Comparative Analysis of Three Systems with Imperfect Coverage and Standby Switching Failures

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***Abstract*—The cloud computing is an emerging new computing paradigm which provides high reliability, high availability, and QoS-guaranteed computing services. The reliability and stability of power supply is one of the most important factors in successful cloud computing. In this paper, we compare three different configurations with imperfect coverage and standby switching failures based on the reliability and availability. The time-to-repair and the time-to-failure for each of the primary and warm standby components are assumed to be exponentially distributed. We derive the explicit expressions for the mean time-to-failure, *MTTF*, and steady-state availability, for three configurations and perform a comparative analysis. Three configurations are ranked based on *MTTF*, steady-state availability, and cost/benefit where benefit is either *MTTF* or steady-state availability.**

Keywords-Reliability; Availability; Imperfect coverage; Standby switching failures

I. INTRODUCTION

Uncertainty is one of the important issues in management decisions. Maintaining a high or required level of reliability and/or availability is especially essential in information industry, communication systems, power plants, etc. With the increasing demand for computing resources, the computing paradigm has evolved from stand-alone computing, distributed computing, grid computing, to cloud computing. Cloud computing hosts and delivers services over the Internet, i.e., information is processed on servers located in cloud data center and cached temporarily on clients via the Internet. A data center usually consists of thousands of servers that are organized in racks and interconnected through gigabit ethernet or other fabrics. Data center consumes a lot of electricity to maintain its normal operation. The power consumption breakdown of a data center includes servers and storage systems, power conditioning equipment, cooling and humidification systems, and networking equipment. In this paper, we discuss the optimal configuration of power electricity for data centers in terms of reliability and availability. Cao [4] first introduced reliability concept into a queueing system with a repairable service station which has exponentially distributed lifetime and generally distributed repair time. The concept of the standby switching failures in the reliability with standby system was first proposed by Lewis [6]. The concept of coverage and its effect on the reliability and/or availability model of a repairable system has been introduced by several authors such as Amari, et al. [1], Arnold [3], Dugan [5], Trivedi [7], and etc. Moreover, the status and trends of imperfect coverage models and its associated reliability analysis techniques were introduced in Amari, et al. [2]. Wang and Chiu [9] investigated the cost benefit analysis of availability systems with warm standby units and imperfect coverage. Wang and Chen [8] performed comparative analysis of availability between three system with general repair times, reboot delay and switching failures. Wang et al. [11] studied the cost benefit analysis of series systems with warm standby components and general repair times. Recently, Wang et al. [10] performed comparisons of reliability and the availability between four systems with warm standby components, reboot delay and standby switching failures.

The problem considered in this paper is more general than the works of Wang et al. [11] and Wang et al. [12]. We first systematically develop the explicit expressions for the $MTTF_i$ and $A_{T_i}(\infty)$ to three configurations with imperfect coverage and standby switching failures. Next, efficient Maple computer programs are utilized to perform a parametric investigation. We provide extensive numerical results to study the effects of various values of system parameters to the cost/benefit ratios. Finally, we rank the configurations for the *MTTF*, the $A_T(\infty)$, and the cost/benefit, based on specific values of distribution parameters, as well as of the costs of the components.

