Tighter Effective Bandwidth Estimation for Multifractal Network Traffic

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***Abstract*—In literature, several studies have shown the presence of fractal nature in a wide variety of traffic and the impact of these phenomena on network performance. In this paper, we derive a new expression for effective bandwidth estimation in order to offer better resource allocation in network planning and design, especially for network traffic with multifractal characteristics. Based on a new construction approach for conservative multiplicative cascades proposed in literature and the corresponding multifractal traffic parameters, a global scaling parameter is determined and used together with the multifractal traffic model parameters for the effective bandwidth computation. The proposed approach was validated in terms of dynamically allocated bandwidths.**

Keywords - Multifractal Traffic; Global Scaling Parameter; Effective Bandwidth.

I. INTRODUCTION

The concept of effective bandwidth provides a way to characterize the resource requirements of a connection, being a useful tool for analysis and description of traffic in networks. It is considered that the effective bandwidth is the rate of transmission of information, usually with the lower limit the average rate and upper limit the peak rate of traffic, given the Quality of Service (QoS) requirements set a priori for a given traffic flow. A very good review and perspective on effective bandwidths can be found in [3].

The effective bandwidth of a source is highly sensitive to the statistical properties of the source which frequently are not known a priori. Accurate effective bandwidth estimation depends on the how faithful is the chosen traffic model. Technically the concept of effective bandwidths is much broader than a simple measure, depending on traffic models, queue disciplines and performance criteria.

Effective bandwidths required to meet the QoS requirements also depend on the traffic characteristics. The characteristics of traffic flows in current networks make their estimation no trivial and difficult using too simplified traffic models such as Markov models. Several methods of effective bandwidth estimation have been developed for broadband network traffic flow mainly based on different traffic modeling approaches. Among them the most representative ones are the following: The estimation of effective bandwidths based on self-similar traffic modeling proposed by Norros [4]; the so-called empirical effective bandwidth proposed by Tartarelli, et al. [5] without assuming any specific statistical traffic model; effective bandwidths for ATM (Automatic Teller Machine) traffic based on Markov multi-class fluid modeling proposed by Kesidis, et al. in [6]; and finally effective bandwidths based on traffic under the VVGM (Variable Variance Gaussian Multiplier) multifractal model proposed by Krishna, et al. in [7].

There are many studies showing the high variability and fast evolution of today´s internet traffic due to new applications and control protocols, i.e., modern traffic flows present variable bursts in a wide range of time scales, in contrast to the old assumptions that bursts of traffic exist only on short time scales [8, 9]. It has been shown that these incidences of multi-scales bursts affect significantly network performance [8, 9].

More realistic modeling attempts appeared, initially for characterizing the self-similarity of Internet and Ethernet traffic [10]. Although the self-similarity has provided a plausible explanation, it has failed to justify some essential local behaviors and statistical measures of real traffic flows. Therefore, the term self-similarity generally refers to those processes which are asymptotically or the second order self-similar, or monofractal [11]. In these cases, the Hurst parameter has been widely used to provide a measure of the degree of self-similarity of traffic processes.

In order to achieve even more realistic traffic modeling, taking into account multiple scaling properties as well as providing robust description of local behavior of modern network traffic, multifractal theory was adapted and used for the building of new network traffic models. Multifractal traffic modeling has enjoyed considerable success due to its theoretical robustness, versatility and generalization capability. Some well-known multifractal models designed and used for modern network traffic modeling are: VVGM [7], VSCM (Variable Scale parameter Cauchy Multiplier) [12], MWM (Multifractal Wavelet Model) [13], AWMM (Adaptive Wavelet Based Multifractal Model) [14], and mBm (multifractional Brownian motion) [15]. No doubt have those traffic models provided a more accurate description of traffic flows and contributed to the improvement in the network simulation and design tools.

The main purpose of this work is to derive and evaluate effective bandwidth for data source under a multifractal model proposed in our previous work [1, 2]. The construction of this model has been based on a new conservative multiplicative binomial cascade with its multipliers determined by a Newton Binomial equation. The major strength of this model is its high capability of capturing major multifractal properties represented by the corresponding scaling function and moment factor.

Therefore, this work also intends to validate this new multifractal traffic model by comparing the efficiency of the derived effective bandwidth expression with others well-established in the literature.

