Numerical Simulation of Ocean Ice Dynamics using Hybrid FE/FV Methods

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***Abstract*— Hybrid Finite Element/Finite Volume (FE/FV) methods have been employed to study oceanic circulation in a simplified domain consisting of both non-moving and moving ice floes. Our hybrid FE/FV flow solver combines the merits of both finite element and finite volume methods, and is highly sophisticated, robust and is a first of its kind approach extended for studying ocean ice dynamics and dispersion. Sea ice dynamics is one of the key components of ocean circulation models. The ultimate goal through the present project is to develop a highly accurate ice dynamics model that can be used to predict ice-edge positions, velocities for offshore operations, short term forecast for waterways, and also for long term global climatic studies. Preliminary results show that hybrid FE/FV methods can be successfully extended for studying ocean ice dynamics, through coupled implementation of automatic mesh movement. Movement of an isolated ice floe through the ocean and also waves impacting multiple dynamic ice floes are successfully simulated while maintaining the mesh integrity.**

Keywords-Shallow water; Hybrid FE/FV; Ice dynamics; Mesh movement.

I. INTRODUCTION

Sea ice plays a crucial role in the Arctic region impacting navigational shipping routes, military, costal guard applications, weather forecasting, and also offshore drilling platforms. With release of greenhouse gases into atmosphere and associated global warming, the Arctic region has become that much more important and accurate studies of Arctic Ocean ice dynamics have become highly imperative. Arctic region can be classified arbitrarily as central Arctic where sea ice is continuous and also into a marginal ice zone (with individual ice floes) which is an interfacial region between Open ocean and Frozen Central Ocean. Marginal Ice Zone (MIZ) is of particular interest due to its proximity to shipping routes and also as a threat to offshore drilling structures. However, modelling sea ice dynamics in MIZ, where individual ice floes are of arbitrary shapes and much more mobile and fluid, is a highly complex and challenging task.

MIZ region has received significant attention over the past few decades and literature in this area is thoroughly discussed in recent reviews by Squire et al. [1][2]. Within the MIZ region, researchers focussed on either continuum ice models, where MIZ is assumed to have certain rheological properties a priori (like a granular material), or on accurately modelling individual ice floes [2]. Within the second group, most of the studies are still limited to theoretical works, numerical models, and recently to spectral methods and Laplace transforms [3][4]. Direct numerical simulation studies in the MIZ region are relatively scarce due to the huge challenges involved in simulating individual ice floes requiring mesh movement, simulating complex wave-floe, floe-floe interactions and also due to the computing power required in realistic simulations of large regions of MIZ. The present work is the first of its kind to the author's knowledge in simulating sea ice dynamics using hybrid finite element/volume (FE/FV) methods. Due to the highly challenging nature of the problem, the present study is being conducted in a systematic way by employing the hybrid FE/FV methodology to ocean ice dynamics with varying degrees of complexity. As a first step, in this work, circular waves impacting both non-moving and moving ice-floes in simplistic oceanic conditions are simulated. Presently, only translation motion is implemented. Section II details the governing equations. Numerical methods that are used to solve the governing equations are discussed in Section III. Results are presented in Section IV and Section V highlights the conclusions and future work.

II. GOVERNING EQUATIONS

A. *Shallow Water Equations*

The Shallow Water Equations (SWEs) [5] are derived by depth-averaging the Reynolds averaged Navier-Stokes equations for a column of fluid with mass and momentum conservation. In SWEs, it is assumed that vertical motions are negligible and that pressure is hydrostatic. The depth and velocity of fluid moving in the domain $\mathbf{x}(x,y)\in\Omega$ with boundary $\partial\Omega=\partial\Omega_g+\partial\Omega_h$ during the time interval $t\in(0,\mathrm{T})$ in non-conservation form can be described by,

$$\frac{\partial H}{\partial t}+\nabla\cdot\ H\mathbf{u}\ =\dot{n} \qquad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t}+\ \mathbf{u}\cdot\nabla\ \mathbf{u}=-g\nabla H-\nabla\left(\frac{p_a}{\rho_0}+gZ\right)+\frac{\nu}{H}\nabla\cdot\nabla\ \mathbf{u}H \qquad (2)$$

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where $H, \dot{n}, \mathbf{u}, g, p_a, \rho_0, \nu$ and Z are the water depth, net source term, velocity, gravity, surface pressure, fluid density, kinematic viscosity and surface elevation, respectively. Figure 1 demonstrates the terminology to describe *H*, *h*, and Z .

