A Localized Hybrid Penalization Method for Simultaneous Data Reconstruction and Biase Field Correction

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***Abstract*—A new variational model is dedicated to simultaneous image reconstruction and bias field rectification, where the image is corrupted by mixed Laplace-Gaussian noise. To solve the model numerically, an adaptive augmented Lagrangian mehod is combined with the expectation maximization strategy. Some numerical results are also presented to validate the proposed model and algorithm.**

Keywords- Localized Regularization Method; Mixed Laplace-Gaussian Noise; ALM; EM; Data Reconstruction; Bias correction

I. INTRODUCTION

This paper addresses the problem of rectifying bias field in images corrupted by mixed noise. Bias field, also known as intensity inhomogeneity, is the spurious smooth intensity variation across the whole image. It is present in many different imaging modalities, such as microscopy, ultrasound and Magnetic Resonance Imaging (MRI), and high-energy radiography (HER) as well (e.g., [1]). In HER, the bias field can be resulted in the intensity distribution of the radiation source, the artifact of the image intensifier and the vignetting effect of the optical lens in the charge-coupled-device (CCD) based imager. Many image analysis processes, including image segmentation and Abel inverse transformation, are highly sensitive to the issue.

In HER, the problem of image denoising has to be considered carefully. In CCD-recorded data, background hot noise is inherent in the current carriers, while pulse spike noise is also encountered due to not only the interaction between transmitted particles and CCD chips, but the tremendous flaw in the transistors as well. In other words, its statistical property is a combination of two different distributions. In this paper, we focus on a special case of noise distribution, that is, Laplace and Gaussian mixture (LGM). Our problem is then to reconstruct an original image from an observation with simultaneous LGM noise removal and bias field correction.

As it is known, finite mixture models are among the most familiar approaches for image segmentation and intensity inhomogeneity elimination (i.e., [2]). They combine the maximum likelihood (ML) or maximum a posterior (MAP) probability criterion with the expectation maximization (EM) algorithm to construct subjective energy functionals. Mixture noise models have also been drawn much research interest in the community of image processing, including Poisson-Gaussian noise and impulse-Gaussian noise (e.g., [3[-[6]). Liu et al. [7] proposed a finite Laplacian mixture model to approximate the impulse noise. The authors combined the first order total variation (FOTV) regularization with the EM method to gain an adaptive restoration method. In [8], Gong, Shen and Toh present a first order regularization model to recover images contaminated by mixed or even unknown noises. More recently, Calatroni et al. [9] proposed a variational model encoding the mixed Poisson-Gaussian noise as an infimal convolution of discrepancy terms of noise distributions.

During the last two decades, numerous approaches have been suggested to correct the bias field, which are mainly divided into two categories. The first one is parameter algorithms, and the second one is non-parametric methods, which do not require any prior knowledge on the intensity probability distribution and the bias field is integrated into a faith energy based on ML or MAP estimation.

In this paper, we propose a variational method by integrating the LGM model with the EM algorithm to investigate the inverse problem in the presence of bias field and mixed noise. The data term in our energy is derived from a localized LGM assumption, and the bias field is regarded as a parameter of the LGM model. Compared with earlier methods based on mixture assumptions, our model contains a higher-order cost term [10], used to eliminate the staircasing fallback of the first-order total variation, and it does not require extra constraints on the bias field. Moreover, the regularization parameters in our model are updated adaptively according to the change of the functional cost. Numerical experiments on MRI images have shown its efficiency.

The remainder of the paper is organized as follows. Section 2 introduces the BiLGM-TVBH model based on the EM algorithm. Section 3 presents briefly the numerical algorithm of our proposed model and some experimental results. Concluding remarks are given in the last section.

II. LGM MODEL AND EM ALGORITHM

Some notations adopted in this paper is listed as follows. Let $\mathbb{R}^n$ be n-dimensional real Euclidean space, $\mathbb{R} = \mathbb{R}^1$. In the following, we denote $u, f : \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$ to be an original gray scale image and an observed image describing the same scene, Ω is open and bounded with Lipschitz boundary.

