Frequency Domain Equalization of CAZAC-OFDM with Transversal Filter using LMS Algorithm

Hiroyuki Yamano, Yoshitsugu Sugai and Masahiro Muraguchi
Department of Electrical Engineering, Tokyo University of Science
6-3-1 Niijuku, Katsushika-ku, Tokyo, 125-0051, Japan
E-mail: 4319579@ed.tus.ac.jp, murag@ee.kagu.tus.ac.jp

***Abstract*— In wireless communications, signals are affected by multipath fading in the transmission channel, which causes amplitude fluctuation and phase fluctuation at the receiver front-end. Unfortunately, the Frequency Domain Equalization (FDE) using pilot subcarriers, which is ordinary used in conventional Orthogonal Frequency Division Multiplex (OFDM) systems, cannot apply the OFDM system with Constant Amplitude Zero Auto-Correlation (CAZAC) precoding, CAZAC-OFDM. In this paper, we propose a method to improve the negative effect of multipath fading by applying transversal filter using Least Mean Square (LMS) algorithm to CAZAC-OFDM systems. We have confirmed that the CAZAC-OFDM system with the proposed transversal filter maintains enough low Bit Error Rate (BER) performance under flat fading and frequency selective fading channels.**

Keywords-OFDM; CAZAC sequence; Frequency domain equalization; Transversal filter; LMS algorithm.

I. INTRODUCTION

In recent years, with the spread of smartphones, tablet terminals and PCs, the demand for wireless communication is expanding. Furthermore, with the advent of video distribution and streaming services, the amount of data has increased rapidly. Therefore, an Orthogonal Frequency Division Multiplex (OFDM) scheme is widely used in wireless communication, due to its great advantage of high spectrum utilization that is about twice spectral efficiency compared with single carrier scheme. Moreover, the OFDM scheme has resistance properties to multipath fading.

Unfortunately, OFDM scheme has high Peak-to-Average Power Ratio (PAPR). Many kinds of method have been proposed to solve this PAPR problem [1]. The OFDM scheme with Constant Amplitude Zero Auto-Correlation (CAZAC) precoding, CAZAC-OFDM, is a known modulation technique aimed at alleviating the PAPR problem [2][3]. It has been reported that one CAZAC sequence in cooperation with Inverse Fast Fourier Transform (IFFT) process converts the PAPR of the M-array Quadrature Amplitude Modulation (M-QAM) OFDM signal into the PAPR of an M-QAM single-carrier signal [4]-[7].

In wireless systems under multipath fading environment, interference between delayed waves makes amplitude fluctuation and phase fluctuation at the receiver front-end. The Frequency Domain Equalization (FDE) using pilot subcarriers, which is ordinary used in conventional OFDM systems, cannot apply the CAZAC-OFDM. In the CAZAC-OFDM scheme, IFFT input signal for each subcarrier includes all QAM signal components, and each subcarrier carries uniformly distributed QAM data. The CAZAC-OFDM has a nature of spread-spectrum just like a Code Division Multiple Access (CDMA) or convolutional coding. Therefore, the CAZAC-OFDM scheme cannot have any pilot subcarriers although it has a frequency diversity effect itself.

In this paper, we study a fading compensation technique by applying a transversal filter using Least Mean Square (LMS) algorithm to the CAZAC-OFDM. A waveform equalizer represented by a transversal filter equalizes the transmission channel characteristics by inserting it in front of the receiver. When the characteristics of the transmission channel are dynamically changing and not in the steady state, an adaptive equalizer must be used. In this case, the training signal should be sent periodically. The tap coefficients of the filter are iteratively updated so that the output signal converges to the transmission signal. One type of adaptive algorithm used in the adaptive equalizer is the LMS algorithm [8]-[10].

We have confirmed that the CAZAC-OFDM system with the proposed transversal filter maintains enough low Bit Error Rate (BER) performance under flat fading and frequency selective fading channels.