II. PROBLEM STATEMENT

For the sake of discussion, we consider a data center require a 30MW power electricity, and assume that the electricity generation capacity of generators is available in units of 30MW, 15MW, and 10MW. To provide reliable and stable power supply, there are standby generators, and all active and standby generators are continuously monitored by a fault detecting device to identify if they fail or not. We also assume that standby generators are allowed to fail while inactive before they are put into full operation. Each of the active components fails independently of the state of the others and has an exponential time-to-failure distribution with parameter λ. Whenever an active component (or warm standby component) fails, it may be immediately detected and located with a coverage probability c, and the failed component is instantly replaced by a warm standby component with switchover time β_1 if any standby is available. We now assume that each of the available standby component fails independently of the state of all the others and has an exponential time-to-failure distribution with parameter α ($0 < \alpha < \lambda$). Moreover, we define the *unsafe failure* state of the system as any one of the breakdowns is *not covered*. We further assume that active-component failure (or standby-component failure) in the *unsafe failure* state is cleared by a reboot. Reboot delay is assumed to be exponentially distributed with parameter β_2 for an active component (or standby component). The system fails when the remaining electricity generation capacity is less than 30MW. We define such situation as the state of *safe failure*. We assume that there is always the possibility of failures during the switching from standby state to active state. Let us assume that the switching component has a failure probability q. Active components and standby components are considered to be repairable. Whenever a primary component or a standby component fails, it is immediately repaired based on a first-come, first-served (FCFS) discipline. The time-to-repair for each of the primary and warm standby components are assumed to be exponentially distributed with parameter μ. Once a component is repaired, it is as good as new. Further, failure times and repair times are independently distributed random variables.

We consider three configurations as follows: the first configuration consists of one 30 MW active component and one 30 MW warm standby component. The second configuration is composed of two 15 MW active components and one 15 MW warm standby component. We assume the standby component can replace either one of the initially working components in case of failure. The third configuration includes of three 10 MW active components and two 10 MW warm standby component.

III. PROBLEM SOLUTIONS

Let $P_{n,m}(t)$ be the probability that exactly n primary components and m standby components are working at time $t (t \geq 0)$, and let $P_{uf_i}(t)$ be the probability that the system is in unsafe failure states, where $i = 1, 2, 3, 4$.

A. Calculations for configuration 1

A.1. MTTF

Using Trivedi's concept (see Trivedi [7]) and Wang et al.' concept (see Wang et al. [10]), the state-transition-rate diagram of configuration 1 is shown in Figure 1. The probability vector $\mathbf{P}(t)$ of configuration 1 is defined as:

$$\mathbf{P}(t) = [P_{1,1}(t), P_{1,0}(t), P_{0,1}(t), P_{uf_1}(t), P_{uf_2}(t), P_{0,0}(t)] .$$

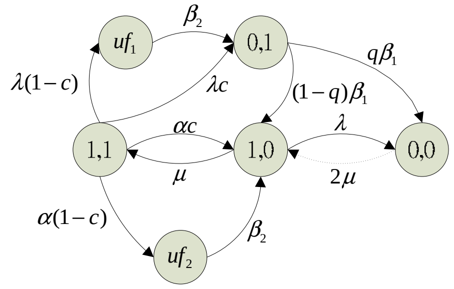

Figure 1. The state-transition-rate diagram of configuration 1

Relating the state of the system at time t and $t + dt$, the steady-state equations for configuration 1 can be expressed as follows:

$$d\mathbf{P}(t)/dt = B_1\mathbf{P}(t), \qquad (1)$$

where

$$B_1 = \begin{pmatrix} -\lambda-\alpha & \mu & 0 & 0 & 0 & 0 \\ \alpha c & -\lambda-\mu & (1-q)\beta_1 & 0 & \beta_2 & 0 \\ \lambda c & 0 & -\beta_1 & \beta_2 & 0 & 0 \\ \lambda(1-c) & 0 & 0 & -\beta_2 & 0 & 0 \\ \alpha(1-c) & 0 & 0 & 0 & -\beta_2 & 0 \\ 0 & \lambda & q\beta_1 & 0 & 0 & 0 \end{pmatrix}.$$