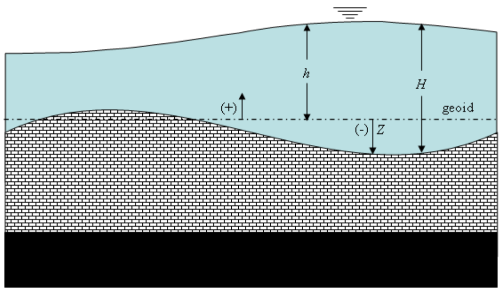

Figure 1. Shallow water problem description.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -g\nabla H - \nabla\left(\frac{p_a}{\rho_0} + gZ\right) + \nu(\nabla\cdot\nabla)\mathbf{u} + \frac{\nu}{H}\mathbf{u}(\nabla\cdot\nabla H) \tag{3}$$

Here, we expanded the viscous term from conservation form to non-conservation form. In our hybrid method we store the variable H at the node and the velocity $\mathbf{u}$ at the element center. Using linear interpolation function will result in constant gradient and zero Laplacian for working variable H . As a result, we can drop the last term in (3). Also, while our solver includes the capabilities for studying wind stress, tide, and Coriolis forces [5] they have been omitted in the present study and also from above equations.

B. Linear Elasticity Equations for Mesh Moving

Mesh moving equations [6] from linear elasticity are described below:

$$\nabla.\sigma = F \tag{4}$$

$$\sigma = \lambda(\nabla.X)I + 2\mu\varepsilon \tag{5}$$

$$\varepsilon = \frac{1}{2}\left[(\nabla X) + (\nabla X)^T\right] \tag{6}$$

where σ is the stress tensor, ε is the strain tensor, F is the body force per unit volume, λ and μ_l are the lame parameters, and X is the displacement vector.

III. NUMERICAL METHOD

A. Hybrid FE/FV Methodology

The time discretization of Eq. (3) using backward difference will yield

$$\frac{\alpha_1\mathbf{u} + \alpha_0\mathbf{u}^n + \alpha_{-1}\mathbf{u}^{n-1}}{\Delta t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \nu(\nabla\cdot\nabla)\mathbf{u} = -g\nabla H - \nabla\left(\frac{p_a}{\rho_0} + gZ\right) \tag{7}$$

where both $\mathbf{u}$ and H are unknowns at time step $n+1$. For first order time accurate scheme, $\alpha_1 = 1.0$, $\alpha_0 = -1.0$ and $\alpha_{-1} = 0.0$ and for second order time accurate scheme $\alpha_1 = 1.5$, $\alpha_0 = -2.0$ and $\alpha_{-1} = 0.5$. The hybrid FE/FV scheme evolves by perturbing H such that

$$H \rightarrow H + H' \tag{8}$$

where H' is very small compared to H . The time discretized momentum equation will lead to (4)

$$\frac{\alpha_1\tilde{\mathbf{u}} - \alpha_1\mathbf{u}}{\Delta t} + \frac{\alpha_1\mathbf{u} + \alpha_0\mathbf{u}^n + \alpha_{-1}\mathbf{u}^{n-1}}{\Delta t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \nu(\nabla\cdot\nabla)\mathbf{u} = -g\nabla H - g\nabla H' - \nabla\left(\frac{p_a}{\rho_0} + gZ\right). \tag{9}$$

Here, we introduced $\tilde{\mathbf{u}}$ which is the final velocity at time $n+1$ in the iterative nonlinear scheme. In this context, $\mathbf{u}$ will be intermediate velocity field during the nonlinear iteration and the balance between $\tilde{\mathbf{u}}$ and $\mathbf{u}$ will be enforced through the gradient of H'. This process is similar to the projection methods commonly used to solve incompressible Navier Stokes equations [7][8][9][10]. Note that as $H' \rightarrow 0$, then $\nabla H' \rightarrow 0$ which ensures $\mathbf{u} \rightarrow \tilde{\mathbf{u}}$. We use the fractional time splitting method to first compute an intermediate velocity from Eq. (3) which is the predictor step and then use the results in the correction phase described as:

$$g\frac{\Delta t}{\alpha_1}\nabla H' = \mathbf{u} - \tilde{\mathbf{u}} \tag{10}$$