The rest of this paper is organized as follows: In Section 2, we describe the CAZAC-OFDM system. In Section 3, we describe the method of channel estimation. In Section 4, we describe the transversal filter. In Section 5, we describe LMS algorithm. In Section 6, we show how convergence is achieved by updating the tap coefficients of the filter. In Section 7, we describe the effect of the proposed transversal filter in the CAZAC-OFDM system. Finally, we conclude this paper in Section 8.

 ISBN: 978-1-61208-727-6

II. CAZAC-OFDM SYSTEM

A. OFDM System

OFDM system is a kind of multi-carrier modulation scheme that digitally modulates and multiplexes a number of orthogonal subcarriers in the frequency domain. In the OFDM system, data signals are mapped by digital modulation such as QAM or PSK. Then, it is converted from frequency domain signal to discrete-time domain signal by N size IFFT. The discrete-time OFDM signal $x[n]$ with N subcarriers is represented as follows.

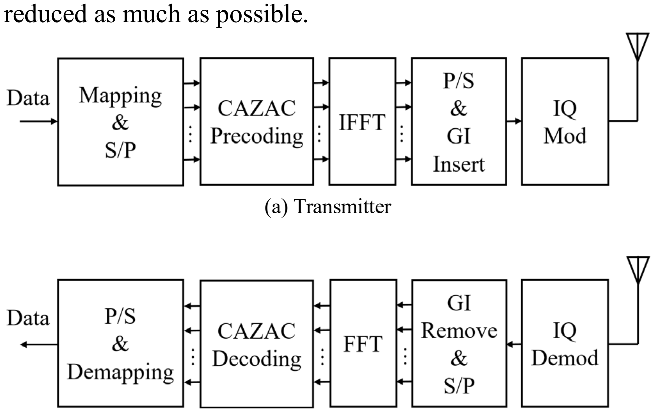
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{N}} \quad (1)$$

where $j = \sqrt{-1}$, n is the discrete-time index and $X[k]$ is the frequency domain signal [11].

In general, OFDM has high PAPR. PAPR is the ratio of peak power to average power and is defined by the following equation.

$$PAPR = 10\log_{10}\frac{\max\limits_{0\le n\le N-1}|x[n]|^2}{\operatorname{mean}\limits_{0\le n\le N-1}|x[n]|^2}\ [\mathrm{dB}] \quad (2)$$

The value of PAPR increases as the peak power increases compared to the average power. As PAPR increases, power consumption of the transmitter increases, so PAPR should be reduced as much as possible.



(a) Transmitter

(b) Receiver

Figure 1. CAZAC-OFDM system.

B. CAZAC Precoding

Figure 1 shows the configurations of CAZAC-OFDM transmitter and receiver. CAZAC-OFDM reduces PAPR by performing precoding using Zadoff-Chu sequence, which is a type of CAZAC sequence. The Zadoff-Chu sequence is expressed by the following equation.

$$c_k = \begin{cases} \exp\left(j\dfrac{\pi rk(k+1)}{L}\right) & (L \text{ is odd}) \\ \exp\left(j\dfrac{\pi rk^2}{L}\right) & (L \text{ is even}) \end{cases} \quad (3)$$
$$k = 0,1,\cdots,L-1$$

where L is the sequence length and r is the sequence number. Assuming that $r = 1$ and $L = N^2$ in the above equation, the Zadoff-Chu sequence is as follows.

$$c_k = \exp\left(j\frac{\pi k^2}{N^2}\right) \quad (4)$$

where N is the number of subcarriers. Using the above equation, CAZAC precoding generates an $N \times N$ square matrix $\boldsymbol{M}$.

$$\boldsymbol{M} = \frac{1}{N}\begin{bmatrix} c_0 & c_1 & \cdots & c_{N-1} \\ c_N & c_{N+1} & \cdots & c_{2N-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)N} & c_{(N-1)N+1} & \cdots & c_{N^2-1} \end{bmatrix} \quad (5)$$