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Assuming no blurring effect on the observed image, the reconstruction problem assumes the following form

$$\text{find u s.t. } f(x) = b(x)\big(Hu(x) + n(x)\big), \quad (1)$$

where $b, n: \Omega \longrightarrow \mathbb{R}$ represent the bias field and the additive noise, respectively. H is a forward map, such as Abel's transform [11][12] given by

$$Hu(x) \cong Hu(s,t) = \int_{|s|}^{+\infty} \frac{ru(r,t)}{\sqrt{r^2-|s|^2}} dr. \quad (2)$$

$n(x)$ is a realization of independent and identically distributed random variable η with PDF

$$p_\eta(z,\Theta_0) = \sum_{k=1}^{2} \gamma_k \, p_k(z;\sigma_k^2), \quad (3)$$

$$p_1(z;\sigma_1^2) = \frac{1}{2\sigma_1^2} e^{-|z|/\sigma_1^2},$$

$$p_2(z;\sigma_2^2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-|z|^2/\sigma_2^2},$$

Θ_0 denotes the parameter set $\{\sigma_k^2, \gamma_k, k = 1,2\}$, γ_k is positive real number acting on mixture radio.

Following the idea in [13] and the references therein, the bias field function $b(x)$ is assumed to be positive, smoothly varying function on the image domain Ωsuch that $b(s) \approx b(x)$ for all $s \in O_x$ which is definitely small neighborhood with center at x. We regard the intensity of the observation is a realization of a random variable ξ for all $x \in \Omega$. Then, by routine computation, there holds

$$p_\xi(z;\Theta) = \frac{\gamma_1}{2b\sigma_1^2} e^{-\frac{|z-bHu|}{b\sigma_1^2}} + \frac{\gamma_2}{\sqrt{2\pi b^2\sigma_2^2}} e^{-\frac{|z-bHu|^2}{2b^2\sigma_2^2}} \quad (4)$$

with $\Theta = \Theta_0 \cup b$. And then, our problem is reformulated to reconstruct u from relation (1) with unknown parameter set Θ.

Based on the framework of Bayesian approach [14][15], the MAP estimation of u can be given by

$$u^* = \arg\min_u \{-\log(f|u) - \log p(u)\} \quad (5)$$

that is, u^* minimizes an energy functional with two different components: the fidelity part, $-\log p(f|u)$, can be further specified by MAP estimation and EM algorithm; the penalty part, $-\log p(u)$, can be designated as the combination of the first and second order total variation [6],[10] for detail preserving and artifact elimination.

Making use of the expression (3) and the assumptions on the bias field function, all the intensities $f(t)$ within a neighborhood O_s share the same PDF $p_\xi(z,\Theta)$. For simplification, we introduce some notations as follows.

$$w_1(s) = \frac{\gamma_1}{2b(s)\sigma_1^2}, w_2(s) = \frac{\gamma_2}{\sqrt{2\pi b^2(s)\sigma_2^2}},$$

and

$$g = g(x,s) = \left|\frac{f(x)}{b(s)} - Hu(x)\right|. \quad (6)$$

By the independency assumption, we have a local negative log-likelihood functional in the neighborhood

$$\mathcal{E}_s(\Theta,u) = -\int_{O_s} \log\left(w_1 e^{-\frac{g}{\sigma_1^2}} + w_2 e^{-\frac{g^2}{2\sigma_2^2}}\right) dx. \quad (7)$$

Then introduce a Gaussian weighting function to identify the contribution of different points in the neighborhood, the local energy becomes

$$\mathcal{E}_s(\Theta,u(s)) = -\int_{O_s} G_\sigma(\mathrm{s}-\mathrm{x}) \log\left(w_1 e^{-\frac{g}{\sigma_1^2}} + w_2 e^{-\frac{g^2}{2\sigma_2^2}}\right) dx. \quad (8)$$

where $G_\sigma(\cdot)$ is a symmetric Gaussian kernel with a standard deviation σ such that $G_\sigma(t) \approx 0$ as $t \notin O_s$.

Then, expanding the local integral domain to the whole domain Ω and considering the global information of the given data, we have the following energy functional

$$\mathcal{E}(\Theta,u) = -\iint_\Omega G_\sigma \log\left(w_1 e^{-\frac{g}{\sigma_1^2}} + w_2 e^{-\frac{g^2}{2\sigma_2^2}}\right) dxds. \quad (9)$$

To perform estimation on Θ, we utilize the EM algorithm. Introducing a vector-valued auxiliary variable $\varphi = (\varphi_1, \varphi_2)$ in

$$\Delta = \{\varphi | 0 < \varphi_i < 1, \textstyle\sum_{i=1}^{2} \varphi_i = 1\}, \quad (10)$$

and a functional

$$\Phi(\Theta,u,\varphi) = \iint G_\sigma \left(\frac{\varphi_1}{\sigma_1^2} g + \frac{\varphi_2}{2\sigma_2^2} g^2 - \varphi_1 \log \frac{\gamma_1}{2b\sigma_1^2} - \varphi_2 \log \frac{\gamma_2}{\sqrt{2\pi b^2 \sigma_2^2}} + \sum_{i=1}^{2} \varphi_i \log \varphi_i \right) dxds$$