The paper is organized as follows. In Section II, we present a brief description of the multifractal model proposed in [1, 2]. In Section III, we show in detail the derivation of the effective bandwidth expression. In Section IV we provide a brief summary of other effective bandwidth estimation methods used for experimental investigation. Section V is dedicated for the presentation and comparison of obtained experimental results. Finally, in Section VI we conclude.

II. MULTIFRACTAL TRAFFIC MODEL

The multifractal traffic model used in this work was proposed in our previous papers [1, 2] and in this section we present this model with enough details in order to be able to understand the follow-up.

Definition 1: A stochastic process $X(t)$ is called multifractal if it has stationary increments and satisfies:

$$E(|X(t)|^q) = c(q)t^{\tau(q)+1} = c(q)t^{\tau_0(q)} \tag{1}$$

for some positive values $q \in Q$, $[0,1] \subseteq Q$, $\tau(q)$ (scaling function) and $c(q)$ (moment factor) are functions on domain Q and are independent of t. The function $\tau(q)$, also known as the partition function, is concave with $\tau(0) = -1$. [16]

A. Multiplicative Cascades

Definition 2. A multiplicative cascade is an iterative process that fragments a given set into smaller and smaller pieces according to a geometric rule and, at the same time, distributes the total mass of the given set according to another scheme.

A.1.The Proposed Binomial Multiplicative Cascade

Based on the Definition 2, the proposed multiplicative binomial cascade distribute its masses according to the Newton Binomial expression $\binom{2^N}{k}(x)^{2^N-k}(1-x)^k$, where N is a positive integer representing the stage number of the cascade and $k = 0,1,\dots,2^N-1$. Without losing the generality, consider an initial interval $I = [0,1]$, and let x be a real-valued random variable uniformly distributed over the interval I.

At the N[th] stage of the cascade, the first subinterval has the mass by applying the following weighting factor on the unit mass:

$$W_{\underbrace{00\dots0}_{N\ digits}} = (x)^{2^N} + (1-x)^{2^N} \tag{2}$$

while for the remaining subintervals the weighting factors are:

$$W_{b_1b_2\dots b_N} = \binom{2^N}{i}(x)^{2^N-i}(1-x)^i\Big|_{i=1,\dots,2^N-1} \tag{3}$$

where $b_1b_2\dots b_N$ is the binary representation of decimal numeral i, also used to denote the corresponding sub-interval at the N[th] stage of the cascade. As consequence, it is easy to see that the cascade is mass conservative in expectation.

Considering the k^{th} stage of the cascade, each subinterval of the (k-1)[th] stage is further divided into two equal length intervals. Thus, at k^{th} stage of the cascade, the mass measure of the first interval $I_k = [0,2^{-k}]$ is equal to:

$$\mu[I_k] = \mu[0,2^{-k}] = \mu[I_{k-1}]W_{\underbrace{00\dots0}_{k\ digits}} = \mu[0,2^{-k+1}]W_{\underbrace{00\dots0}_{k\ digits}} = \mu[0,2^{-k+1}]\left[(x_{k-1})^{2^k} + (1-x_{k-1})^{2^k}\right] \tag{4}$$

For the other intervals, we have:

$$\mu[I_k] = \mu[I_{k-1}]W_{b_1b_2\dots b_k} = \mu[I_{k-1}]\binom{2^k}{i}(x_{k-1})^{2^k-i}(1-x_{k-1})^i\Big|_{i=1,\dots,2^k-1} \tag{5}$$

Notice that $x_1, x_2, x_3, \dots$ are i.i.d. random variables uniformly distributed on $[0,1]$.

Let Δt_k denote the length of each subinterval at the k[th] stage of the cascade. Thus, the mass measure of the multifractal process on the dyadic interval of length Δt_k starting at $t = 0.b_1\dots b_k = \sum_{i=1}^{k} b_i 2_i$ calculated as:

$$\mu(\Delta t_k) = R(b_1)R(b_1b_2)\dots R(b_{1,\dots,}b_k) \tag{6}$$

where $R(b_{1,\dots,}b_i)$ is the multiplier of the corresponding sub-interval at the stage i of the cascade. As the multipliers are independently and identically distributed (i.i.d.), it can be shown that the expectation measurement satisfies the following scaling relationship:

$$E(X(\Delta t_k)^q) = (E(R^q))^k = \Delta t_k^{-log_2 E(R^q)} \tag{7}$$

Therefore, the multifractal process can be characterized through its scaling function defined by $\tau(q) = -log_2 E(R^q)$.