The transmission data sequence $\boldsymbol{X}$ with N subcarriers is expressed as follows.

$$\boldsymbol{X} = \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} \quad (6)$$

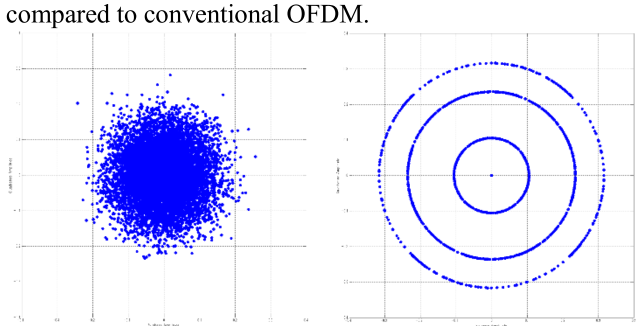
When the transmission data sequence $\boldsymbol{X}$ is multiplied by the square matrix $\boldsymbol{M}$, the output by CAZAC precoding $\boldsymbol{P}$ is expressed by the following equation.

$$\boldsymbol{P} = \boldsymbol{MX}$$
$$= \frac{1}{N}\begin{bmatrix} c_0 & c_1 & \cdots & c_{N-1} \\ c_N & c_{N+1} & \cdots & c_{2N-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)N} & c_{(N-1)N+1} & \cdots & c_{N^2-1} \end{bmatrix}\begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} \quad (7)$$

When IFFT is performed on the output by CAZAC precoding $\boldsymbol{P}$, the time signal of CAZAC-OFDM s_n is expressed by the following equation.

$$s_n = c_{\left(\frac{N}{2}-n\right)modN} \cdot X_{\left(\frac{N}{2}-n\right)modN} \quad (8)$$

Therefore, as shown in Figure 2, the time domain signal of CAZAC-OFDM is obtained by rotating the phase of the mapping data while maintaining the amplitude of the mapping data. As a result, CAZAC-OFDM has reduced PAPR compared to conventional OFDM.



(a) Conv. OFDM (b) CAZAC-OFDM

Figure 2. Time domain signal.

In the conventional OFDM, the mapped value $\boldsymbol{X}$ is directly input to the IFFT. Therefore, in the frequency domain, each data is allocated to each subcarrier as shown in Figure 3a. On the other hand, in CAZAC-OFDM, $\boldsymbol{P}$ obtained by multiplying the mapped value $\boldsymbol{X}$ by the matrix $\boldsymbol{M}$ generated from the Zadoff-Chu sequence is input to the IFFT. In this case, the data loaded on the subcarrier is represented by the sum of the components of $\boldsymbol{X}$ with different phase rotations. Therefore, in the frequency domain, as shown in Figure 3b, all data are included in each subcarrier. This is the frequency diversity effect.

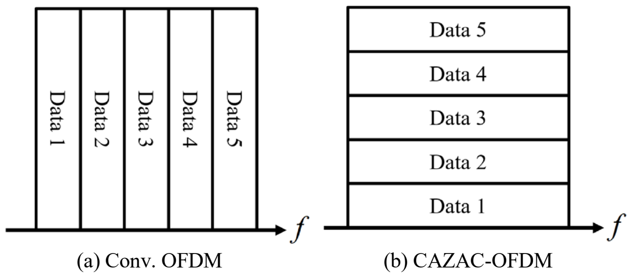


Figure 3. State of data.

III. Channel Estimation

A. Channel Estimation in Conventional OFDM

In wireless communication, amplitude fluctuation and phase fluctuation occur in the transmission channel due to the influence of multipath fading. Therefore, the error rate of data becomes large. So, in the conventional OFDM, pilot subcarriers are used as a method to estimate the characteristics of multipath fading. The pilot subcarriers are known signals on the receiving side added for channel estimation, and is periodically inserted into the data subcarriers as shown in Figure 4. By examining how much the pilot subcarriers are fluctuated on the receiving side, it becomes possible to estimate the characteristics of multipath fading at that time.