Given Θ^0, u^0, compute the minimizer of $\Phi(\Theta,u,\varphi)$ via the following alternating minimization scheme

$$\begin{cases} \varphi^{v+1} = \arg min_{\varphi\in\Delta} \Phi(\Theta^v, u^v, \varphi) \\ (\Theta^{v+1}, u^{v+1}) = \arg min_{\Theta,u} \Phi(\Theta, u, \varphi^{v+1}) \end{cases} \quad (11)$$

where v denotes the inner iteration number.

It can be verified that the sequence defined by (11) has the following properties. $\mathcal{E}(\Theta,u)$ decreases with respect to v, that is,

$\mathcal{E}(\Theta^{v+1}, u^{v+1}) \le \mathcal{E}(\Theta^v, u^v)$.

Moreover, $\mathcal{E}(\Theta,u)$ and $\Phi(\Theta,u,\varphi)$ possess a same global minimizer of Θ.

In our context, the original image is always piecewise smooth, we then define a weighted regularizer in the form of

$$R(\nabla u, \nabla^2 u) = \int_\Omega \big(w(x)|\nabla u| + \big(1 - w(x)\big)|\nabla^2 u|\big) dx, \quad (12)$$

$w(x)$ is a weighting function based on edge detection

Combining (12) with $\Phi(\Theta,u,\Psi)$ in (11), denoted by $\Psi(\Theta,u,\varphi^{v+1})$, we obtain an alternating minimization model (BiLGM-TVBH) for image reconstruction and bias field correction, i.e.,

$$\begin{cases} \varphi^{v+1} = \arg min_{\varphi\in\Delta} \Psi(\Theta^v, u^v, \varphi) \\ (\Theta^{v+1}, u^{v+1}) = \arg min_{\Theta,u} \Psi(\Theta, u, \varphi^{v+1}) \end{cases} \quad (13)$$

III. ALGORITHM AND NUMERICAL EXPERIMENTS

The algorithm of our proposed alternating minimization model is based on the variable splitting technique and the alternating direction method of multipliers (e.g., [16][17]). The minimization problem for(Θ,u) can be further separated into two stages, that is, parameters and bias field estimation and image reconstruction.

By introducing auxiliary variables, the proposed unconstrained minimization problem can be reformulated by a constrained sequence of convex optimization subproblems, which can be solved separately by the augmented Lagran-

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gian method with some modification on its penalty parameters. Our main idea of the modification is to employ anisotropic diffusion by introducing some adaptive terms in the equation related with the reconstruction. And then, the corresponding minimization problem of the Lagrangian functional is further divided into several sub-problems, from which the parameters Θ and the reconstructed version of u can be solved iteratively.

By routine computation, we get

$$\varphi_1^{v+1} = \frac{\frac{\gamma_1^v}{2(\sigma_1^2)^v}\exp\left(-\frac{|F^v|}{(\sigma_1^2)^v}\right)}{\frac{\gamma_1^v}{2(\sigma_1^2)^v}\exp\left(-\frac{|F^v|}{(\sigma_1^2)^v}\right)+\frac{\gamma_2^v}{\sqrt{2\pi(\sigma_2^2)^v}}\exp\left(-\frac{|K^v|}{2(\sigma_2^2)^v}\right)}, \tag{14}$$

$$\varphi_2^{v+1} = 1 - \varphi_1^{v+1}. \tag{15}$$

where

$$F^v = f\int_\Omega \frac{G_\sigma}{b^v}ds - Hu^v,$$

$$K^v = \int_\Omega G_\sigma\left|\frac{f}{b^v} - Hu^v\right|^2 ds.$$

For simplicity, we only mention the solution of the subproblem for updating b

$$b^{v+1} = \frac{-A_2^{v+1}+\sqrt{\left(A_2^{v+1}\right)^2+4B_2^{v+1}}}{2}, \tag{16}$$

where

$$A_2^{v+1} = \frac{1}{2}\int_\Omega \sum_{i=1}^2 \frac{\varphi_i^{v+1}}{(\sigma_1^2)^v} G_\sigma \frac{f^2}{b^v}dx, \tag{17}$$

$$B_2^{v+1} = \frac{1}{2}\int_\Omega \sum_{i=1}^2 \frac{\varphi_i^{v+1}}{(\sigma_1^2)^v} G_\sigma f^2 dx, \tag{18}$$

Some numerical experiments are performed to validate our proposed model and its algorithm on both bias field correction and image enhancement. Figures 1-4 show the results for bias field correction and detail preserving. We find that our method can also preserve details and no ringing artifacts occur near the edges in the reconstructed MR images.