A.2. Capture of Multifractal Characteristics

From the Definition 1, multifractal traffic modeling consists of determination of scaling function $\tau(q)$ and the moment factor $c(q)$ [17]. This can be achieved by the product of a cascade and i.i.d. positive random variables Y's. More specifically, a multifractal traffic process model can be interpreted as the product of the random peak rate of the flow Y and the measure of burstiness $\mu(\Delta t_N)$ at the modelled time scale Δt_N. The variable Y is chosen to be independent of the cascade measure $\mu(\Delta t_k)$, then the obtained series, denoted by $X(\Delta t_N)$, satisfies the following equation:

$$E\left(X(\Delta t_N)^q\right) = E(Y^q)E(\mu(\Delta t_N)^q) = E(Y^q)\Delta t_N^{\tau_0(q)} \tag{8}$$

Analyzing Equation (8) with the definition of multifractal processes Equation (1) we can show that R and Y should be related with $\tau(q)$ and c (q), respectively, as the following:

$$\begin{cases} -log_2\left(E(R)^q\right) = \tau_0(q) \\ E\left(Y^q\right) = c(q) \end{cases} \tag{9}$$

The scaling function $\tau(q)$ can be accurately modeled by assuming that R is a random variable on [0,1] with a beta distribution $\mathrm{Beta}(\alpha, \beta)$. The beta distribution is a family of continuous probability distributions defined on the interval $[0,1]$ parameterized by two positive, typically denoted by α and β. The beta distribution can be suited to the statistical modeling of proportions in applications where values of proportions equal to 0 or 1 do not occur. Thus, the function $\tau_0(q) \coloneqq \tau(q) + 1$ related to the scaling function $\tau(q)$, can be written as [1, 2]:

$$\tau_0(q) = \log_2 \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+q)}{\Gamma(\alpha)\Gamma(\alpha+\beta+q)} \tag{10}$$

where $\Gamma(.)$ denotes the Gamma function.

In [14] and [18] the authors show that the random variable Y can be considered as having a lognormal distribution defined by its two parameters m and v. Therefore the q^{th} moment of Y is explicitly given by $E(Y^q) = e^{mq+v^2q^2/2}$. Consequently the moment factor $c(q)$ for the processes is given by [1] and [2]:

$$c(q) = e^{mq+v^2q^2/2} 2^{N\left(\log_2 \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+q)}{\Gamma(\alpha)\Gamma(\alpha+\beta+q)}\right)} \tag{11}$$

Analyzing the Equations (10) and (11), one can notice that the proposed multifractal model is fully characterized by a set of four model parameters (α, β, m, v), and the mean and variance of this traffic process are related to the model parameters, respectively, as follows:

$$E[X(t)] = e^{m+v^2/2} \tag{12}$$

$$\mathrm{var}[X(t)] = e^{2m+2v^2} 2^{2N} \left(\frac{(\alpha+\beta)(\alpha+\beta+1)}{(\alpha+1)\alpha}\right)^N e^{2m+v^2} \tag{13}$$

III. PROPOSED EFFECTIVE BANDWIDTH ESTIMATION

It is well known that there is another popular way to characterize a multifractal process which is through its local Hölder exponent function [19]. The Hölder exponent also can be interpreted as a generalization of a global scaling parameter of a fractal process known as The Hurst parameter. Frequently traffic flows are assumed holding only monofractal characteristics in order to make queuing analysis simpler, i.e., adopting a simplified traffic model parameter for multifractal traffic arrivals.

A self-similar process $X(t)$ with Hurst parameter H with mean zero and variance σ^2 obeys a scaling relation of the form:

$$log\{var[X^m]\} = (2H-2)log\{m\} + log\{\sigma^2\} \tag{14}$$

where m is the aggregating parameter [20]. In particular, it was shown in [2], for the proposed cascade modeling process, one can obtain the following expression:

$$log_2\{var[X^m]\} = log_2\left\{e^{2m+2v^2}\right\} + \\ + \left\{\log_2\left(\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta-1)}\right)^N\right\} + \left\{\log_2\left(\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}\right)^{-\log_2 m}\right\} \tag{15}$$

In terms of multifractal model parameter, α, β, m, v.

Comparing (15) with (14), we can establish the following equality:

$$log\{m\}(2H-2) = -\,log\{m\}\,log_2\left(\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}\right) \tag{16}$$

Therefore,

$$H_{EG} \triangleq H = 1 - \frac{1}{2} log_2\left(\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}\right) \tag{17}$$

In Equation (17) we define a global parameter H_{EG} for multifractal traffic processes, similar to the Hurst parameter H in monofractals cases. More details see [2]. Thus, considering that there is a global scaling parameter for multifractal processes, given by Equation (17), next we derive an analytical expression for effective bandwidth in terms of multifractal model parameters.

