Call Blocking Probabilities of Elastic and Adaptive Traffic with Retrials

Ioannis D. Moscholios*, Vassilios G. Vassilakis†, John S. Vardakas† and Michael D. Logothetis†
*Dept. of Telecommunications Science and Technology, University of Peloponnese, 221 00 Tripolis, Greece.
Email: idm@uop.gr
†Dept. of Electrical and Computer Engineering, University of Patras, 265 04 Patras, Greece.
{vasilak, jvardakas, mlogo}@upatras.gr

***Abstract*—We study a single-link multirate loss system, which accommodates both elastic and adaptive traffic of Poisson arriving calls, with exponentially distributed service time and flexible bandwidth requirements. If the available link bandwidth is not enough to accept a new call with its peak-bandwidth requirement, then the call can retry one or more times (single/multi-retry loss models) to be connected in the system with reduced bandwidth. If its last bandwidth requirement remains higher than the available link bandwidth, the call can still be accepted in the system, by compressing the bandwidth of all in-service calls together with its last bandwidth requirement. The proposed models do not have a product form solution, and therefore we propose approximate recursive formulas for the efficient calculation of call blocking probabilities. The consistency and the accuracy of our models are verified by simulation, and found to be very satisfactory.**

***Keywords*-Poisson process; elastic/adaptive traffic; call blocking; Markov chains; recurrent formula.**

I. Introduction

The call-level QoS assessment in modern telecom networks remains an open issue, due to the existence of elastic and adaptive traffic. By the term "elastic traffic" we mean calls that can compress their bandwidth, while simultaneously increasing their service time, during their lifetime in the system, so that the product *service time* by *bandwidth* remains constant. In the case of "adaptive traffic", calls can compress their bandwidth but they do not alter their service time. The call-level modeling of elastic and adaptive traffic is mostly based on the classical Erlang Multirate Loss Model (EMLM) ([[1]- [2]) which has been widely used in wired (e.g. [3]- [5]), wireless (e.g. [6]- [7]) and optical networks (e.g. [8]- [10]) to model systems that accommodate calls of different service-classes.

In the EMLM, Poisson arriving calls of different service-classes compete for the available link bandwidth under the complete sharing policy (all calls compete for all bandwidth resources). Calls are blocked and lost if their required bandwidth is higher than the available link bandwidth. Accepted calls remain in the link for a generally distributed service time [1]. The fact that the steady-state probabilities of the EMLM have a Product Form Solution (PFS) leads to an accurate calculation of Call Blocking Probabilities (CBP) via the Kaufman-Roberts recursive formula [1], [2]. In [11], the EMLM is extended to include retrials. Blocked calls can immediately retry one or more times (Single- and Multi-Retry loss Model, SRM and MRM, respectively) to be connected in the system by requiring less bandwidth units (b.u.). A retry call is blocked and lost if its last bandwidth requirement is higher than the available link bandwidth.

In this paper, we consider a system supporting elastic and adaptive traffic with single/multi retrials. If the available link bandwidth is less than or equal to the last bandwidth requirement of a retry call, the system compresses it, down to a minimum proportion (common to all calls) of its required last bandwidth, together with the bandwidth of all in-service calls. If the resulting bandwidth is less than the available link bandwidth, the new call is accepted; otherwise is blocked and lost. When a call, whose bandwidth is compressed, departs from the system, then all in-service calls expand their bandwidth. Due to retrials/compression, the models (single and multi-retry model) do not have a PFS. However, we propose approximate recursive formulas for the link occupancy distribution, and consequently CBP, calculation. Simulation results validate the proposed models and show very good accuracy. If only elastic traffic exists in the link, then the proposed models coincide with the models of [12]. In the case of no retrials for calls of all service-classes, the proposed models coincide with the model of [13], which is named, herein, Extended EMLM (E-EMLM). In [14], elastic/adaptive calls have several bandwidth requirements and request for bandwidth, upon their arrival, according to the occupied link bandwidth (i.e. calls do not retry).

This paper is organized as follows. In Section II, we review the E-EMLM, the SRM and the MRM. In Section III, we present the proposed models and provide formulas for the approximate calculation of CBP. In Section IV, we present analytical and simulation results to evaluate the accuracy and consistency of our models. We conclude in Section V.

II. Review of the E-EMLM and Multirate Loss Models with Retrials

A. Review of the E-EMLM

Consider a link of capacity C b.u. that accommodates K service-classes and let $T{>}C$ be the limit that determines the maximum permitted bandwidth compression among calls; the higher parameter T, the higher permitted compression. A service-class k ($k = 1, \ldots, K$) can be elastic or adaptive.

