# StaCo: Stackelberg-based Coverage Approach in Robotic Swarms

Kateřina Staňková, Bijan Ranjbar-Sahraei, Gerhard Weiss, Karl Tuyls
Department of Knowledge Engineering
Maastricht University
Email: {k.stankova,b.ranjbarsahraei,gerhard.weiss,k.tuyls}@maastrichtuniversity.nl

***Abstract*—The Lloyd algorithm is a key concept in multi-robot Voronoi coverage applications. Its advantages are its simplicity of implementation and asymptotic convergence to the robots' optimal position. However, the speed of this convergence cannot be guaranteed and therefore reaching the optimal position may be very slow. Moreover, in order to ensure the convergence, the Hessian of the corresponding cost function has to be positive definite all the time. Validation of this condition is mostly impossible and, as a consequence, for some problems the standard approach fails and leads to a non-optimal positioning. In such situations more advanced optimization tools have to be adopted. This paper introduces Stackelberg games as such a tool. The key assumption is that at least one robot can predict short-term behavior of other robots. We introduce the Stackelberg games, apply them to the multi-robot coverage problem, and show both theoretically and by means of case studies how the Stackelberg-based coverage approach outperforms the standard Lloyd algorithm.**

***Keywords*—*Swarm robots; Coverage control; Lloyd algorithm; Game theory; Stackelberg games***

## I. Introduction

In recent years many researchers in robotics, control, and computer science have focused on swarm robotics and have developed solutions of fundamental *swarm robotic* problems (see [1] for solving flocking control problem, [2] for aggregation, [3] for multi-robot coverage, and [4] for formation). However, most of the proposed solution methods encounter difficulties in real-world applications, such as finding only sub-optimal solutions and the inability of the algorithms to account for non-convex environments. Subsequently, despite the wide range of existing works in the domain of multi-robot coverage [3], [5]–[9], there are still only very few in-field deployments, due to a wide gap between the theory of multi-robot coverage systems and the practice.

The Stackelberg Coverage (StaCo) approach proposed in this paper addresses the deficiencies of the existing works in multi-robot coverage, by adding one or more relatively advanced robots, called leaders, to the swarm. In other words, we assume a priori a heterogeneous robotic swarm, similar to that shown in Figure 1. In this figure, two intelligent robots act as the leaders, which can perceive the environment globally, and a large swarm of simple robots following simple local rules. The main advantage of such a heterogeneous approach is preserving the simplicity of the major population of the robotic swarm, while a small group of robots can predict behavior of the others and act so that the desired behavior is achieved faster and with a higher precision. The main building block of our approach is the so-called *Stackelberg game theory* [10], [11], which belongs to the more general noncooperative game theory [10], [12]. Game theory has been successfully applied in various fields; its known applications in the robotic field relate to pursuit-evasion and search problems [13], [14]. However, application of the Stackelberg games in the multi-robot coverage is new.

The remainder of the paper is structured as follows: A motivation example of classical coverage limitations will be presented in Section II. In Section III we will briefly review the game-theoretic preliminaries and introduce Stackelberg games. In Section IV the Voronoi-based coverage problem will be defined as a Stackelberg game and its properties will be discussed. The simulation setup and the results of applying the proposed approach will be presented in Section V. In Section VI we will discuss the advantages of the StaCo approach and give concluding remarks.

## II. Motivation

A motivaton example, which illustrates the limitations of classical approaches in mutli-robot coverage, is shown in Figure 2. The group of robots, initiated in the position depicted in Figure 2a, moves based on the standard coverage approach suggested in [3]. With this approach, the robots are driven to the final configuration shown in Figure 2b. However, this configuration is sub-optimal (The globally optimal solution will be found adopting the StaCo approach proposed in this paper in Section V, Figure 7c). The problem of being enmeshed

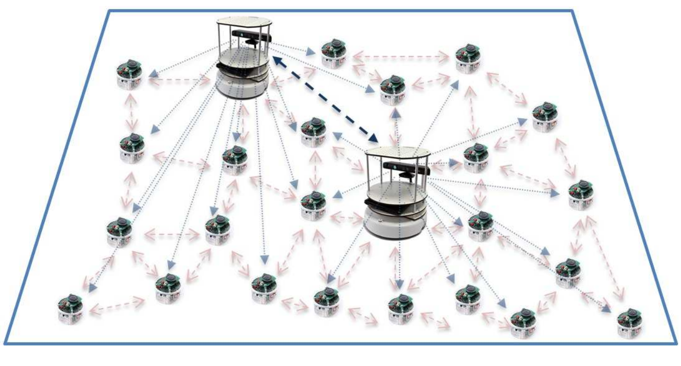

Figure 1. A heterogeneous robotic swarm with 2 leading robots and 34 following robots. The followers can collect information only from their neighbors, while the leaders are capable of collecting information from the entire robotic swarm. The leaders may be able to predict possible future reactions of the followers and to enforce their own decisions on the followers.