Let $X(t)$ the traffic arrival process under the proposed multifractal modeling with global scale given by H_{EG}. Assuming the stage number N in the generation of the cascade is large enough and using the fBm statistical model, we can express the mean by $E[X(\delta)] = \mu\delta$ and the variance by $var[X(\delta)] = \sigma^2\delta^{2H_{EG}}$. The moment generating function of X(t) in terms of parameters θ and δ is [21]:

$$G(\theta, \delta) = \left(e^{\mu\delta\theta + \frac{\sigma^2\delta^{2H_{EG}}\theta^2}{2}}\right) \tag{18}$$

Thus the effective bandwidth may be given as:

$$e_{b_x}(\theta, \delta) = \frac{1}{\theta\delta} log G(\theta, \delta) \tag{19}$$

Substituting the relation (18) into (19), we have:

$$e_{b_x}(\theta, \delta) = \frac{1}{\theta\delta} log\left(e^{\mu\delta\theta + \frac{\sigma^2\delta^{2H_{EG}}\theta^2}{2}}\right) \tag{20}$$

Thus,

$$e_{b_x}(\theta,\delta) = \frac{1}{\theta\delta}\mu\delta\theta + \frac{\sigma^2\delta^{2H_{EG}}\theta^2}{2} = \mu + \frac{\theta\sigma^2}{2}\delta^{2H_{EG}-1} \quad (21)$$

Therefore

$$e_{b_x}(\theta,\delta) = \mu + \frac{\theta\sigma^2}{2}\delta^{\left(2-log_2\left(\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}\right)\right)-1} \quad (22)$$

where θ represents the asymptotic exponential decay rate of the distribution of the queue size and δ the time scale.

IV. OTHER METHODS OF EFFECTIVE BANDWIDTH

A. Norros Effective Bandwidth

Norros et al. [4] proposed an expression for effective bandwidth estimation by considering traffic with the fBm self-similar characteristics, that is:

$$\alpha = m + K(H)\sqrt{-2ln(P_{loss})}^{1/H} a^{\frac{1}{2H}} b^{-(1-H)/H} m^{\frac{1}{2H}} \quad (23)$$

where m represents the average rate of the traffic flow in (bit/s), $K(H) = H^H(1-H)^{1-H}$, a the coefficient of variance, P_{loss} the overflow probability of buffer, H Hurst Parameter and b the buffer size.

The effective bandwidth suggested by (23) takes into account the self-similarity property of traffic through its Hurst parameter, which is an appropriate alternative for most traffic flows holding long-range dependence (LRD) characteristics. The effective bandwidth estimates become much tighter when buffer size become large. For more details, see [4].

B. Empírical Effective Bandwidth

The effective bandwidth estimation by Equation (24) Proposed in [5], known as Empirical Effective Bandwidth, does not assumes any specific traffic flow model.

$$\alpha(s,t,N) = \frac{1}{st} log\hat{E}_{N_t}\left[e^{sX(0,t)}\right] \quad 0 < s; 0 < t < N_t \quad (24)$$

where $X(0,t)$ indicates the aggregated amount of arrived traffic data within a time interval t and $\hat{E}_{N_t}\left[e^{sX(0,t)}\right]$ is the data-measured moment generating function from the traffic trace with N_t samples. For both Poisson and On-Off processes, the empirical effective bandwidths are very close to their respective analytical effective bandwidths. For more details, see [5].

C. Kesidis Effective Bandwidth

In [6] Kesidis et.al. derived an expression of effective bandwidth for fluids Markov multi-class and other types of source models under ATM traffic. The authors showed that when traffic sources share a buffer system with deterministic service rate, a constraint on the tail of the buffer occupancy distribution is a linear constraint on the number of sources, i.e., for a small loss probability one can assume that each source transmits at a fixed rate called effective bandwidth.

Let m be the average rate of traffic, $s = ln(P_{loss}/B)$, P_{loss} the overflow probability of buffer and B the buffer size. The Effective Bandwidth (EB) is given by:

$$EB = m\frac{e^s - 1}{s} \quad (25)$$

For more details, see [6].