Let K_e and K_a be the set of elastic and adaptive service-classes ($K_e + K_a = K$), respectively. Service-class k calls follow a Poisson process with rate λ_k, request b_k b.u. (peak-bandwidth requirement) and have an exponentially distributed service time with mean μ_k^{-1}. Let j be the occupied link bandwidth, $j = 0, 1, \ldots, T$, when a service-class k call arrives in the link. If $j + b_k \leq C$, the call is accepted in the link with b_k, μ_k^{-1}. If $j + b_k > T$ the call is blocked and lost. If $T \geq j + b_k > C$ the call is accepted in the link by compressing its bandwidth and the bandwidth of all in-service calls. The compressed bandwidth of the new service-class *k* call is given by:

$$b_k^{'} = r b_k = \frac{C}{j^{'}} b_k \tag{1}$$

where r is the compression factor (common to all service-classes) given by $r \equiv r(\boldsymbol{n}) = C/j$, $j^{'} = j + b_k = \boldsymbol{nb} + b_k$, $\boldsymbol{n} = (n_1, n_2, ..., n_k, ..., n_K)$, $\boldsymbol{b} = (b_1, b_2, ..., b_k, ..., b_K)$ and n_k is the number of in-service calls of service-class k in steady state.

Similarly, all in-service calls compress their bandwidth to $b_i^{'} = \frac{C}{j^{'}} b_i$ for $i = 1, ..., K$. After the compression of all calls the link state is $j = C$. So, the link operates at its full capacity and all calls share this capacity in proportion to their peak-bandwidth requirement. The minimum bandwidth that a service-class k call (new or in-service) tolerates is:

$$b_{k,\min}^{'} = r_{\min} b_k = \frac{C}{T} b_k \tag{2}$$

where $r_{\min}$ is the minimum proportion of peak-bandwidth.

When a service-class *k* call, with bandwidth $b_k^{'}$, departs from the system, the remaining in-service calls of each service-class i $(i = 1, ..., K)$, expand their bandwidth in proportion to their peak-bandwidth b_i. After bandwidth compression/expansion, all elastic service-class k calls ($k = 1, ..., K_e$) increase/decrease their service time so that the product *service time* by *bandwidth* remains constant. Adaptive service-class calls do not alter their service time.

The bandwidth compression mechanism destroys reversibility in the model and therefore no PFS exists. However, in [13] an approximate recursive formula is proposed for the calculation of the link occupancy distribution, $G(j)$:

$$G(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{\min(j,C)} \sum\limits_{k \in K_e} \alpha_k b_k G(j - b_k) + \frac{1}{j} \sum\limits_{k \in K_a} \alpha_k b_k G(j - b_k) & \\ & \text{for } j = 1, ..., T \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $\alpha_k = \lambda_k \mu_k^{-1}$ is the offered traffic-load (in erl) of service-class k calls.

The proof of (3) is based on a reversible Markov chain which approximates the bandwidth compression/expansion mechanism of the E-EMLM. The local balance (LB) equations of this Markov chain are of the form [13]:

$$\lambda_k P(\boldsymbol{n}_k^-) = n_k \mu_k \phi_k(\boldsymbol{n}) P(\boldsymbol{n}) \tag{4}$$

where $P(\boldsymbol{n})$ is the probability distribution of state $\boldsymbol{n}$, $P(\boldsymbol{n}_k^-)$ is the probability distribution of $\boldsymbol{n}_k^- = (n_1, n_2, ..., n_{k-1}, n_k - 1, n_{k+1}, ..., n_K)$ and $\phi_k(\boldsymbol{n})$ is a state dependent factor which describes: i) the bandwidth compression factor and ii) the increase factor of service time of service-class k $(k = 1, ..., K)$ calls in state $\boldsymbol{n}$. In other words, $\phi_k(\boldsymbol{n})$ has the same role with *r* in (1) but it may be different for each service-class. The values of $\phi_k(\boldsymbol{n})$ are determined by:

$$\phi_k(\boldsymbol{n}) = \begin{cases} 1 & , \text{ for } \boldsymbol{nb} \leq C, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ \frac{x(\boldsymbol{n}_k^-)}{x(\boldsymbol{n})}, & \text{for } C < \boldsymbol{nb} \leq T \ \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ 0 & , \text{ otherwise} \end{cases} \tag{5}$$

where $\boldsymbol{\Omega} = \{\boldsymbol{n} : 0 \leq \boldsymbol{nb} \leq T\}$ and $\boldsymbol{nb} = \sum_{k=1}^{K} n_k b_k$.

In (5), $x(\boldsymbol{n})$ is a state multiplier, whose values are chosen so that (4) holds, [13]:

$$x(\boldsymbol{n}) = \begin{cases} 1 & , \text{ when } \boldsymbol{nb} \leq C, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ \frac{1}{C} \left(\sum\limits_{k \in K_e} n_k b_k x(\boldsymbol{n}_k^{-1}) + r(\boldsymbol{n}) \sum\limits_{k \in K_a} n_k b_k x(\boldsymbol{n}_k^{-1}) \right) & \\ \quad \text{when } C < \boldsymbol{nb} \leq T, \ \boldsymbol{n} \in \boldsymbol{\Omega} & \\ 0 & , \text{ otherwise} \end{cases} \tag{6}$$

Having determined $G(j)$ we calculate CBP of service-class k calls, B_k, as follows:

$$B_k = \sum_{j = T - b_k + 1}^{T} G^{-1} G(j) \tag{7}$$

where $G = \sum_{j=0}^{T} G(j)$ is the normalization constant.