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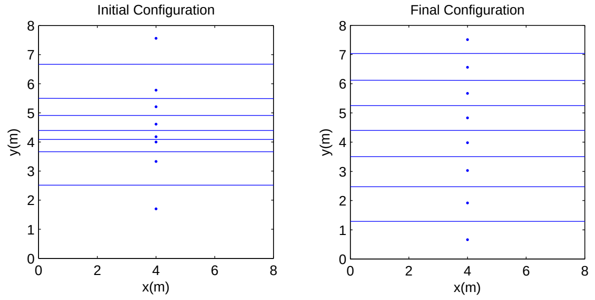


(a) (b)

Figure 2. Example of the problem in which the standard coverage approach leads to only a locally optimal but not the globally optimal configuration (dots denote robot locations and lines denote boundaries of the Voronoi regions): (a) initial configuration, (b) final configuration achieved by approach suggested in [3]; this final configuration is suboptimal

in a local optimum can also be seen in non-convex environments (e.g. in presence of obstacles). With the motivation to avoid such complications and to speed up the procedure of finding the global optimum, we introduce the StaCo approach. Adopting this approach, the majority of the swarm consists of simple robots following local rules introduced in [3], while one or two more advanced robots (leaders) improve the system performance by taking different actions, taking the decisions of the others into account. Consequently, the decentralized behavior of the swarm and the simplicity of most robots is preserved, while overall system performance is significantly improved.

## III. Basics of Stackelberg Games

Let us explain basics of Stackelberg games by the following static example.

**Example III.1.** *(Two-player static game) Let two players* $L$ *and* $F$ *have decision variables* $u_L \in \mathbb{R}$ *and* $u_F \in \mathbb{R}$*, respectively. Let functions* $J_L : \mathbb{R}^2 \to \mathbb{R}$ *and* $J_F : \mathbb{R}^2 \to \mathbb{R}$ *be smooth and strictly convex on* $\mathbb{R}^2$*. Player* $L$ *chooses* $u_L \in \mathbb{R}$ *in order to minimize her cost* $J_L(u_L, u_F)$*, while player* $F$ *minimizes* $J_F(u_L, u_F)$ *by choosing* $u_F \in \mathbb{R}$*. Illustration of this situation is given in Figure 3, where level curves (contours) of* $J_L$ *and* $J_F$ *are depicted in* $(u_L, u_F)$*-plane. If there is no hierarchy between* $L$ *and* $F$ *(i.e., if they either act simultaneously or if they do not know how the other player acts), the Nash equilibrium applies [15]. In such a situation,* $L$ *and* $F$ *would pick* $u_L^{(N)}$ *and* $u_F^{(N)}$*, respectively, where* $u_L^{(N)} = \arg\min_{u_L} J_L\big(u_L, u_F^{(N)}\big)$*,* $u_F^{(N)} = \arg\min_{u_F} J_F\big(u_L^{(N)}, u_F\big)$*. The outcome of the game would be* $J_L\big(u_L^{(N)}, u_F^{(N)}\big)$ *and* $J_F\big(u_L^{(N)}, u_F^{(N)}\big)$ *for* $L$ *and* $F$*, respectively (i.e., the values of* $J_L$ *and* $J_F$ *evaluated at point* $N = \big(u_L^{(N)}, u_F^{(N)}\big)$*, which is the* Nash solution (equilibrium) *of the game. Note that in Figure 3, point* $N = \big(u_L^{(N)}, u_F^{(N)}\big)$ *lies on the intersection of the curves* $R_L(u_F)$ *and* $R_F(u_L)$ *defined by* $\frac{\partial J_L}{\partial u_L} = 0$ *(bold dotted curve) and* $\frac{\partial J_F}{\partial u_F} = 0$ *(bold dashed curve), respectively.*