To evaluate the *MTTF*, we take the transpose matrix of B_1 and delete the rows and columns for the absorbing state(s). The new matrix is called A_1. The expected times to reach an absorbing states is obtained from

$$E[T_{P(0)\to P(\text{absorbing})}] = \mathbf{P}(0)(-A_1^{-1})[1,1,1,1,1]^T, \qquad (2)$$

where the initial conditions are given by

$$\mathbf{P}(0) = [P_{1,1}(0), P_{1,0}(0), P_{0,1}(0), P_{uf_1}(0), P_{uf_2}(0)] = [1,0,0,0,0],$$

and

$$A_1 = \begin{pmatrix} -\lambda-\alpha & \alpha c & \lambda c & \lambda(1-c) & \alpha(1-c) \\ \mu & -\lambda-\mu & 0 & 0 & 0 \\ 0 & (1-q)\beta_1 & -\beta_1 & 0 & 0 \\ 0 & 0 & \beta_2 & -\beta_2 & 0 \\ 0 & \beta_2 & 0 & 0 & -\beta_2 \end{pmatrix}.$$

For configuration 1, the explicit expression for the $MTTF_1$ is given by

$$E[T_{P(0)\to P(\text{absorbing})}] = MTTF_1 .$$

This implies that

$$MTTF_1 = \frac{\Lambda_1}{\lambda\Delta} - \frac{-\alpha-\lambda+\lambda q}{\lambda\Delta} + \frac{\Lambda_1}{\beta_1\Delta} - \frac{\Lambda_1\Lambda_2}{\beta_2\Delta} - \frac{\alpha\Lambda_1\Lambda_2}{\lambda\beta_2\Delta}, \qquad (3)$$

where $\Lambda_1 = \lambda + \mu$, $\Lambda_2 = -1 + c$, and $\Delta = \mu q + \lambda + \alpha$.

A.2. Availability

To discuss the availability case of configuration 1, we use the following procedure to obtain the steady-state availability. In steady-state, the derivatives of the state probabilities become zero. Thus we have

$$\begin{pmatrix} -\lambda-\alpha & \mu & 0 & 0 & 0 & 0 \\ \alpha c & -\lambda-\mu & (1-q)\beta_1 & 0 & \beta_2 & 2\mu \\ \lambda c & 0 & -\beta_1 & \beta_2 & 0 & 0 \\ \lambda(1-c) & 0 & 0 & -\beta_2 & 0 & 0 \\ \alpha(1-c) & 0 & 0 & 0 & -\beta_2 & 0 \\ 0 & \lambda & q\beta_1 & 0 & 0 & -2\mu \end{pmatrix} \begin{pmatrix} P_{1,1}(\infty) \\ P_{1,0}(\infty) \\ P_{0,1}(\infty) \\ P_{uf_1}(\infty) \\ P_{uf_2}(\infty) \\ P_{0,0}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{4}$$

Solving (4) and using the following normalizing condition

$$P_{1,1}(\infty)+P_{1,0}(\infty)+P_{0,1}(\infty)+P_{uf_1}(\infty)+P_{uf_2}(\infty)+P_{0,0}(\infty)=1,$$

we then obtain $P_{uf_1}(\infty)$, $P_{uf_2}(\infty)$, and $P_{0,0}(\infty)$.

Let T_1 represent the time-to-failure of the system for configuration 1. The explicit expression for the $A_{T_1}(\infty)=1-P_{uf_1}(\infty)-P_{uf_2}(\infty)-P_{0,0}(\infty)$ is given by

$$A_{T_1}(\infty)=\frac{2\beta_2\mu\left(\mu\beta_1+\beta_1\lambda+\beta_1\alpha+\mu\lambda\right)}{\beta_1\beta_2\lambda^2+2\mu\beta_1\beta_2\alpha+2\mu^2\Delta_1+\lambda\Delta_2} \tag{5}$$

where $\Delta_1=\beta_1\beta_2+\beta_1\alpha-\beta_1\alpha c+\beta_1\lambda-\beta_1\lambda c+\beta_2\lambda$ and $\Delta_2=\beta_1\beta_2(2\mu+\alpha+\mu q)$.