B. *Review of multirate loss models with retrials*

Consider again a link of capacity *C* b.u. that accommodates Poisson arriving calls of *K* service-classes. Calls of service-class k $(k = 1, ..., K)$ have an arrival rate λ_k and request b_k b.u. If b_k b.u. are available, a call of service-class *k* remains in the system for an exponentially distributed service-time with mean μ_k^{-1}. Otherwise, the call is blocked and retries to be connected with parameters (b_{kr}, μ_{kr}^{-1}) where $b_{kr} < b_k$ and $\mu_{kr}^{-1} > \mu_k^{-1}$. The SRM does not have a PFS and therefore the determination of $G(j)$, is based on an approximate recursive formula, [11]:

$$G(j) = \begin{cases} 1 & , \text{ for } j = 0 \\ \frac{1}{j} \sum\limits_{k=1}^{K} \alpha_k b_k G(j - b_k) + & \\ \frac{1}{j} \sum\limits_{k=1}^{K} \alpha_{kr} b_{kr} \gamma_{kr}(j) G(j - b_{kr}) & \text{for } j = 1, ..., C \\ 0 & , \text{ otherwise} \end{cases} \tag{8}$$

where: $\alpha_k = \lambda_k \mu_k^{-1}$, $\alpha_{kr} = \lambda_k \mu_{kr}^{-1}$, $\gamma_{kr}(j) = 1$ when $j > C - (b_k - b_{kr})$.

The proof of (8) is based on two assumptions: 1) the application of LB, which exists only in PFS models and 2) the application of *Migration Approximation* (MA) which assumes that the occupied link bandwidth from retry calls

 ISBN: 978-1-61208-199-1

is negligible when $j \leq C-(b_k-b_{kr})$. The variable $\gamma_{kr}(j)$ expresses the MA in (8). The blocking probability of a retry service-class k call, B_{kr}, is given by:

$$B_{kr} = \sum_{j=C-b_{kr}+1}^{C} G^{-1}G(j) \tag{9}$$

where $G = \sum_{j=0}^{C} G(j)$ is the normalization constant.

In the MRM, a blocked service-class *k* call may retry many times with parameters $(b_{kr_l}, \mu_{kr_l}^{-1})$ for $l=1,...,s(k)$, where $b_{kr_{s(k)}} < ... < b_{kr_1} < b_k$ and $\mu_{kr_{s(k)}}^{-1} > ... > \mu_{kr_1}^{-1} > \mu_k^{-1}$. The MRM does not have a PFS and therefore the calculation of *G*(*j*), is based on an approximate recursive formula [11]:

$$G(j)=\begin{cases} 1 & \text{for } j=0 \\ \frac{1}{j}\sum_{k=1}^{K}\alpha_k b_k G(j-b_k)+ & \\ \frac{1}{j}\sum_{k=1}^{K}\sum_{l=1}^{s(k)}\alpha_{kr_l} b_{kr_l}\gamma_{kr_l}(j)G(j-b_{kr_l}) & \text{for } j=1,...,C \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where: $\alpha_{kr_l}=\lambda_k\mu_{kr_l}^{-1}$ and $\gamma_{kr_l}(j)=1$, if $C \geq j > C-(b_{kr_{l-1}}-b_{kr_l})$.

The blocking probability of a retry service-class *k* call with its last bandwidth requirement, $B_{kr_{s(k)}}$, is given by:

$$B_{kr_{s(k)}} = \sum_{j=C-b_{kr_{s(k)}}+1}^{C} G^{-1}G(j) \tag{11}$$

If calls of a service-class *k* do not have retry parameters, then their blocking probability, B_k, is determined by:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1}G(j) \tag{12}$$

III. Multirate Loss Models of Elastic & Adaptive Traffic With Retrials

A. The elastic-adaptive single-retry loss model

The proposed Elastic-Adaptive Single-Retry loss Model (EA-SRM) is a non-PFS model that combines the characteristics of the E-EMLM and the SRM. In order to prove an approximate but recursive formula for the calculation of *G*(*j*), the following example is presented.

Consider a link of capacity *C* b.u. that accommodates Poisson arriving calls of two service-classes. The 1^{st} service-class is adaptive and the 2^{nd} is elastic. Only calls of the 2^{nd} service-class have retry parameters. The traffic parameters are: $(\lambda_1, \mu_1^{-1}, b_1)$ and $(\lambda_2, \mu_2^{-1}, \mu_{2r}^{-1}, b_2, b_{2r})$ with $b_{2r} < b_2$ and $\mu_{2r}^{-1} > \mu_2^{-1}$. Bandwidth compression is permitted for calls of both service-classes up to a limit *T*. Although the EA-SRM is a non-PFS model we use the LB of (4), initially for calls of the 1^{st} service-class:

$$\lambda_1 P(\boldsymbol{n}_1^-) = n_1\mu_1\phi_1(\boldsymbol{n})P(\boldsymbol{n}), \quad 1 \leq \boldsymbol{nb} \leq T \tag{13}$$

where $\boldsymbol{n}=(n_1,n_2,n_{2r}), \boldsymbol{n}_1^- = (n_1-1, n_2, n_{2r})$ with $n_1 \geq 1$ and

$$\phi_1(\boldsymbol{n}) = \begin{cases} 1 & , \text{ for } \boldsymbol{nb} \leq C, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ \frac{x(\boldsymbol{n}_1^-)}{x(\boldsymbol{n})}, & \text{for } C < \boldsymbol{nb} \leq T, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ 0 & , \text{ otherwise} \end{cases} \tag{14}$$

with $\boldsymbol{nb} = j = n_1b_1 + n_2b_2 + n_{2r}b_{2r}$.