*Let us now consider a different situation: Player* $L$ *(in this new situation referred to as* leader*) knows* $R_F(u_L)$ *(bold dashed curve) in advance and can act first. In such a situation it is better for the leader to choose* $u_L^{(S)} = \arg\min_{u_L} J_L\big(u_L, R_F(u_L)\big)$ *instead of* $u_L^{(N)}$*. Subsequently, follower* $F$ *chooses* $u_F^{(S)}$ *(there is no other* $u_F$ *for which* $J_F\big(u_L^{(S)}, u_F\big) < J_F\big(u_L^{(S)}, u_F^{(S)}\big)$*). Point* $S = \big(u_L^{(S)}, u_F^{(S)}\big)$ *is then the* Stackelberg solution (equilibrium) *of the game and* $J_L\big(u_L^{(S)}, u_F^{(S)}\big)$*,* $J_F\big(u_L^{(S)}, u_F^{(S)}\big)$ *are Stackelberg outcomes of this game for the leader and the follower, respectively [10], [11].*

We will now generalize the example. Let us state first the assumptions that we raise on the cost functions and decision spaces in the static game:

(A1) Let $\Gamma_L$ and $\Gamma_F$ be convex compact sets, referred to as decision spaces for the leader and follower, respectively.

(A2) Let $J_L : \Gamma_L \times \Gamma_F \to \Gamma_L$ and $J_F : \Gamma_L \times \Gamma_F \to \Gamma_F$ be strictly convex smooth functions on $\Gamma_L \times \Gamma_F$, referred to as costs for the leader and follower, respectively.

Imposing assumptions (A1) and (A2), we provide following definitions:

**Definition III.2.** *(Optimal response set in the static game) Under assumptions (A1) and (A2), the set* $R(u_L) \subset \Gamma_F$ *defined for each strategy* $u_L \in \Gamma_L$ *of* $L$ *by* $R(u_L) = \{\xi \in \Gamma_F : J_F(u_L, \xi) \le J_F(u_L, u_F), \forall u_F \in \Gamma_F\}$ *is the* optimal response set *for* $F$*.*

**Definition III.3.** *(Stackelberg strategy in the static game) Under assumptions (A1) and (A2) and with* $R(u_L)$ *unique for each* $u_L \in \Gamma_L$*, strategy* $u_L^{(S)} \in \Gamma_L$ *is called a Stackelberg equilibrium strategy for* $L$ *if* $J_L\big(u_L^{(S)}, R(u_L^{(S)})\big) = \min_{u_L \in \Gamma_L} J_L\big(u_L, R(u_L)\big)$*.*

The existence and uniqueness of Stackelberg strategy is discussed in following lemma:

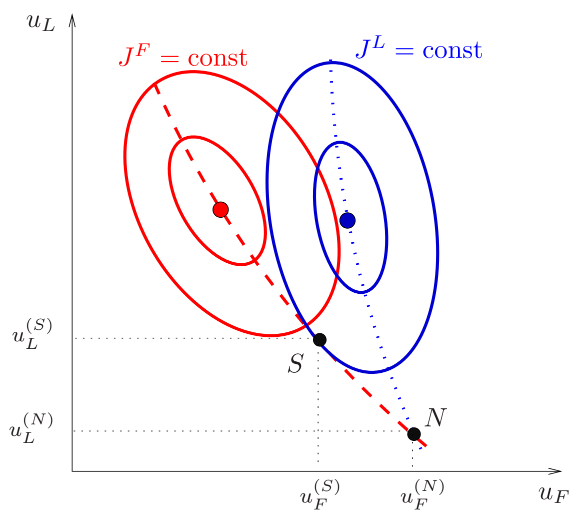


Figure 3. Illustration of the difference between Stackelberg (S) and Nash (N) equilibrium solutions. When compared to the Nash equilibrium, under the same conditions of the game the Stackelberg equilibrium never leads to higher costs for the leader (provided that they both exist). Moreover, there are situations in which the Stackelberg equilibrium concept might be more profitable for the follower as well, as this figure illustrates.

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**Lemma III.4.** *(Existence and uniqueness of Stackelberg strategy) Every two-person static game with leader $L$ and follower $F$, where (A1) and (A2) hold, admits a unique Stackelberg strategy for the leader.*

**Proof**. *If $\Gamma_L$ and $\Gamma_F$ are convex compact sets and $J_L$ : $\Gamma_L \times \Gamma_F \to \Gamma_L$ and $J_F : \Gamma_L \times \Gamma_F \to \Gamma_F$ are strictly convex smooth cost functions, then $R(u_L) \subset \Gamma_F$ by Definition III.2. Existence and uniqueness of the Stackelberg strategy directly follow from Definition III.3.*

To conclude this short introduction, we state the obvious property of the Stackelberg outcome.