B. Calculations for configuration 2

B.1. MTTF

Using Trivedi's concept (see Trivedi [7]) and Wang et al.' concept (see Wang et al. [10]), the state-transition-rate diagram of configuration 2 is shown in Figure 2. The $\mathbf{P}(t)$ of configuration 2 is defined as:

$$\mathbf{P}(t)=[P_{2,1}(t),P_{2,0}(t),P_{1,1}(t),P_{uf_1}(t),P_{uf_2}(t),P_{1,0}(t)].$$

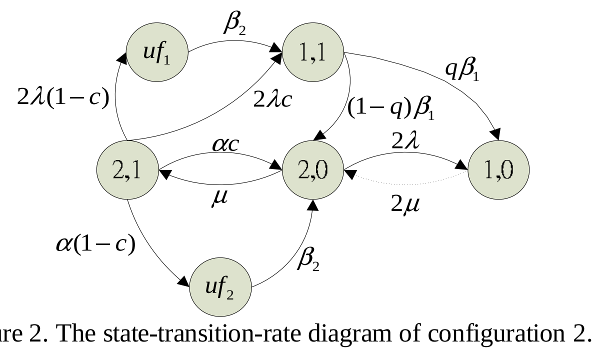

Figure 2. The state-transition-rate diagram of configuration 2.

Relating the state of the system at time t and $t+dt$, the steady-state equations for configuration 2 can be expressed as follows:

$$d\mathbf{P}(t)/dt=B_2\mathbf{P}(t), \tag{6}$$

where

$$B_2=\begin{pmatrix} -2\lambda-\alpha & \mu & 0 & 0 & 0 & 0 \\ \alpha c & -2\lambda-\mu & (1-q)\beta_1 & 0 & \beta_2 & 0 \\ 2\lambda c & 0 & -\beta_1 & \beta_2 & 0 & 0 \\ 2\lambda(1-c) & 0 & 0 & -\beta_2 & 0 & 0 \\ \alpha(1-c) & 0 & 0 & 0 & -\beta_2 & 0 \\ 0 & 2\lambda & q\beta_1 & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

To evaluate the *MTTF*, we take the transpose matrix of B_2 and delete the rows and columns for the absorbing state(s). The new matrix is called A_2. The expected times to reach an absorbing states is obtained from

$$E[T_{P(0)\to P(\text{absorbing})}]=\mathbf{P}(0)(-A_2^{-1})[1,1,1,1,1]^T, \tag{8}$$

where the initial conditions are given by

$$\mathbf{P}(0)=[P_{2,1}(0),P_{2,0}(0),P_{1,1}(0),P_{uf_1}(0),P_{uf_2}(0)]=[1,0,0,0,0].$$

For configuration 2, the explicit expression for the $MTTF_2$ is given by

$$E[T_{P(0)\to P(\text{absorbing})}]=MTTF_2.$$

This implies that

$$MTTF_2=\frac{1}{2}\left(\frac{\Lambda_1}{\lambda\Delta_1}-\frac{-\alpha-2\lambda+2\lambda q}{\lambda\Delta_1}+\frac{2\Lambda_1}{\beta_1\Delta_1}-\frac{2\Lambda_1\Lambda_2}{\beta_2\Delta_1}-\frac{\alpha\Lambda_1\Lambda_2}{\lambda\beta_2\Delta_1}\right), \tag{9}$$

where $\Lambda_1=2\lambda+\mu$, $\Lambda_2=-1+c$, and $\Delta_1=\mu q+2\lambda+\alpha$.

B.2. Availability

For the availability case of configuration 2, we use the same procedure in 3.1.2 to obtain the steady-state availability. In steady-state, the derivatives of the state probabilities become zero. Thus we have

$$\begin{pmatrix} -2\lambda-\alpha & \mu & 0 & 0 & 0 & 0 \\ \alpha c & -2\lambda-\mu & (1-q)\beta_1 & 0 & \beta_2 & 2\mu \\ 2\lambda c & 0 & -\beta_1 & \beta_2 & 0 & 0 \\ 2\lambda(1-c) & 0 & 0 & -\beta_2 & 0 & 0 \\ \alpha(1-c) & 0 & 0 & 0 & -\beta_2 & 0 \\ 0 & 2\lambda & q\beta_1 & 0 & 0 & -2\mu \end{pmatrix} \begin{pmatrix} P_{2,1}(\infty) \\ P_{2,0}(\infty) \\ P_{1,1}(\infty) \\ P_{uf_1}(\infty) \\ P_{uf_2}(\infty) \\ P_{1,0}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{10}$$