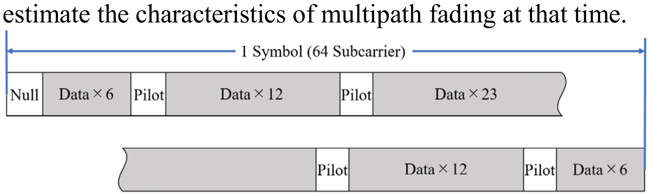


Figure 4. The example of pilot subcarriers.

B. Pilot Subcarriers in CAZAC-OFDM

As described above, in the conventional OFDM, the characteristics of multipath fading are estimated by inserting pilot subcarriers, and correction is performed accordingly. However, this method cannot be used in CAZAC-OFDM. This is due to the frequency diversity effect of CAZAC-OFDM. In the IFFT output after CAZAC precoding, all data in the time domain will be distributed to each subcarrier. Therefore, even if pilot subcarriers are inserted as shown in Figure 5, in the IFFT output, the information as the pilot subcarriers is dispersed to all the subcarriers, so that it cannot function as pilot subcarriers. Therefore, CAZAC-OFDM cannot correct the effect of multipath fading. Therefore, it is necessary to correct the influence of multipath fading without using pilot subcarriers.

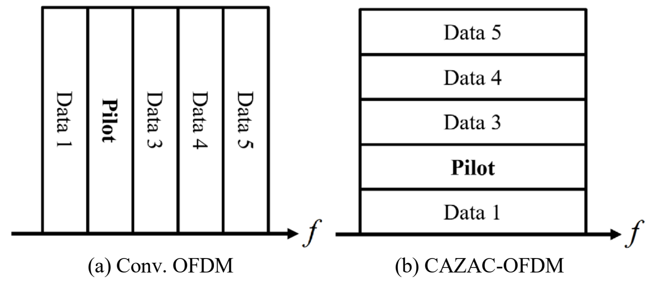


Figure 5. Pilot subcarriers insertion.

IV. Transversal Filter

A. Structure

The model of the transversal filter is shown in Figure 6. The transmitted signal is received under the influence of multipath fading. Therefore, by inserting the transversal filter in front of the receiver on the receiving side, it is possible to estimate the transmission signal and correct the influence of multipath fading.

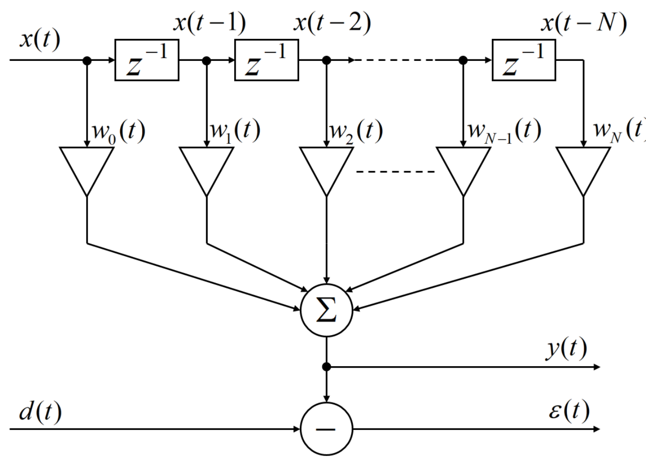


Figure 6. Transversal filter.

