of 0.23 accuracy at 5 landmarks. As more landmarks were added the accuracy fell slowly. For the Internet like topology the 1D LMDS index was again the worst performer for any numbers of landmarks we have tested, getting a peak accuracy score of 0.1 which again fell slowly as the number of landmarks increased.

The 8D network coordinates produced by LMDS showed almost optimal performance in the mesh topologies, with accuracy rising up to 0.97 after 17 landmarks. In the Internet like topologies, however, it performed significantly worse, achieving a maximum of 0.58 at 49 landmarks. The curve shows a steep improvement up to around 8 landmarks, at which point the rate of improvement slows considerably. This is also another indirect confirmation of the intrinsic dimensionality of this type of topologies.

The non optimal performance of the 8D network coordinates system in the Internet-like topologies may be due to using a triangulation based technique (which is essentially how LMDS works) in a network space which violates the triangular inequality (the communication latency from A to B might be more than from A to C to B). The mesh topology however is much closer to Euclidean space and so the inequality would hold true in most cases.

The effectiveness of the VLHC 1D node indexing scheme is considerably greater in the mesh topology than in the Autonomous Systems topology. Indeed it scores only 12% less than a brute force approach in the mesh topology which makes it suitable for most applications. The utility in an Internet like topology will be considerably more dependant on the problem domain, as the peak accuracy of 41% may not be sufficient for some applications where a high degree of geographical accuracy is needed, but rather for ones where a notion of locality in a scalar index outweighs the overheads of adopting a more accurate but more complex multidimensional network coordinate scheme.

VI. Conclusion

This work has presented the application of the concept of virtual landmarks to the problem of generating locality aware peer identifiers by means of space filling curves. Quantitative evidence has suggested that applying LMDS in conjunction with a Hilbert Curve produces a superior mapping to a 1D index in terms of locality preserving properties, when compared to either technique applied independently. This agrees with the postulation that a small number of virtual landmarks are sufficient to capture the intrinsic dimensionality of Internet-like networks. More experimental work is required to investigate the sensitivity of the proposed method with respect to this parameter in real network topologies.

In contrast to previous applications of the Hilbert Curve, a larger numbers of landmarks can be employed by exploiting a decentralised LMDS algorithm. This not only increases accuracy but also allows for more robustness.

The experimental analysis has shown that, as expected, 1D indices are less accurate than multidimensional network coordinates. In general, applications should consider what accuracy level of nearest neighbour preservation is needed before adopting either a 1D scheme or a multidimensional scheme, as the cost of simplicity is still substantial.

Further work will include using graphs extracted from real network topologies to support the results obtained via simulation, the implementation and testing of the outlined Gossip-based approach and the adaptation of the algorithm to handle a dynamic environment with churn rate for nodes and landmarks and with time-varying latencies.

References

[1] H. Shen and C. Xu, "Hash-based proximity clustering for load balancing in heterogeneous dht networks," *Journal of Parallel and Distributed Computing*, vol. 65, no. 5, pp. 686–702, May 2005.

[2] M. Castro, P. Druschel, Y. C. Hu, and A. Rowstron, "Exploiting network proximity in peer-to-peer overlay networks," Microsoft Research, Cambridge, England, Tech. Rep. MSR-TR-2002-82, May 2002.

[3] S. Ratnasamy, M. Handley, R. Karp, and S. Shenker, "Topologically-aware overlay construction and server selection," in *Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies Proceedings*, New York, USA, Jun. 2002, pp. 1190–1199.

[4] Y. Zhu and Y. Hu, "Towards efficient load balancing in structured P2P systems," in *18th International Parallel and Distributed Processing Symposium (IPDPS'04)*, Santa Fe, USA, april 2004, p. 20.

[5] M. Cai and M. Frank, "RDFPeers: a scalable distributed RDF repository based on a structured peer-to-peer network," in *Proceedings of the 13th international conference on World Wide Web*. ACM, 2004, pp. 650–657.

[6] S. Savarimuthu, M. Purvis, M. Purvis, and B. Savarimuthu, "Mechanisms for the self-organization of peer groups in agent societies," *Multi-Agent-Based Simulation XI*, pp. 93–107, 2011.

[7] Z. Li, G. Xie, and Z. Li, "Locality-aware consistency maintenance for heterogeneous P2P systems," in *IEEE International Parallel and Distributed Processing Symposium (IPDPS)*, 2007, pp. 1–10.