Solving (10) and using the following normalizing condition

$$P_{2,1}(\infty)+P_{2,0}(\infty)+P_{1,1}(\infty)+P_{uf_1}(\infty)+P_{uf_2}(\infty)+P_{1,0}(\infty)=1,$$

we then obtain $P_{uf_1}(\infty)$, $P_{uf_2}(\infty)$, and $P_{1,0}(\infty)$.

Let T_2 represent the time-to-failure of the system for configuration 2. The explicit expression for the $A_{T_2}(\infty)=1-P_{uf_1}(\infty)-P_{uf_2}(\infty)-P_{1,0}(\infty)$ is given by

$$A_{T_2}(\infty)=\frac{\beta_2\mu\left(\mu\beta_1+2\beta_1\lambda+\beta_1\alpha+2\mu\lambda\right)}{\beta_1\beta_2\Delta_1+\beta_1\mu^2\Delta_2+2\beta_2\mu^2\lambda} \tag{11}$$

where $\Delta_1 = \mu^2 + 2\mu\lambda + \mu\alpha + 2\lambda^2 + \lambda\alpha + \mu q\lambda$ and $\Delta_2 = \alpha - \alpha c + 2\lambda - 2\lambda c$.

C. Calculations for configuration 3

C.1. MTTF

Using Trivedi's concept (see Trivedi [7]) and Wang et al.' concept (see Wang et al. [10]), the state-transition-rate diagram of configuration 3 is shown in Figure 3. The $\mathbf{P}(t)$ of configuration 3 is defined as:

$$\mathbf{P}(0) = [P_{3,2}(t), P_{3,1}(t), P_{2,2}(t), P_{3,0}(t), P_{2,1}(t),$$
$$P_{uf_1}(t), P_{uf_2}(t), P_{uf_3}(t), P_{uf_4}(t), P_{2,0}(t)].$$

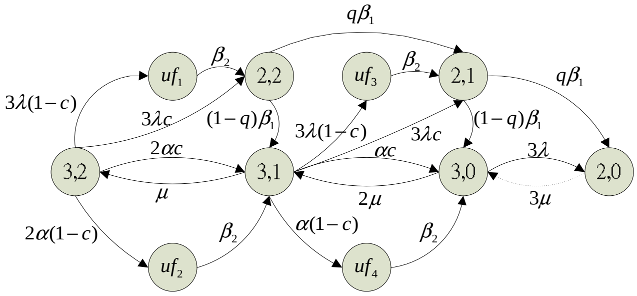

Figure 3. The state-transition-rate diagram of configuration 3.

For the reliability case, the initial conditions are:

$$\mathbf{P}(0) = [P_{3,2}(0), P_{3,1}(0), P_{2,2}(0), P_{3,0}(0), P_{2,1}(0),$$
$$P_{uf_1}(0), P_{uf_2}(0), P_{uf_3}(0), P_{uf_4}(0)]$$
$$= [1, 0, 0, 0, 0, 0, 0, 0, 0, 0].$$

The following differential equations written in matrix form can be obtained:

$$d\mathbf{P}(t)/dt = B_3\mathbf{P}(t). \quad (12)$$

Hence the matrix B_3 is an (10×10) square matrix whose last column is zero. The matrix B_3 is too spacious to be shown here. For the *MTTF*, we take the transpose matrix of B_3 and delete the rows and columns for the absorbing state(s). The new matrix shall be called A_3. The expected times to reach an absorbing states can now be calculated from

$$E[T_{P(0)\to P(\text{absorbing})}] = \mathbf{P}(0)(-A_3^{-1})[1,1,1,1,1,1,1,1]^T. \quad (13)$$

For configuration 3, the explicit expression for the $MTTF_3$ is given by

$$MTTF_3 = E[T_{P(0)\to P(\text{absorbing})}].$$

The mean time to system failure for configuration 3 $MTTF_3$ is too ample to be shown here.