Based on (14) and by multiplying both sides of (13) with b_1 and $r(\boldsymbol{n})$ we have:

$$\alpha_1 b_1 x(\boldsymbol{n}) r(\boldsymbol{n}) P(\boldsymbol{n}_1^-) = n_1 b_1 x(\boldsymbol{n}_1^-) r(\boldsymbol{n}) P(\boldsymbol{n}), \ 1 \leq \boldsymbol{nb} \leq T \tag{15}$$

where $\alpha_1 = \lambda_1\mu_1^{-1}$ and $r(\boldsymbol{n}) = \min(1, C/j)$.

The LB equations of the 2^{nd} service-class calls are derived as follows:

a) If a call arrives in the system and $j+b_2 \leq C$ then it is accepted with b_2 b.u. Thus, the following LB equation holds:

$$\lambda_2 P(\boldsymbol{n}_2^-) = n_2\mu_2\phi_2(\boldsymbol{n})P(\boldsymbol{n}), \quad 1 \leq \boldsymbol{nb} \leq C \tag{16}$$

where $\phi_2(\boldsymbol{n}) = \frac{x(\boldsymbol{n}_2^-)}{x(\boldsymbol{n})} = 1$, when $1 \leq \boldsymbol{nb} \leq C$ and $\boldsymbol{n}_2^- = (n_1, n_2-1, n_{2r})$ with $n_2 \geq 1$.

Multiplying both sides of (16) with b_2 we have:

$$\alpha_2 b_2 x(\boldsymbol{n}) P(\boldsymbol{n}_2^-) = n_2 b_2 x(\boldsymbol{n}_2^-) P(\boldsymbol{n}), \ 1 \leq \boldsymbol{nb} \leq C \tag{17}$$

where $\alpha_2 = \lambda_2\mu_2^{-1}$.

b) If a call arrives in the system and $j+b_2 > C$ then the call is blocked with b_2 and immediately retries with $b_{2r} < b_2$. Now if: 1) $j+b_{2r} \leq C$ the retry call is accepted in the system with b_{2r}, 2) $j+b_{2r} > T$ the retry call is blocked and lost and 3) $C < j+b_{2r} \leq T$ the retry call is accepted in the system by compressing its bandwidth requirement b_{2r} together with the bandwidth of all in-service calls. The compressed bandwidth of the retry call is $b_{2r}' = rb_{2r} = \frac{C}{j+b_{2r}}b_{2r}$. Thus,

$$\lambda_2 P(\boldsymbol{n}_{2r}^-) = n_{2r}\mu_{2r}\phi_{2r}(\boldsymbol{n})P(\boldsymbol{n}), \text{ for } C-b_2+b_{2r} < \boldsymbol{nb} \leq T \tag{18}$$

where $P(\boldsymbol{n}_{2r}^-)$ is the probability distribution of state $\boldsymbol{n}_{2r}^- = (n_1, n_2, n_{2r}-1)$ with $n_{2r} \geq 1$ and

$$\phi_{2r}(\boldsymbol{n}) = \begin{cases} 1 & , \text{ for } \boldsymbol{nb} \leq C, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ \frac{x(\boldsymbol{n}_{2r}^-)}{x(\boldsymbol{n})}, & \text{for } C < \boldsymbol{nb} \leq T, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ 0 & , \text{ otherwise} \end{cases} \tag{19}$$

Based on (19) and by multiplying both sides of (18) with b_{2r} we have:

$$\alpha_{2r} b_{2r} x(\boldsymbol{n}) P(\boldsymbol{n}_{2r}^-) = n_{2r} b_{2r} x(\boldsymbol{n}_{2r}^-) P(\boldsymbol{n}), \text{ for } C-b_2+b_{2r} < \boldsymbol{nb} \leq T \tag{20}$$

where $\alpha_{2r} = \lambda_{2r}\mu_{2r}^{-1}$.

Eqs. (15), (17) and (20) lead to a system of equations:

$$\begin{aligned} &\alpha_1 b_1 x(\boldsymbol{n}) r(\boldsymbol{n}) P(\boldsymbol{n}_1^-) + \alpha_2 b_2 x(\boldsymbol{n}) P(\boldsymbol{n}_2^-) \\ &= (n_1 b_1 x(\boldsymbol{n}_1^-) r(\boldsymbol{n}) + n_2 b_2 x(\boldsymbol{n}_2^-)) P(\boldsymbol{n}) \end{aligned} \tag{21}$$

for $1 \leq \boldsymbol{nb} \leq C - b_2 + b_{2r}$.