B. Principle

In the transversal filter of Figure 6, the output signal $y(t)$ is expressed by the following equation.

$$y(t) = \boldsymbol{x}^T(t)\boldsymbol{w}(t) \tag{9}$$

The input signal $x(t)$ and the tap coefficient $w(t)$ are respectively expressed as follows.

$$\boldsymbol{x}(t) = \begin{bmatrix} x(t) \\ x(t-1) \\ \vdots \\ x(t-N) \end{bmatrix} \tag{10}$$

$$\boldsymbol{w}(t) = \begin{bmatrix} w_0(t) \\ w_1(t) \\ \vdots \\ w_N(t) \end{bmatrix} \tag{11}$$

 ISBN: 978-1-61208-727-6

where the number of taps is $N+1$. The input signal is assumed to be in steady state. Here, let the training signal be $d(t)$. An error signal $\varepsilon(t)$, which is the difference between the output signal and the training signal is expressed by the following equation.

$$\varepsilon(t) = d(t) - y(t) \tag{12}$$

The tap coefficient $\boldsymbol{w}$ is set such that the error signal $\varepsilon(t)$ is minimized. As a result, the optimal tap coefficient $\boldsymbol{w_{opt}}$ is as follows [10].

$$\boldsymbol{w_{opt}} = \boldsymbol{R}^{-1}\boldsymbol{P} \tag{13}$$

where $\boldsymbol{R}$ and $\boldsymbol{P}$ are the following formulas, respectively.

$$\boldsymbol{R} = E[\boldsymbol{x}(t)\boldsymbol{x}^T(t)] \tag{14}$$

$$\boldsymbol{P} = E[d(t)\boldsymbol{x}(t)] \tag{15}$$

where $E[\cdot]$ is the ensemble average. This equation is the solution of the Wiener filter. From the above equation, the optimal tap coefficient can be determined by examining the autocorrelation of the input signal and the cross-correlation between the input signal and the training signal.

However, to determine the correlation of signals, it is necessary to collect the input signal for a long time, so it takes a long time to determine the tap coefficient. In addition, it is assumed that the signal is stationary, and it is decided to be the only optimal tap coefficient, so it cannot cope with the case where the characteristics of the transmission channel change dynamically. Therefore, these problems are solved by using an adaptive filter.

V. LMS ALGORITHM

In the adaptive filter, by iteratively updating the tap coefficient, it gradually approaches the optimal value. Therefore, if the change of the signal characteristics is slower than the convergence time of the adaptive filter, it can correspond to the change.

The steepest descent method is used as a method to update the tap coefficient iteratively so as to minimize the error. This is to update the tap coefficient in the direction of decreasing the gradient of $E[\varepsilon^2(t)]$ at a certain time t, and is expressed by the following equation.

$$\begin{aligned}\boldsymbol{w}(t+1) &= \boldsymbol{w}(t) - \frac{\mu \partial E[\varepsilon^2(t)]}{2\partial \boldsymbol{w}(t)} \\ &= \boldsymbol{w}(t) + \mu E[\varepsilon(t)\boldsymbol{x}(t)]\end{aligned} \tag{16}$$

where μ is the step size parameter, which is the parameter that determines the size of the slope down in one update. As the above equation shows, it requires ensemble average. Therefore, frequent updating of the tap coefficient cannot be performed. Therefore, LMS algorithm uses instantaneous estimates instead. As a result, the equation for updating the tap coefficient is as follows [9].

$$\begin{aligned}\boldsymbol{w}(t+1) &= \boldsymbol{w}(t) - \frac{\mu \partial \varepsilon^2(t)}{2\partial \boldsymbol{w}(t)} \\ &= \boldsymbol{w}(t) + \mu\varepsilon(t)\boldsymbol{x}(t)\end{aligned} \tag{17}$$

The convergence condition of the LMS algorithm is as follows [10].

$$0 < \mu < \frac{2}{N^*} \tag{18}$$

where N^* is the maximum value of the eigenvalues of the correlation matrix of the input signal. When the step size is increased, the convergence speed becomes faster but the steady-state error after convergence is increased. On the other hand, when the step size is reduced, the steady-state error after convergence is decreased but the convergence speed becomes slower. Therefore, it is necessary to decide the step size in consideration of the tradeoff between the convergence speed and the steady-state error.