C.2. Availability

For the availability case of configuration 3, the initial conditions are

$$\mathbf{P}(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0].$$

The following differential equations written in matrix form can be obtained from

$$d\mathbf{P}(t)/dt = B_3\mathbf{P}(t), \quad (14)$$

where the matrix B_3 can be formulated in a way similar to (12). It is an (10×10) square matrix whose last column, rather than being zero as in (12), is appropriately modified. The resulting matrix is too spacious to be shown here. In steady-state, the derivatives of the state probabilities become zero. That allows us to calculate the steady-state probabilities $P_{uf_1}(\infty)$, $P_{uf_2}(\infty)$, $P_{uf_3}(\infty)$, $P_{uf_4}(\infty)$, and $P_{2,0}(\infty)$ with the following normalizing condition

$$P_{3,2}(\infty)+P_{3,1}(\infty)+P_{2,2}(\infty)+P_{3,0}(\infty)+P_{2,1}(\infty)$$
$$+\sum_{i=1}^{4}P_{uf_i}(\infty)+P_{2,0}(\infty)=1.$$

Let T_3 represent the time-to-failure of the system for configuration 3. Again, the explicit expression for the $A_{T_3}(\infty) = 1 - P_{uf_1}(\infty) - P_{uf_2}(\infty) - P_{uf_3}(\infty) - P_{uf_4}(\infty) - P_{2,0}(\infty)$ is too spacious to be shown here.

IV. COMPARATIVE ANALYSIS

A. Comparison for the MTTF

The main purpose of this section is to present specific numerical comparisons for the $MTTF$. Using an efficient Maple program, three configurations will be compared in terms of their $MTTF_i$ $(i = 1, 2, 3)$ with the following values:

$$1/\lambda = 50 \text{ days},\ 1/\alpha = 200 \text{ days}, \text{ and } 1/\mu = 2 \text{ days},$$

or $\lambda = 0.02$, $\alpha = 0.005$, and $\mu = 0.5$.

We consider the following two cases to perform a comparison for the $MTTF_i$ of the configurations 1, 2, and 3.

Case 1: We fix α=0.005, μ=0.5, $q = 0.1$, $c = 0.9$, $\beta_1 = 3.0$, $\beta_2 = 2.4$ and vary λ from 0.02 to 0.1.

Case 2: We fix λ=0.01, α=0.005, $q = 0.1$, $c = 0.9$, $\beta_1 = 3.0$, $\beta_2 = 2.4$ and vary μ from 0.001 to 0.5.

The numerical results of $MTTF_i$ for each configuration i $(i = 1, 2, 3)$ are shown in Table 1 for cases 1 and 2.

Table 1. Comparison of the configurations 1, 2, 3 for $MTTF_i$

	Result
Range of λ	
$0.02 < \lambda < 0.04391$	$MTTF_3 > MTTF_1 > MTTF_2$
$0.04391 < \lambda < 0.1$	$MTTF_1 > MTTF_3 > MTTF_2$
Range of μ	
$0.001 < \mu < 0.12652$	$MTTF_1 > MTTF_3 > MTTF_2$
$0.12652 < \mu < 0.5$	$MTTF_3 > MTTF_1 > MTTF_2$

B. Comparison for the $A_{T_i}(\infty)$

In this section, we consider the following two cases to compare the $A_T(\infty)$ of the configurations 1, 2, and 3.

Case 1: We fix α=0.0005, μ=0.1, $q = 0.1$, $c = 0.9$,

$\beta_1 = 3.0$, $\beta_2 = 2.4$ and vary λ from 0.001 to 0.1.