$$\alpha_1 b_1 x(\boldsymbol{n}) r(\boldsymbol{n}) P(\boldsymbol{n}_1^-) + \alpha_2 b_2 x(\boldsymbol{n}) P(\boldsymbol{n}_2^-) + \alpha_{2r} b_{2r} x(\boldsymbol{n}) P(\boldsymbol{n}_{2r}^-) = (n_1 b_1 x(\boldsymbol{n}_1^-) r(\boldsymbol{n}) + n_2 b_2 x(\boldsymbol{n}_2^-) + n_{2r} b_{2r} x(\boldsymbol{n}_{2r}^-)) P(\boldsymbol{n}) \quad (22)$$

for $C - b_2 + b_{2r} < \boldsymbol{nb} \leq C$.

$$\alpha_1 b_1 x(\boldsymbol{n}) r(\boldsymbol{n}) P(\boldsymbol{n}_1^-) + a_{2r} b_{2r} x(\boldsymbol{n}) P(\boldsymbol{n}_{2r}^-) = (n_1 b_1 x(\boldsymbol{n}_1^-) r(\boldsymbol{n}) + n_{2r} b_{2r} x(\boldsymbol{n}_{2r}^-)) P(\boldsymbol{n}) \quad (23)$$

for $C < \boldsymbol{nb} \leq T$.

Eqs. (21)-(23) are combined in one equation by assuming that calls with b_{2r} are negligible when $1 \leq \boldsymbol{nb} \leq C - b_2 + b_{2r}$ (MA) and calls with b_2 are negligible when $C < \boldsymbol{nb} \leq T$:

$$\alpha_1 b_1 x(\boldsymbol{n}) r(\boldsymbol{n}) P(\boldsymbol{n}_1^-) + \alpha_2 b_2 \gamma_2(\boldsymbol{nb}) x(\boldsymbol{n}) P(\boldsymbol{n}_2^-) + \alpha_{2r} b_{2r} \gamma_{2r}(\boldsymbol{nb}) x(\boldsymbol{n}) P(\boldsymbol{n}_{2r}^-) = (n_1 b_1 x(\boldsymbol{n}_1^-) r(\boldsymbol{n}) + n_2 b_2 x(\boldsymbol{n}_2^-) + n_{2r} b_{2r} x(\boldsymbol{n}_{2r}^-)) P(\boldsymbol{n}) \quad (24)$$

for $0 \leq \boldsymbol{n} \leq T$,
where $\gamma_2(\boldsymbol{nb}) = 1$ for $1 \leq \boldsymbol{nb} \leq C$, otherwise $\gamma_2(\boldsymbol{nb}) = 0$ and $\gamma_{2r}(\boldsymbol{nb}) = 1$ for $C - b_2 + b_{2r} < \boldsymbol{nb} \leq T$, otherwise $\gamma_{2r}(\boldsymbol{nb}) = 0$.

At this point, we derive a formula for $x(\boldsymbol{n})$ by making the following assumptions:

1) When $C < \boldsymbol{nb} \leq T$, $\boldsymbol{n} \in \boldsymbol{\Omega}$, the bandwidth of all in-service calls should be compressed by $\phi_k(\boldsymbol{n})$, $k = 1, 2$, so that:
$$n_1 b_1' + n_2 b_2' + n_{2r} b_{2r}' = C \quad (25)$$

2) We keep the product *service time* by *bandwidth* of service-class k calls (elastic or adaptive) in state $\boldsymbol{n}$ of the irreversible Markov chain equal to the corresponding product in the same state $\boldsymbol{n}$ of the reversible Markov chain:
$$\begin{aligned} \frac{b_1 r(\boldsymbol{n})}{\mu_1} &= \frac{b_1'}{\mu_1 \phi_1(\boldsymbol{n})} \quad \text{or } b_1' = b_1 \phi_1(\boldsymbol{n}) r(\boldsymbol{n}) \\ \frac{b_2 r(\boldsymbol{n})}{\mu_2 r(\boldsymbol{n})} &= \frac{b_2'}{\mu_2 \phi_2(\boldsymbol{n})} \quad \text{or } b_2' = b_2 \phi_2(\boldsymbol{n}) \\ \frac{b_{2r} r(\boldsymbol{n})}{\mu_{2r} r(\boldsymbol{n})} &= \frac{b_{2r}'}{\mu_{2r} \phi_{2r}(\boldsymbol{n})} \quad \text{or } b_{2r}' = b_{2r} \phi_{2r}(\boldsymbol{n}) \end{aligned} \quad (26)$$

By substituting (26) in (25) we obtain:

$$n_1 b_1 \phi_1(\boldsymbol{n}) r(\boldsymbol{n}) + n_2 b_2 \phi_2(\boldsymbol{n}) + n_{2r} b_{2r} \phi_{2r}(\boldsymbol{n}) = C \quad (27)$$

where $\phi_1(\boldsymbol{n})$, $\phi_2(\boldsymbol{n})$ are given by (14) and $\phi_{2r}(\boldsymbol{n})$ by (19).