VI. CONVERGENCE BY UPDATING TAP COEFFICIENTS

We simulated the convergence of constellation and time domain signals on the receiving side by updating the tap coefficient using MATLAB / Simulink. The simulation results are shown in Figures 7 and 8, respectively. Note that as the constellation and the time domain signals progress from Figure 7a to 7d, it indicates that time has elapsed.

The following figure shows that the constellation and the time domain signals converge as the tap coefficients are updated iteratively. As a result, communication becomes possible as usual only after convergence.

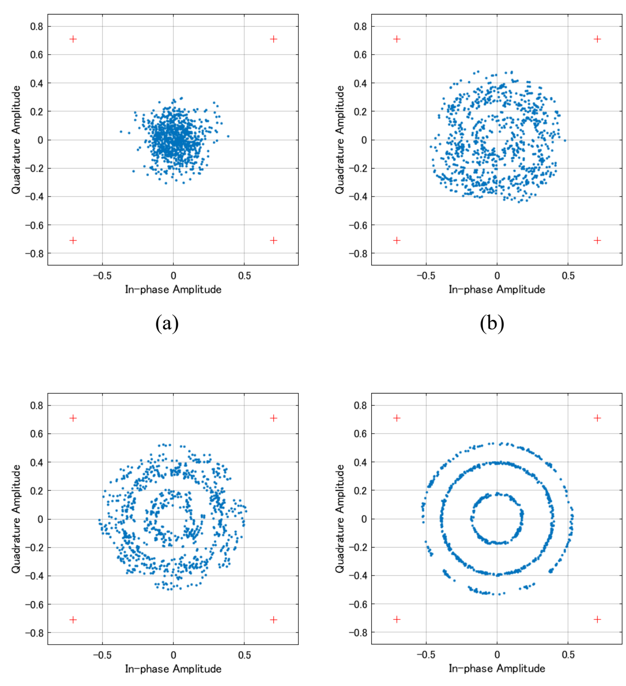


Figure 7. Convergence of time domain signals of CAZAC-OFDM.

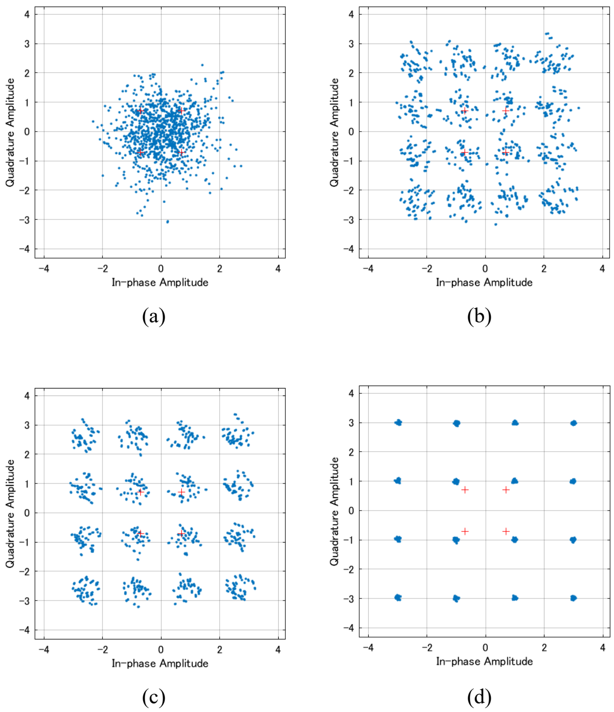


Figure 8. Convergence of 16QAM constellation.

VII. PERFORMANCE EVALUATION BY SIMULATION

A. Simulation Specification

In this paper, in order to confirm how much the error rate of the signal is improved by using the transversal filter, the BER performance of CAZAC-OFDM and conventional OFDM are compared using MATLAB / Simulink.