Case 2: We fix λ=0.01, α=0.0005, $q = 0.1$, $c = 0.9$,

$\beta_1 = 3.0$, $\beta_2 = 2.4$ and vary μ from 0.01 to 0.5.

The numerical results of $A_{T_i}(\infty)$ for each configuration i $(i = 1, 2, 3)$ are shown in Table 2 for cases 1 and 2.

Table 2. Comparison of the configurations 1, 2, 3 for $A_{T_i}(\infty)$

	Result
Range of λ	
$0.0 < \lambda < 0.00005$	$A_{T_1}(\infty) > A_{T_3}(\infty) > A_{T_2}(\infty)$
$0.00005 < \lambda < 0.01656$	$A_{T_3}(\infty) > A_{T_1}(\infty) > A_{T_2}(\infty)$
$0.01656 < \lambda < 0.1$	$A_{T_1}(\infty) > A_{T_3}(\infty) > A_{T_2}(\infty)$
Range of μ	
$0.01 < \mu < 0.058489$	$A_{T_1}(\infty) > A_{T_3}(\infty) > A_{T_2}(\infty)$
$0.058489 < \mu < 0.5$	$A_{T_3}(\infty) > A_{T_1}(\infty) > A_{T_2}(\infty)$

C. Comparison of all configurations based on their cost/benefit ratios

The cost (C_i) of the configuration i $(i = 1, 2, 3)$ are listed in the following:

$$C_1 = \$48 \times 10^6,\ C_2 = \$39 \times 10^6,\ C_3 = \$42 \times 10^6$$

Consider the following two cases, we perform a comparison for the cost/benefit ratios, namely, $C_i / MTTF_i$ and $C_i / A_{T_i}(\infty)$ for each configuration i $(i = 1, 2, 3)$. The results are depicted in Figures 4-7, respectively.

Case 1: We fix α=0.0005, μ=0.1, $q = 0.1$, $c = 0.9$,

$\beta_1 = 3.0$, $\beta_2 = 2.4$ and vary λ from 0.001 to 0.1.

Case 2: We fix λ=0.01, α=0.0005, $q = 0.1$, $c = 0.9$,

$\beta_1 = 3.0$, $\beta_2 = 2.4$ and vary μ from 0.01 to 0.5.

Figure 4 and Figure 5 show that the $C_i / MTTF_i$ and $C_i / A_{T_i}(\infty)$ increase as λ increases for any configuration. We observe from Figure 4 that the optimal configuration using the $C_i / MTTF_i$ value depends on the value of λ. When $\lambda < 0.0574$, the optimal configuration is configuration 3, but when $\lambda > 0.0574$, the optimal configuration is configuration 1. One observes from Figure 5 that the optimal configuration using the $C_i / A_{T_i}(\infty)$ value depends on the value of λ. When $\lambda < 0.0418$, the optimal configuration is configuration 2, when $0.0418 < \lambda < 0.0757$, the optimal configuration is configuration 3, and when $\lambda > 0.0757$, the optimal configuration is configuration 1.

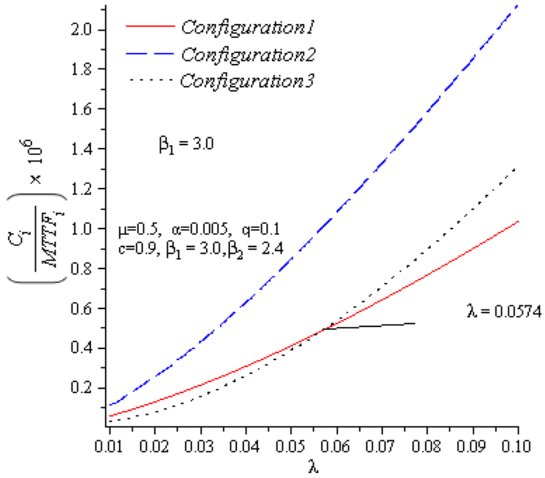

Figure 4. $C_i / MTTF_i$ versus λ.