Eq. (27), due to (14) and (19), is written as:

$$x(n) = \begin{cases} 1 & , \text{ for } \boldsymbol{nb} \leq C, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ \frac{1}{C}[n_1 b_1 x(\boldsymbol{n}_1^-) r(\boldsymbol{n}) + n_2 b_2 x(\boldsymbol{n}_2^-) + n_{2r} b_{2r} x(\boldsymbol{n}_{2r}^-)] & \text{for } C < \boldsymbol{nb} \leq T, \ \boldsymbol{n} \in \boldsymbol{\Omega} \\ 0 & , \text{ otherwise} \end{cases} \quad (28)$$

Based on (28), we consider again (24). Since $x(\boldsymbol{n}) = 1$, when $0 \leq \boldsymbol{nb} \leq C$, it is proved in [11] that:

$$\alpha_1 b_1 G(j - b_1) + \alpha_2 b_2 G(j - b_2) + \alpha_{2r} b_{2r} \gamma_{2r}(j) G(j - b_{2r}) = jG(j) \quad (29)$$

for $1 \leq j \leq C$ and $\gamma_{2r}(j) = 1$ for $C - b_2 + b_{2r} < j$.

To prove (29), the MA is needed, which assumes that 2nd service-class retry calls do not exist in states $j \leq C - b_2 + b_{2r}$.

When $C < \boldsymbol{nb} \leq T$, we have $\gamma_2(\boldsymbol{nb}) = 0$ and based on (28), we can write (24) as follows:

$$\frac{C}{j} \alpha_1 b_1 P(\boldsymbol{n}_1^-) + \alpha_{2r} b_{2r} \gamma_{2r}(\boldsymbol{nb}) P(\boldsymbol{n}_{2r}^-) = CP(\boldsymbol{n}) \quad (30)$$

since $r(\boldsymbol{n}) = C/j$, when $C < \boldsymbol{nb} \leq T$.

To introduce the link occupancy distribution $G(j)$ in (30), we sum both sides of (30) over $\boldsymbol{\Omega}_j = \{\boldsymbol{n} \in \boldsymbol{\Omega} \,|\, \boldsymbol{nb} = j\}$:

$$\frac{C}{j} \alpha_1 b_1 \sum_{\{\boldsymbol{n}|\boldsymbol{nb}=j\}} P(\boldsymbol{n}_1^-) + \alpha_{2r} b_{2r} \gamma_{2r}(j) \sum_{\{\boldsymbol{n}|\boldsymbol{nb}=j\}} P(\boldsymbol{n}_{2r}^-) = C \sum_{\{\boldsymbol{n}|\boldsymbol{nb}=j\}} P(\boldsymbol{n}) \quad (31)$$

Since $\sum_{\boldsymbol{n} \in \boldsymbol{\Omega}_j} P(\boldsymbol{n}) = G(j)$, (31) is written as:

$$\frac{C}{j} \alpha_1 b_1 G(j - b_1) + \alpha_{2r} b_{2r} \gamma_{2r}(j) G(j - b_{2r}) = CG(j) \quad (32)$$

where $\gamma_{2r}(j) = 1$ for $C - b_2 + b_{2r} < j \leq T$.

The combination of (29) and (32) gives an approximate recursive formula for the calculation of $G(j)$ (for $1 \leq j \leq T$) when the 1^{st} service-class is adaptive and the 2^{nd} service-class is elastic with retrials:

$$G(j) = \frac{1}{j} \alpha_1 b_1 G(j - b_1) + \frac{1}{\min(j, C)} [\alpha_2 b_2 \gamma_2(j) G(j - b_2) + \alpha_{2r} b_{2r} \gamma_{2r}(j) G(j - b_{2r})] \quad (33)$$

where $\gamma_2(j) = 1$ for $1 \leq j \leq C$, $\gamma_{2r}(j) = 1$ for $C - b_2 + b_{2r} < j \leq T$.

In the case of K service-classes and assuming that all service-classes may have retry parameters, (33) becomes:

$$G(j) = \begin{cases} 1 & \text{for } j = 0 \\ \frac{1}{j}\left[\sum_{k \in K_a} \alpha_k b_k \gamma_k(j) G(j - b_k) + \sum_{k \in K_a} \alpha_{kr} b_{kr} \gamma_{kr}(j) G(j - b_{kr})\right] + \\ \frac{1}{\min(C, j)}\left[\sum_{k \in K_e} \alpha_k b_k \gamma_k(j) G(j - b_k) + \sum_{k \in K_e} \alpha_{kr} b_{kr} \gamma_{kr}(j) G(j - b_{kr})\right] \\ \quad \text{for } j = 1, ..., T \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

where: $\alpha_k = \lambda_k \mu_k^{-1}$, $\alpha_{kr} = \lambda_k \mu_{kr}^{-1}$,

$$\gamma_k(j) = \begin{cases} 1 \text{ for } 1 \leq j \leq C \text{ and } b_{kr} > 0 \\ 1 \text{ for } 1 \leq j \leq T \text{ and } b_{kr} = 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\gamma_{kr}(j) = \begin{cases} 1 \text{ for } C - b_k + b_{kr} < j \leq T \\ 0 \text{ otherwise} \end{cases}$$

The blocking probability of a retry service-class k call, B_{kr}, is given by:

$$B_{kr} = \sum_{j = T - b_{kr} + 1}^{T} G^{-1} G(j) \quad (35)$$

where $G = \sum_{j=0}^{T} G(j)$ is the normalization constant.