Figures 9 and 10 show the configurations of CAZAC-OFDM transmitter and receiver, respectively. In addition, the transmitter and receiver configurations of the conventional OFDM, which does not use CAZAC precoding are structures, which removed the block of CAZAC precoding from Figures 9 and 10.

On the transmitting side, mapping (16QAM) is first performed on transmission data. Then, CAZAC-OFDM signal is generated by performing parallelization by serial-to-parallel conversion, CAZAC precoding and IFFT. At this time, if IFFT is performed without CAZAC precoding, a conventional OFDM signal is generated. After that, guard intervals are inserted, serialized by parallel-to-serial conversion, quadrature modulation is performed, and the signal is transmitted.

On the receiving side, fading in the transmission channel is corrected by passing through a transversal filter after quadrature demodulation. Then, received data is generated by serial-to-parallel conversion, removal of guard intervals, FFT, CAZAC decoding, parallel-to-serial conversion, and demapping.

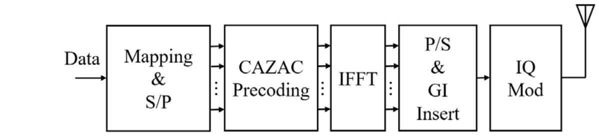


Figure 9. Transmitter configuration.

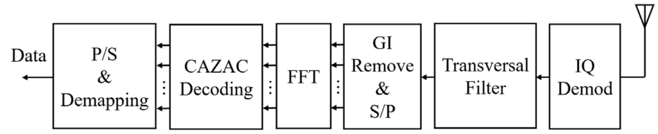


Figure 10. Receiver configuration.

Table I shows the simulation specification.

TABLE I. SIMULATION SPECIFICATION.

Mapping	16QAM
FFT Size	64
Carrier Frequency	120 MHz
Data Rate	32 Mbps
Number of Data Subcarriers	59
Guard Interval	16
Step Size	0.1
Channel Model	Flat Fading (Rayleigh)
	Frequency Selective Fading (Rayleigh)
	AWGN

B. Simulation Results

We compared the BER performance without fading channels. The simulation results are shown in Figure 11. From this result, it is proved that CAZAC-OFDM can obtain almost the same BER performance as conventional OFDM when the transversal filter is used.

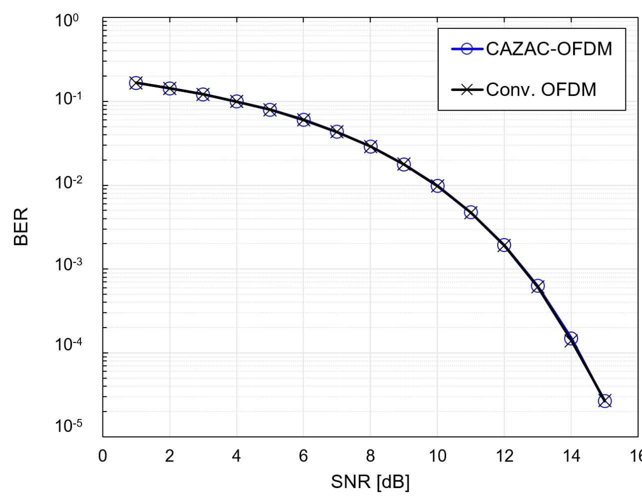


Figure 11. BER performance (without fading).

We compared the BER performance under flat fading and frequency selective fading channels. The simulation results are shown in Figure 12. From this result, it is proved that CAZAC-OFDM can obtain almost the same BER performance as that of the conventional OFDM by using the transversal filter even under fading environment. That is, it is concluded that CAZAC-OFDM is effective even under flat fading and frequency selective fading channels by using the transversal filter.