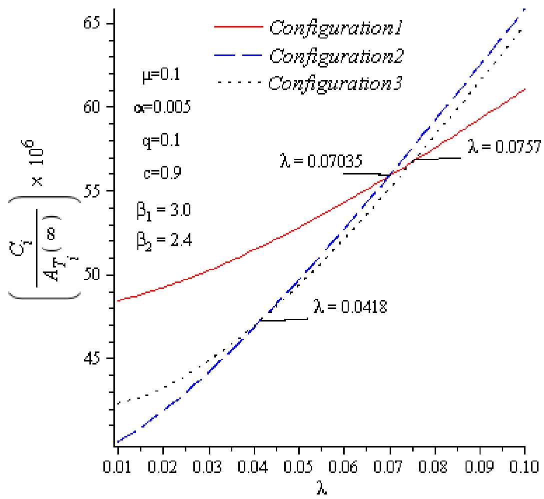

Figure 5. $C_i / A_{T_i}(\infty)$ versus λ.

We can easily see from Figure 6 and Figure 7 that the $C_i / MTTF_i$ and $C_i / A_{T_i}(\infty)$ decrease as μ increases for any configuration. Figure 6 reveals that the optimal configuration using the $C_i / MTTF_i$ value depends on the value of μ. When $\mu < 0.0979$, the optimal configuration is configuration 1, but when $\mu > 0.0979$, the optimal configuration is configuration 3. We observe from Figure 7 that the optimal configuration using the $C_i / A_{T_i}(\infty)$ value depends on the value of μ as well. When $\mu > 0.0241$, the optimal configuration is configuration 2.

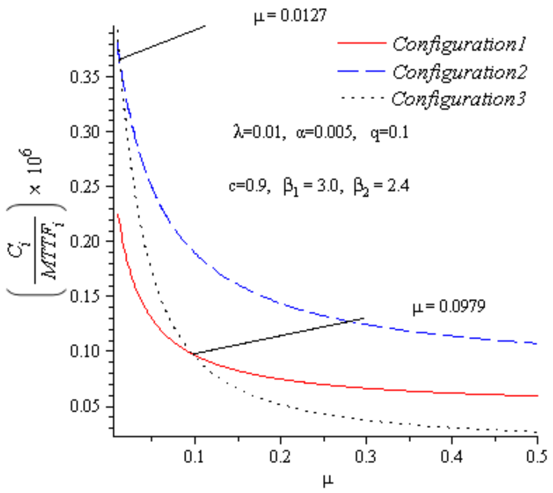

Figure 6. C_i / $MTTF_i$ versus μ .

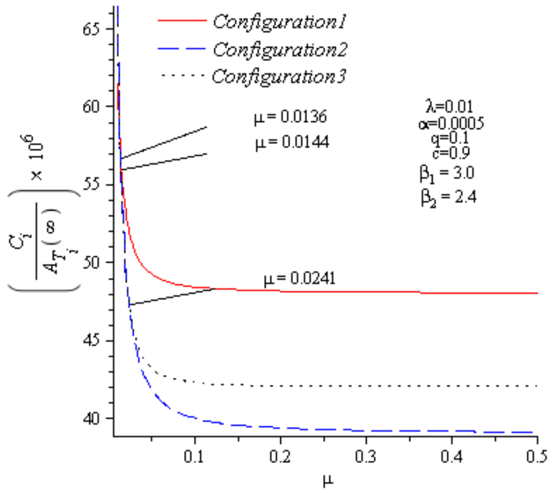

Figure 7. C_i / $A_{T_i}(\infty)$ versus μ .

V. CONCLUSIONS

In this paper, we analyzed three different configurations with imperfect coverage and standby switching failures to study the cost/benefit analysis of three configurations under uncertainty. For each configuration, we present the explicit expressions for the $A_{T_i}(\infty)$ and the *MTTF*. We rank three configurations based on the $A_{T_i}(\infty)$, the *MTTF*, and the cost/benefit where benefit is either steady-state availability or *MTTF*

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