B. *The elastic-adaptive multi-retry loss model*

In the proposed Elastic-Adaptive Multi-Retry loss Model (EA-MRM) a blocked service-class k call may have many retry parameters $(b_{kr_l}, \mu_{kr_l}^{-1})$ for $l = 1, ..., s(k)$, with $b_{kr_{s(k)}} <$

$... < b_k$ and $\mu^{-1}_{kr_{s(k)}} > ... > \mu^{-1}_k$. The EA-MRM does not have a PFS and therefore the calculation of *G(j)* is based on an approximate formula whose proof is similar to that of (34):

$$G(j)=\begin{cases} 1 \quad \text{for } j=0 \\ \frac{1}{j}\sum\limits_{k\in K_a} \alpha_k b_k \gamma_k(j) G(j-b_k) + \\ \frac{1}{j}\sum\limits_{k\in K_a}\sum\limits_{s=1}^{s(k)} \alpha_{kr_s} b_{kr_s} \gamma_{kr_s}(j) G(j-b_{kr_s}) + \\ \frac{1}{\min(C,j)}\sum\limits_{k\in K_e} \alpha_k b_k \gamma_k(j) G(j-b_k) + \\ \frac{1}{\min(C,j)}\sum\limits_{k\in K_e}\sum\limits_{s=1}^{s(k)} \alpha_{kr_s} b_{kr_s} \gamma_{kr_s}(j) G(j-b_{kr_s}) \\ \qquad \text{for } j=1,...,T \\ 0 \qquad \text{otherwise} \end{cases} \tag{36}$$

where: $\alpha_{kr} = \lambda_k \mu^{-1}_{kr}$ and

$$\gamma_k(j) = \begin{cases} 1 \text{ for } 1 \le j \le C \text{ and } b_{kr_s} > 0 \\ 1 \text{ for } 1 \le j \le T \text{ and } b_{kr_s} = 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\gamma_{kr_s}(j) = \begin{cases} 1 \text{ for } C - b_{kr_{s-1}} + b_{kr_s} < j \le C \text{ and } s \ne s(k) \\ 1 \text{ for } C - b_{kr_{s-1}} + b_{kr_s} < j \le T \text{ and } s = s(k) \\ 0 \text{ otherwise} \end{cases}$$

If the link accommodates only elastic service-classes, then (36) is written as [12]:

$$G(j)=\begin{cases} 1 \quad \text{for } j=0 \\ \frac{1}{\min(C,j)}\sum\limits_{k\in K_e} \alpha_k b_k \gamma_k(j) G(j-b_k) + \\ \frac{1}{\min(C,j)}\sum\limits_{k\in K_e}\sum\limits_{s=1}^{s(k)} \alpha_{kr_s} b_{kr_s} \gamma_{kr_s}(j) G(j-b_{kr_s}) \\ \qquad \text{for } j=1,...,T \\ 0 \qquad \text{otherwise} \end{cases} \tag{37}$$

The blocking probability, $B_{kr_{s(k)}}$, of a retry service-class k call with its last bandwidth requirement, is given by:

$$B_{kr_{s(k)}} = \sum_{j=T-b_{kr_{s(k)}}+1}^{T} G^{-1} G(j) \tag{38}$$

IV. Application example - Evaluation

For evaluation, we present an application example and compare the analytical CBP with those obtained by simulation [15]. Since the reliability ranges of the measurements (95% confidence interval) are very small, we present only mean values (from 7 runs).

A link of capacity $C = 80$ b.u. accommodates three service-classes whose calls follow a Poisson process. Calls of the 1^{st} and 2^{nd} service-class are adaptive and do not retry, while calls of the 3^{rd} service-class are elastic and may retry. Their bandwidth requirements are b_1=1 b.u., b_2=2 b.u. and b_3=6 b.u., respectively. The reduced bandwidth of the 3^{rd} service-class calls, for two retrials (at most), are: $b_{3_{r1}} = 5$ b.u. and $b_{3_{r2}}$=4 b.u. The call holding time is exponentially distributed with mean value $\mu_1^{-1} = \mu_2^{-1} = \mu_3^{-1} = 1$. The initial values of the offered traffic-load are: $\alpha_1 = 20$ erl,

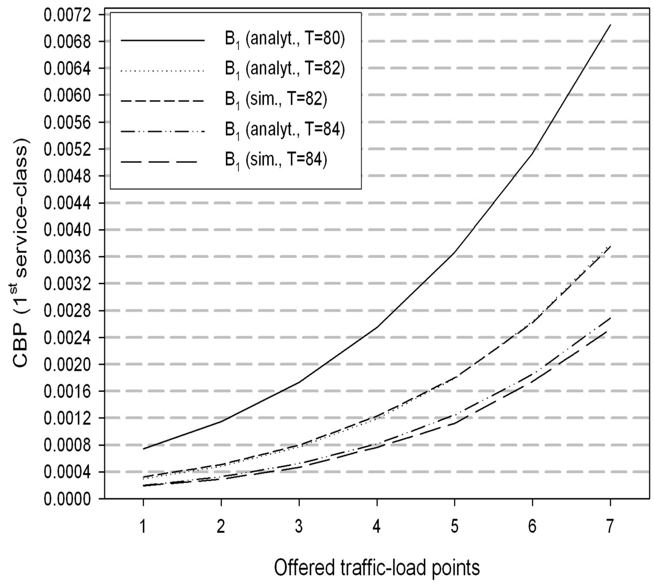

Figure 1. CBP (1^{st} service-class, adaptive).