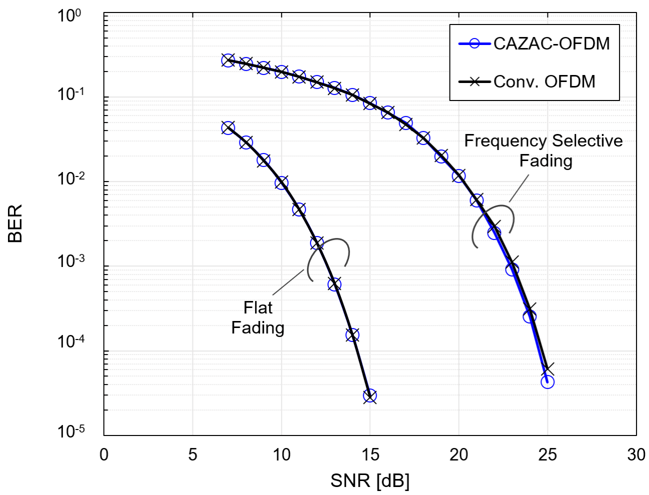


Figure 12. BER performance (with fading).

VIII. CONCLUSION

In this paper, we proposed a method to improve the negative effect of multipath fading by applying the transversal filter to CAZAC-OFDM system. Moreover, it has been confirmed by simulation that the CAZAC-OFDM system with the transversal filter maintains enough low BER performance under flat fading and frequency selective fading channels. The result shows that the CAZAC-OFDM system is effective under fading environment.

REFERENCES

[1] R. Ishioka, T. Kimura, and M. Muraguchi, "A Proposal for a New OFDM Wireless System using a CAZAC Equalization Scheme," Proc. AICT 2017, pp. 47-51, June 2017.

[2] Z. Feng, *et al.*, "Performance-Enhanced Direct Detection Optical OFDM Transmission With CAZAC Equalization," IEEE Photonics Technology Letters, vol. 27, no. 14, pp. 1507-1510, May 2015.

[3] I. Baig and V. Jeoti, "PAPR Reduction in OFDM Systems: Zadoff-Chu Matrix Transform Based Pre/Post-Coding Techniques," Second International Conference on Computational Intelligence, Communication Systems and Networks, pp. 373-377, July 2010.

[4] K. Miyazawa, T. Kimura, and M. Muraguchi, "Proposal of visible light OFDM system with CAZAC equalization," 23rd Asia-Pacific Conference on Communications (APCC), pp. 491-496, December 2017.

[5] Y. Sugai, Y. Shirato, T. Kimura, and M. Muraguchi, "PAPR and Spectral Control Procedure for OFDM Wireless Systems Using CAZAC Equalization," in AICT'2018, pp. 75-80, 2018.

[6] T. Onoda, R. Ishioka, and M. Muraguchi, "Proposal of Power Saving Techniques for Wireless Terminals," in AICT'2018, pp. 115-120, 2018.

[7] T. Kazama, K. Miyazawa, and M. Muraguchi, "Time-domain signal management for OFDM signals," 5th International Conference on Wireless and Mobile Network (WiMNeT 2018), pp. 23-35, November 2018.

[8] Z. Qingqing, Y. Xiao, H. Ling, and D. Sanlei, "The LMS channel estimation algorithm of low-voltage power line communication," Power Engineering and Automation Conference, pp. 541-544, September 2012.

[9] M. M. Rana, "Performance comparison of LMS and RLS channel estimation algorithms for 4G MIMO OFDM systems," Proceedings of 14th International Conference on Computer and Information Technology, pp. 635-639, December 2011.

[10] K. Elangovan, "Comparative study on the Channel Estimation for OFDM system using LMS, NLMS and RLS algorithms," Proceedings of the International Conference on Pattern Recognition, pp. 359-363, March 2012.

[11] H. Seung, Hee and L. Jae, Hong, "An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission," IEEE Wireless Communications, vol. 12, no. 2, pp. 56-65, April 2005.