[8] M. Gharib, Z. Barzegar, and J. Habibi, "A novel method for supporting locality in peer-to-peer overlays using hypercube topology," in *International Conference on Intelligent Systems, Modelling and Simulation (ISMS)*, 2010, pp. 391–395.

[9] A. Allan and G. Di Fatta, "Effectiveness of landmark analysis for establishing locality in P2P networks," in *The Second International Conference on Advances in P2P Systems (AP2PS 2010)*, October 2010.

[10] C. Gotsman and M. Lindenbaum, "On the metric properties of discrete space-filling curves," *IEEE Transactions on Image Processing*, vol. 5, no. 1, pp. 794–797, Jan. 1996.

[11] B. Moon, H. Jagadish, C. Faloutsos, and J. Saltz, "Analysis of the clustering properties of the Hilbert space-filling curve," *IEEE Transactions on Knowledge and Data Engineering*, vol. 13, no. 1, pp. 124–141, Jan. 2001.

[12] J. Sammon, "A nonlinear mapping for data structure analysis," *IEEE Transactions on Computers*, vol. C-18, no. 5, pp. 401–409, May 1969.

[13] L. Tang and M. Crovella, "Geometric exploration of the landmark selection problem," *Passive and Active Network Measurement*, pp. 63–72, 2004.

[14] V. De Silva and J. Tenenbaum, "Sparse multidimensional scaling using landmark points," *Dept. Math., Stanford University, Stanford, CA, Tech. Rep*, 2004.

[15] C. Faloutsos and Y. Rong, "Spatial access methods using fractals: Algorithms and performance evaluation," University of Maryland, Maryland, USA, Tech. Rep. UMIACS-TR-89-31, Mar. 1989.

[16] H. Jagadish, "Linear clustering of objects with multiple attributes," *ACM SIGMOD Record*, vol. 19, no. 2, pp. 332–342, Jun. 1990.

[17] L. Tang and M. Crovella, "Virtual landmarks for the internet," in *Proceedings of the 3rd ACM SIGCOMM conference on Internet measurement*. ACM, 2003, pp. 143–152.

[18] W. Torgeson, "Multidimensional scaling of similarity," *Psychometrika*, vol. 30, pp. 379–393, 1965.

[19] R. Shepard, "Analysis of proximities: Multidimensional scaling with an unknown distance function I & II," *Psychometrika*, vol. 27, pp. 125–140, 219–246, 1962.

[20] J. B. Tenenbaum and V. de Silva, "A global geometric framework for nonlinear dimensionality reduction," *Science*, vol. 290, no. 5500, pp. 2319–2323, 2000.

[21] V. de Silva and J. B. Tenenbaum, "Sparse multidimensional scaling using landmark points," University of Stanford, Standford, USA, Tech. Rep. CSE-TR-456-02, jun 2004.

[22] J. C. Platt, "Fastmap, metricmap, and landmark mds are all nystrom algorithms," in *In Proceedings of 10th International Workshop on Artificial Intelligence and Statistics*. IEEE, 2005, pp. 261–268.

[23] S. Baskakov, "Landmarks selection algorithm for wireless sensor networks," in *Self-Adaptive and Self-Organizing Systems, 2008. SASO'08. Second IEEE International Conference on*. IEEE, 2008, pp. 361–369.

[24] Q. Cao and T. Abdelzaher, "Scalable logical coordinates framework for routing in wireless sensor networks," *ACM Transactions on Sensor Networks (TOSN)*, vol. 2, no. 4, pp. 557–593, 2006.

[25] D. Moore. (2011) Fast hilbert curve generation, sorting, and range queries. Rice University. Texas, USA. [Online]. Available: http://www.tiac.net/∼sw/2008/10/Hilbert/moore/hilbert.c

[26] A. Butz, "Alternative algorithm for Hilbert's space-filling curve," *IEEE Transactions on Computers*, vol. C-20, no. 4, pp. 424–426, april 1971.

[27] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms (3rd Ed.)*. MIT Press, 2009.

[28] A. Medina, A. Lakhina, I. Matta, and J. Byers, "BRITE: An approach to universal topology generation," in *Proc. of the International Workshop on Modeling, Analysis and Simulation of Computer and Telecommunications Systems (MASCOTS'01)*, 2001, pp. 346–353.

[29] B. Waxman, "Routing of multipoint connections," *Selected Areas in Communications, IEEE Journal on*, vol. 6, no. 9, pp. 1617–1622, 1988.

 ISBN: 978-1-61208-173-1