We can observe that Equation (3) used in the predictor phase is time discretized momentum equation in its original form. Clearly, the predictor phase satisfies consistency criteria and conserves the momentum. To derive the continuity wave equation, we multiply Eq. (10) by H and then take the divergence to obtain

$$g\frac{\Delta t}{\alpha_1}\nabla\cdot(H\nabla H') = \nabla\cdot(H\mathbf{u}) - \nabla\cdot(H\tilde{\mathbf{u}}) \tag{11}$$

Since the last term in Eq. (11) includes the final velocity at time step $n+1$, we can replace it by its equivalent in

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continuity equation. The time discretization of continuity equation with perturbed H is

$$\nabla\cdot(H\tilde{\mathbf{u}}) = \dot{n} - \frac{\alpha_1 H + \alpha_1 H' + \alpha_0 H^n + \alpha_{-1} H^{n-1}}{\Delta t} \tag{12}$$

We combine (11) and (12) and obtain time-discretized wave equation. The results can be written as

$$\begin{aligned} &H' + \frac{\Delta t}{\alpha_1}\nabla\cdot(H'\mathbf{u}) - \frac{\Delta t^2}{\alpha_1^2}\nabla\cdot(C^2\nabla H') \\ &= -\frac{\Delta t}{\alpha_1}\left[\frac{\alpha_1 H + \alpha_0 H^n + \alpha_{-1} H^{n-1}}{\Delta t} + \nabla\cdot(H\mathbf{u}) - \dot{n}\right]. \end{aligned} \tag{13}$$

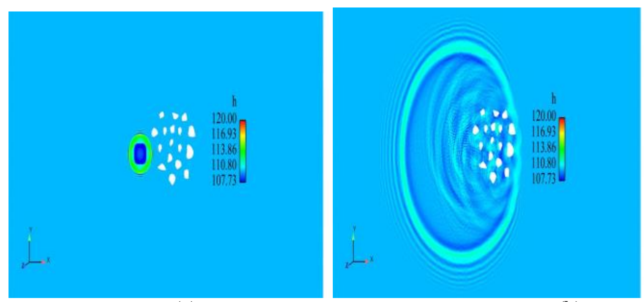
where $C = \sqrt{gH}$ is the wave speed. It can be seen that the right hand side of (13) is weighted by time discretized continuity equation. Therefore, as $H' \rightarrow 0$, (13) will yield zero residual for continuity equation. Clearly, Eq. (13) satisfies consistency criteria and conserves the mass. We use the cell-centered finite volume method (FV) to solve the momentum equation for the intermediate velocity and the node-based Galerkin finite element method (FE) to solve the wave equation and also for the elasticity equations. From velocity and water depth, forces acting on the individual ice floes are calculated which are used to solve the linear elasticity equations for mesh displacement using finite element method. In our deployment, the velocity unknowns are put at the cell centers and water depth variable is put at the mesh vertices. This deployment makes it convenient to compute the gradients of water depth using local finite element basis function, which is required in solving the momentum equations. Previous numerical results have shown that our hybrid implementation is super convergent in terms of the spatial convergence rates [7][8][9]. Unlike our previous compressible/incompressible flow solvers, the present hybrid FE/FV has not yet been parallelized. For realistic simulations of large regions of MIZ however, the flow solver will eventually be parallelized in future studies.

IV. RESULTS

To test our hybrid FE/FV methodology, non-moving ice floes were first studied, followed by moving ice-floe simulations. In the present simplistic study, idealized conditions are maintained by ignoring wind, tide, and Coriolis forces. Additionally, the ocean bed elevation was assumed to be flat ($Z = 0$).

A. Non-moving Ice-Floes

Wave-ice interactions in a 10 Sq-Km ocean domain with simplified initial/boundary conditions are studied utilizing our hybrid FE/FV flow solver. At the far open ocean boundary, symmetric boundary conditions are applied on all four sides. Ice floes are initially assumed to be rigid, non-moving, and wave effects are analysed on both uniform and also non-uniform randomly placed ice floes. Different simplified artificial forcing mechanisms are imposed to study wave effects on ice floes. Figure 2 below shows circular tsunami type waves impacting rigid, non-uniform, randomly placed ice floes, where water height is plotted at different non-dimensional times. Present hybrid FE/FV extensions combine the merits of both finite element and finite volume methods and are particularly suitable for high aspect ratio grids around ice floes and also for solving incompressible flows. As shown in Figure 2, wave features are well resolved bouncing back after impacting the ice floes. Validation and benchmark comparisons of our hybrid FE/FV methodology for shallow water equations can be found in the work of Aliabadi et al. [9].