D. Krishna Effective Bandwidth

Krishna et al. [7] propose an expression for calculating effective bandwidth based on the multifractal VVGM model. Also, they assumed that traffic can be characterized as fBm processes. The effective bandwidth (EB), given by (26), is written in function of the parameters θ (asymptotic exponential decay rate of the distribution of the queue size) and δ (time scale), traffic average rate m and variance σ^2, and the global scaling exponent of the VVGM model, H_{eff}.

$$EB = m + \frac{\theta\sigma^2}{2}\delta^{(2H_{eff}-1)} \quad (26)$$

For more details, see [7]

V. EXPERIEMENTAL EVALUATION

In this section, we evaluate the efficiency of the proposed effective bandwidth estimation method. Instead of obtaining a unique static bandwidth estimate for the entire traffic trace, dynamically effective bandwidth is estimated instantaneously using only traffic samples inside a sliding time window and used as the current server transmission rate.

Three real traffic traces were used in our simulation: a TCP / IP traffic trace called “lbl_tcp_3” [22], a video traffic flow called “The Simpsons” [23] (high quality video) and a traffic trace collected in a wireless network collected during the ACM SIGCOMM08 conference [24], namely “Sigcomm08”. The traffic samples were aggregated under a time scale on which all three traffic flows exhibit multifractal characteristics [25]. Service is conservative, i.e., server will never remain idle if there is one or more jobs in the service node.

Table I shows some statistical information (means, variances and number of samples) of three traffic traces.

For performance comparison purposes, we also evaluate the queue system using four effective and width estimation approaches described in the previous section: the effective bandwidth proposed by Norros [4], the empirical effective bandwidth proposed by S. Tartarelli et al. [5], the effective bandwidth proposed by Kesidis [6], and the effective bandwidth proposed by Krishna [7].

Table II shows the global scaling parameter values obtained under the proposed modeling method and compares with the Hurst parameters estimated through a Whittle Estimator [26] for all three mentioned traffic traces (namely lbl_tcp_3, The Simpsons and Sigcomm08). It can be seen that numerically two global scaling parameters are close. As a result, the global scaling parameter H_{EG} can be viewed as an alternative measure for self-similarity.

TABLE I. MEAN, VARIANCE, SAMPLES

Traffic Trace	Mean	Variance	Samples
lbl_tcp_3	136.3555	5.7062x10^4	1.789.995
The Simpsons	6.5137x10^3	7.2420x10^6	30.334
Sigcomm08	451.9165	2.3723x10^5	1.358.782

TABLE II. HURST AND GLOBAL PARAMETER

Traffic Trace	Hurst Paramenter (H) Whittle Estimator	Global Scaling Parameter H_{EG}
lbl_tcp_3	0.8420	0.8691
The Simpsons	0.7130	0.7262
Sigcomm08	0.7650	0.7567

Figure 1 shows the necessary effective bandwidth values obtained using Equation (22) and also those using other cited methods for the lbl_tcp_3 traffic trace, 10^{-6} loss probability system performance, 64Kbytes buffer size and a sliding time window of 500 traffic samples. Notice that the proposed method outperforms other approaches requiring the lowest service rate. However, the improvement is relatively small with respect to the methods proposed in [5] and [7], and considerably remarkable in comparison with those by Norros [4] and Kesidis et.al. [6].

It is noteworthy that the method proposed by Kesidis et.al. [6] is based on Markovian traffic modeling, and it is a well-known fact that Markovian Modeling cannot fully represent traffic with multifractal characteristics [11]. As a result, the Markovian based effective bandwidth estimates may be too conservative.

Figure 2 shows the performance curves for the video traffic trace (The Simpsons), considering 10^{-6} loss probability system performance, 32Kbytes buffer size and a sliding time window of 100 traffic samples. Again, the proposed approach shows considerably better performances.

Figure 3 shows the performance curves for a wireless traffic trace (Sigcomm08), considering 10^{-6} loss probability system performance, 64Kbytes buffer size and a sliding time window of 500 traffic samples. Similar results are also observed and, once again, the proposed method prevails.