IV. CONCLUSION AND FUTURE WORK

We presented a new higher-order variational model that is supposed to reconstruct images which are corrupted by mixed noises, and adjust the intensity inhomogeneity in the images simultaneously.

The alternating minimization model is solved by an efficient ADMM-based algorithm, which reduces the solution to a sequence of convex optimization subproblems. Numerical experiments demonstrate the effectiveness of the proposed model, especially for images in MR.

The main advantage of our proposed model is that it provides a framework for reconstructing images corrupted by mixed noise and degraded by bias field. This is of importance since in some real applications the physics of image acquisition yields noise distributions are the outcome of several noise sources and bias field correction is also indispensable for the image analysis.

Future work will perform experimental comparison with the variational models on the same objective. Furthermore, the possibility to extend the proposed model to deal with Poisson noise and to color images will be considered.

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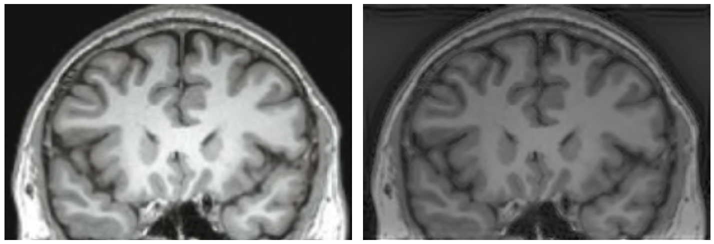
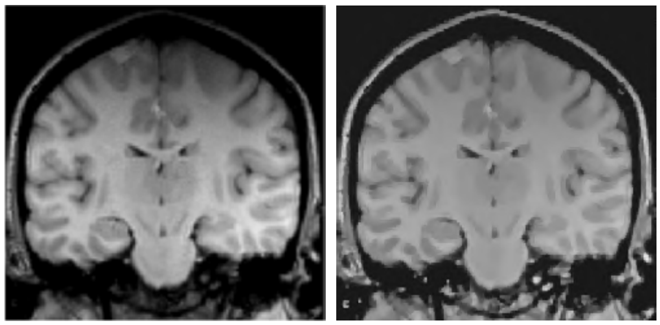


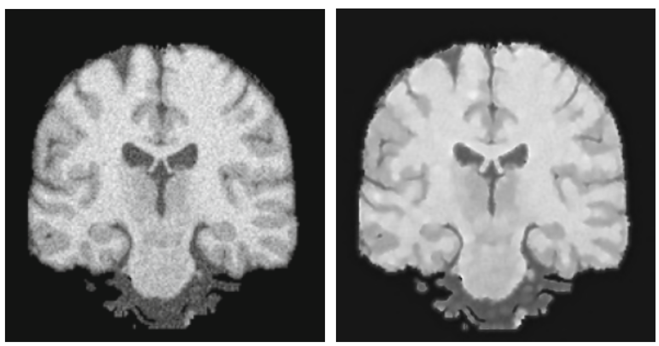
(a) (b)

Figure 1. Numerical experiment. (a):Original MRI image, (b): A reconstructed version of (a) obtained by our proposed model.



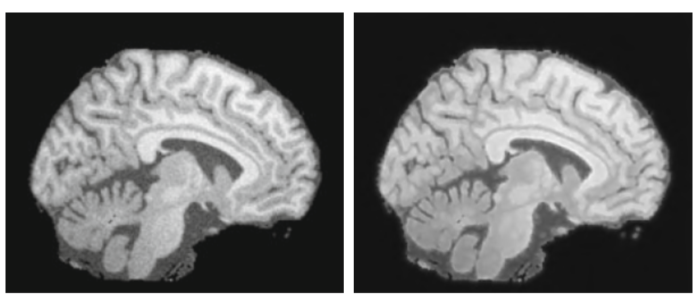
(a) (b)

Figure 2. Numerical experiment. (a):Original MRI image, (b): A reconstructed and enhanced version of (a) obtained by our proposed model.



(a) (b)

Figure 3. Numerical experiment. (a):Original MRI image, (b):A restored and corrected version of (a) obtained by our proposed model.



(a) (b)

Figure 4. Numerical experiment. (a):Original MRI image, (b):An enhanced version of (a) obtained by our proposed model.

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