(a) (b)

Figure 2. Circular waves impacting randomly distributed non-uniform, non-moving ice floes at different non-dimensional time's.

B. Moving Ice-Floes (Automatic Mesh Movement)

Having tested the flow solver on non-moving ice floes, automatic mesh movement was implemented in the flow solver by solving linear elasticity equations. Figure 3 below shows the grid around an isolated circular ice floe moving with a given constant velocity. As it can be seen from the figure, mesh refinement around the circular floe is well maintained as it moves in the ocean domain.

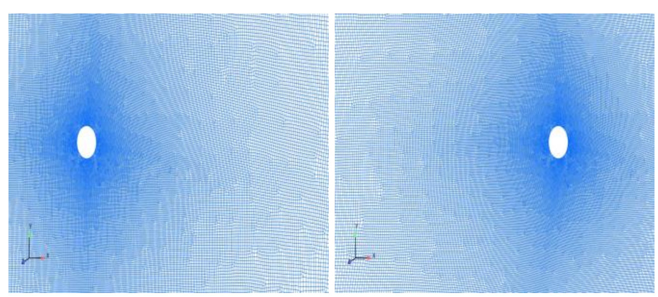
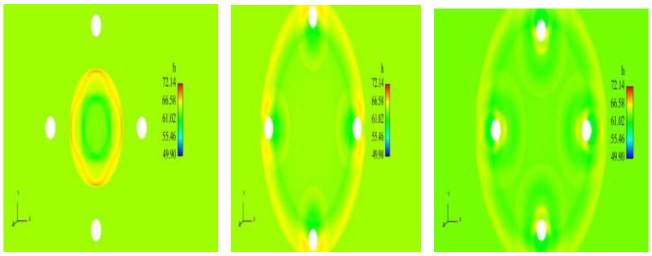


Figure 3. Isolated circular ice floe moving with a given constant velocity.

Circular waves impacting multiple circular ice floes are shown in Figure 4, where zoomed in portion of the ocean domain around the ice floes is shown. It can be seen that the waves sway the ice floes back and forth as they pass through.



(a) (b) (c)

Figure 4. Circular waves impacting uniformly placed movable ice floes at different non-dimensional time's (a) t = 2, (b) t = 5, (c) t = 6

V. CONCLUSIONS AND FUTURE WORK

Hybrid FE/FV methods have been successfully extended for studying ocean ice dynamics in idealized conditions. Automatic mesh movement was also successfully implemented, with well resolved and captured wave features and also grid refinement around the ice floes. Present hybrid FE/FV simulations are the first of its kind in direct simulation of individual ice floe-wave interactions. Mesh moving studies that are presently being conducted are based on the movement of a single and also multiple ice floes perturbed using different forcing mechanisms. Ongoing and potential future work in this highly challenging area is described below (not in any particular order).

- Moving isolated ice floes with different shapes (circular, square, non-uniform) and sizes will be conducted to more thoroughly understand the influence of shape/size on the ice motion through impinging waves.
- While experimental data in large MIZ regions is severely limited, select recent works have focussed on calculating the drift velocity for individual ice floes through laboratory experiments [11][12] and also by theoretical studies [13][14][15]. Single floe studies are crucial for thoroughly analysing the kinematic response of ice floes with characteristic lengths comparable or lower than the impact wavelengths. Isolated ice-floe impacts with offshore structures are observed to be one of the common ice-structure interaction events [11][16]. However, literature and guidance on the impact forces, based on which off-shore structures can be designed, appears to be severely limited. Results from the present study will therefore be compared with available data. Ice floe dynamics studies, such as in the present work, can play a significant role in estimating the forces from single/multi-year ice impacts on offshore platforms.
- Multiple ice-floe studies need to account for floe-floe interactions requiring constitutive relationships.
- Apart from the ocean waves, ice mobility in the MIZ, can also be influenced by wind stress, current, pressure and tides which will be incorporated in future studies.
- Realistic simulation of large regions in the MIZ will require tremendous computing power. Therefore, parallelizing the hybrid FE/FV shallow water flow solver is an important goal for future studies.

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