$\alpha_2 = 6$ erl and $\alpha_3 = 2$ erl. For the retrials of the 3^{rd} service-class note that: $\alpha_3 b_3 = \alpha_{3_{r1}} b_{3_{r1}} = \alpha_{3_{r2}} b_{3_{r2}}$. In the x-axis of all figures, we let $\alpha_3 = 2$ erl, while α_1, α_2 increase in steps of 1.0 and 0.5 erl, respectively. The last values are: $\alpha_1 = 26$ erl, $\alpha_2 = 9$ erl. We consider three values of T: a) $T = C = 80$ b.u., where no bandwidth compression takes place and the EA-MRM gives the same CBP results with the MRM, b) $T = 82$ b.u. where $r_{\min} = C/T = 80/82$ and c) $T = 84$ b.u. where $r_{\min} = C/T = 80/84$. In Figs. 1, 2 and 3, we present the analytical and simulation CBP results of the 1st, 2nd and 3rd service-class (CBP of calls with $b_{3_{r2}}$), respectively, for all values of T. All figures show that our analytical models are: i) of absolutely satisfactory accuracy (compared to simulation) and ii) consistent, since the increase of T results in a CBP decrease, due to bandwidth compression.

V. Conclusion

We propose multirate loss models for a link with elastic and adaptive traffic. When Poisson arriving calls are blocked, with their initial bandwidth, have the ability to retry to be connected in the system one (EA-SRM) or more times (EA-MRM). If a retry call is blocked with its last bandwidth, it can still be accepted in the system by compressing its last bandwidth together with the bandwidth of all in-service calls. We propose approximate but recursive formulas for the efficient CBP calculation. Simulation results verify the analytical results and show that our models are accurate and consistent.

Acknowledgment

Work supported by the Research Program Caratheodory of the Research Committee of the University of Patras, Greece.

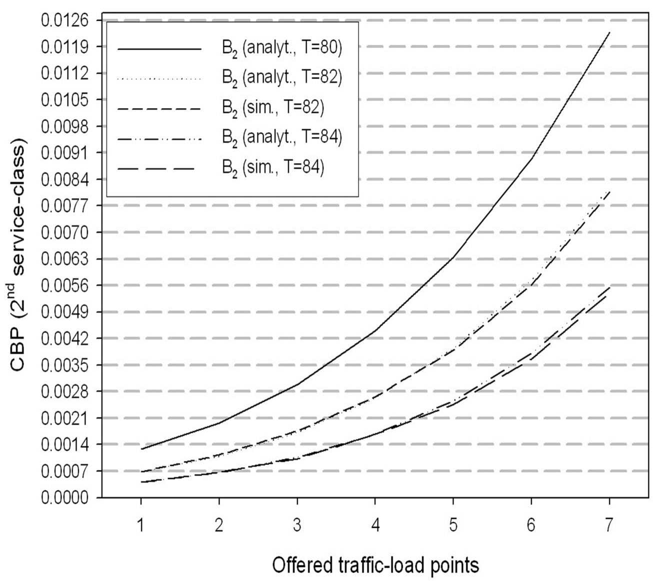

Figure 2. CBP (2^{nd} service-class, adaptive).

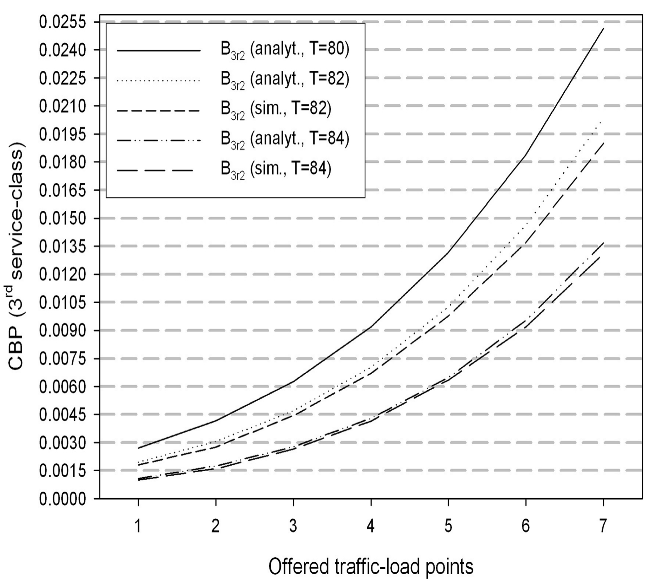

Figure 3. CBP of retry calls with b_{3r_2} (3^{rd} service-class, elastic).

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