**Lemma III.5.** *(Stackelberg outcome versus Nash outcome in a static game) For a two-person static game with leader $L$ and follower $F$, where assumptions (A1) and (A2) hold, $J_L\big(u_L^{(S)}, u_F^{(S)}\big) \leq J_L\big(u_L^{(N)}, u_F^{(N)}\big)$.*

If the decisions of the players and the state of the system evolve in time, while each of these decisions and the state of the system influence (also future) decisions and states, we refer to the game as the *dynamic* game. Without going into too much detail, we state that theory introduced in this section for static games can be extended into the dynamic setting, in both discrete-time dynamic and continuous dynamic cases, under additional assumptions on the system dynamics. For an overview of theory of Stackelberg games with varying information each of the players might know, see [10], [11], [16], [17]. Moreover, a Stackelberg game can also be played among one leader $L$ and multiple followers $F_1, \ldots, F_M$, where the leader, having complete information about the state, cost functions, and dynamics of the followers can impose her decision on the followers at each time step $k \in \{1, \ldots, N\}$ (resp. each time $t \in [0, T]$) in the discrete and continuous case, respectively.

## IV. StaCo approach

In this section we formulate multi-robot coverage problem as a dynamic Stackelberg game with one leader and multiple followers. The approach proposed in this section will be referred to as StaCo: Stackelberg-based Coverage Approach.

Let us consider $M$ robots (players) positioned at time $t = 0$ in convex polytope $\Omega \subset \mathbb{R}^2$. One of the players, denoted for the sake of simplicity as player 1, is the *leader*, other players, denoted by $2, \ldots, M$, are the *followers*. Let $\mathbf{x}(t) \stackrel{\text{def}}{=} \{x_1(t), x_2(t), \ldots, x_M(t)\}$ be the configuration of the robots at time $t$, with $t \in [0, T]$, $\mathbf{x}(0) = \{x_1(0), x_2(0), \ldots, x_M(0)\}$ being the a priori given initial configuration of the robots and $\mathbf{x}(T) = \{x_1(T), x_2(T), \ldots, x_M(T)\}$ being their final configuration at final time $T$, with $x_i(t) \neq x_j(t)$ if $i \neq j$. Let $V_i(t)$ indicate the Voronoi region (cell) in which $i$-th robot is located at time $t$. For each $\mathbf{x}(t)$ the Voronoi regions are defined by the *Voronoi partition* of $\Omega$, $\mathcal{V}(t) = \{V_1(t), \ldots, V_M(t)\}$ generated by the points $\mathbf{x}(t) = \big(x_1(t), \ldots, x_M(t)\big) : V_i(t) = \{\omega \in \Omega : \|\omega - x_i(t)\| \leq \|\omega - x_j(t)\|, \quad \forall j \neq i\}$. System dynamics (with state variable $\mathbf{x}$) are given by the following system of ordinary differential equations:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \ldots, M \tag{1}$$

where $u_i(t)$ is the control (decision) of the $i$-th robot at time $t$. The cost functions for the leader (robot 1) at time $t$ is given by

$$C_1(t) = \sum_{i \in \{1, \ldots, M\}} \int_{V_i(t)} \|\omega - x_i(t)\|^2 \mathrm{d}\omega. \tag{2}$$

Let $T$ be the stopping time, i.e. the minimal time such that for each $\tau > T$ the cost $C_1(\tau)$ does not change: $T = \min\{t : C_1(\tau) = C_1(T) \text{ for } \forall\ \tau > T\}$. Then the leader minimizes $C_1(T)$. The cost function for the follower $j \in \{2, \ldots, M\}$ at time $t$ is

$$C_j(t) = \int_{V_j(t)} \|\omega - x_j(t)\|^2 \mathrm{d}\omega. \tag{3}$$

The problem of the leader (robot 1) can be then defined as

$$(P_{\text{StaCo}}) \begin{cases} \text{Find} \quad u_1^{(S)}(t) = \underset{u_1(t)}{\arg\min}\, C_1(T), \text{ w.r.t.} \\ u_j(t) = \underset{u_j(t)}{\arg\min} \int_{V_j(t)} \|\omega - x_j(t)\|^2 \mathrm{d}\omega. \\ \dot{x}_i(t) = u_i(t), \end{cases}$$

with $j = 2, \ldots, N$, $i = 1, \ldots, N$.

The solution strongly depends on the so-called information pattern, i.e., on the amount of information that each player knows and recalls over her own state, state of the others, and action made by herself and the others during the game.