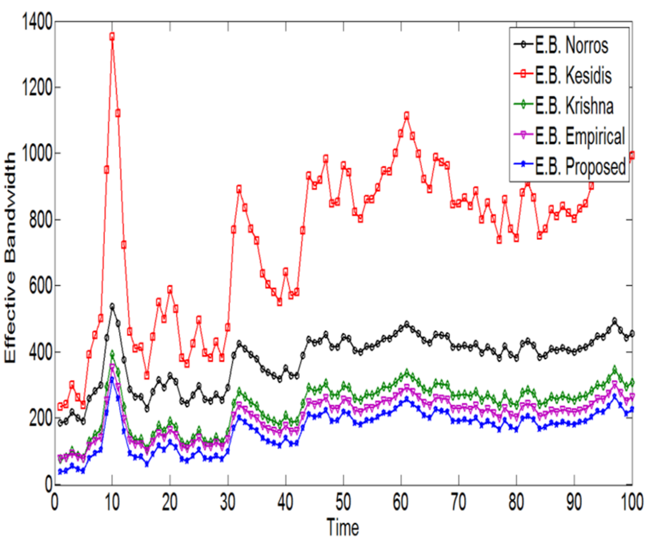

Fig.1. Effective Bandwidth for Traffic Trace lbl_tcp_3.

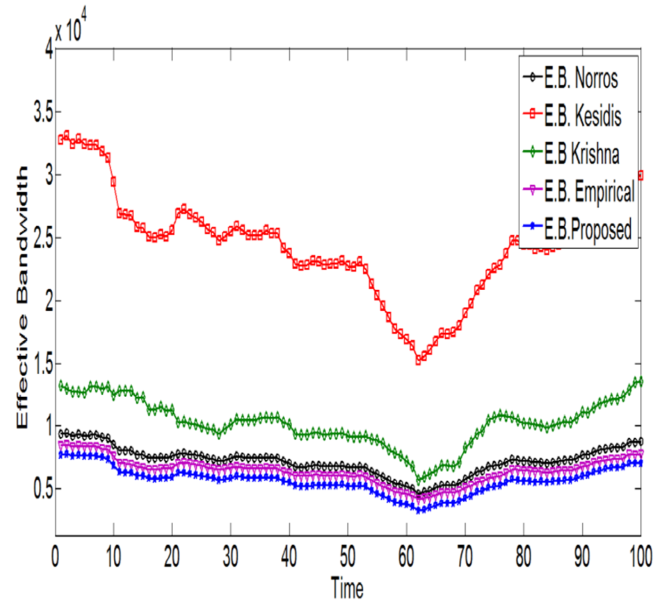

Fig.2. Effective Bandwidth for Traffic Trace Video

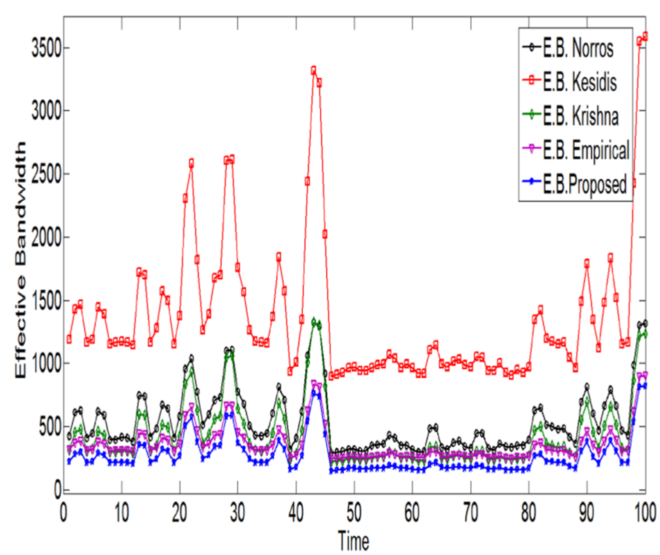

Fig.3. Effective Bandwidth for Traffic Trace Sigcomm08.

VI. CONCLUSION AND FUTURE WORK

In this work, we derived a global scaling parameter based on the multifractal traffic model presented in our recent previous work. In addition, by using this global scaling coefficient we derived an analytical expression of effective bandwidth, which take into account traffic's fractal behavior e characteristics. Experimental investigation results validated our approach showing its outstanding performance in terms of network resource usage. We also believe this global scaling parameter can be used alternatively as a measure of traffic's self-similarity.

For future work, we may investigate how efficient this global scaling parameter is in comparison with the Hurst parameter. The testing results encourage us to pursue further investigation on our derived effective bandwidth expression in terms of its susceptibility and robustness with respect to the variation of traffic modeling and queue system parameters. Based on this new multifractal traffic model, as well as the experience from effective bandwidth investigation acquired from this work, new schemes for network resource allocation and admission control, possibly in real time, will be also our future research issues.

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