**Proposition IV.1.** *Let at time $t$ each player $i$ know only state $x_i(t)$ and corresponding $V_i(t)$ and let Hessian of* (2) *be positive definite at each $t$. Then the so-called continuous-time Lloyd descent [3]*

$$u_i^*(t) = \kappa \left( \frac{\int_{V_i(t)} \mathbf{x}\, \mathrm{d}x_i}{\int_{V_i(t)} \mathrm{d}x_i} - x_i(t) \right), \tag{4}$$

*$\kappa > 0$, asymptotically converges to minimal $C_1(T)$ for player 1 and to minimal $C_j(T)$ for $j = 2, \ldots, M$.*

*Proof:* As shown in [3], $u_i^*(t)$ defined by (4) with respect to $\dot{z}_i(t) = u_i(t)$ converges asymptotically to the set of critical points of (2). The critical points of (2) coincide with critical points of (3). If corresponding $V_i$ is finite, this solution is global due to positive definiteness of (2) [18]. ∎

**Remark IV.2.** *Note that validating the positive definiteness of* (2) *is an open problem [3] and even if the convergence to the global optimum is guaranteed, in general no guarantees on the speed of this convergence exist. This leads us to the question whether there exist algorithms that perform better than the classical Lloyd algorithm if we allow the leader (robot 1) to have more information about the state and decisions of the followers.*

**Proposition IV.3.** *Let player 1 know $x_j(\tau)$ and $u_j(\tau)$ (for all $j \neq 1$) for $\tau \in [t, t+\Delta]$, with $\Delta > 0$, where $u_j(t)$ is defined by* (4). *Let $u_1^{(S)}(t)$ denote the optimal control of player 1, possibly dependentent on $u_j(\tau)$, $\tau \in [t, t + \Delta]$. Let $T^\Delta$, and $C_1^\Delta(T^\Delta)$ denote the corresponding stopping time and the final payoff for player 1 in such a situation, respectively. Then $T^\Delta \leq T$ and $C_1^\Delta(T^\Delta) \leq C_1(T)$.*

*Proof:* The leader's decision is not bounded by any restrictions. Setting this decision to (4) leads to $T^\Delta = T$,

$C_1^{\Delta}(T^{\Delta}) = C_1(T)$. Note that the Hessian of (2) might not be positive definite with the leader's decision defined by (4). Thus, $u_1^{*,S}(t)$ either coincides with (4) or, if this choice would lead to only sub-optimal solution, $u_1^{*,S}(t)$ differs from (4) and leads to a better outcome. This result also follows from extension of Lemma III.5 into dynamic setting with the state equation (1). ∎

Giving more information to the leader almost always leads to the better outcome for the leader also in a very general setting [10], [11], while the StaCo approach never leads to the outcome worse than that reached by standard methods [3]. In the next section we will illustrate that when the classical Lloyd algorithm fails and leads to only a local optimum, the StaCo approach can find the global solution. For the case studies in the next section, the time and space are discretized and therefore the leader can choose from a limited number of decisions at each time step $k$.

## V. Case Studies

In this section, we will study the performance of the proposed StaCo approach in comparison with the classical Voronoi-based coverage approach.

### A. Simulation Setup

To simulate StaCo and compare it with the standard approach, we have developed a 2D robot simulator. This simulator is written in Java and supports simple massless robot motion. The environment $\Omega$ to be covered in all simulations is a $8$ m $\times$ $8$ m square and the speed of each robot is limited to $4$ cm/s. The time discretization of the system is $0.4$ s.

The designed simulator supports Voronoi cell computation for each robot. In each time step, firstly the locations of robots $x$ are used to compute the Voronoi cell of each robot and subsequently the centroid of each cell is computed and used by the robots to find the gradient descent direction (4). With the StaCo approach the robot closest to the center of $\Omega$ is considered as the leader. In each time step, instead of following the gradient descent direction (4), the leader first discretizes its surrounding space into a limited number of accessible locations (in our simulations 8 points on a circle of radius $1.5$ cm, with equal distances to each other). Then for moving to each of these locations, the leader predicts the possible moves of other robots, in one or two time steps, and chooses the movement, which minimizes $C_1$ (i.e. the best possible response to the other robots).

In order to measure the performance of the StaCo approach and to compare it with the performance of the classical coverage techniques, we introduce the *Settling Time* as the time required for the cost function (2) of the whole swarm to enter and remain within a prespecified error boundary. More precisely, we define the settling time $T_s$ as

$$T_s \overset{\text{def}}{=} \min \left\{ T_s \in [0, T_f] \middle| \forall t > T_s : \left| \frac{C_1(T_f) - C_1(t)}{C_1(T_f) - C_1(0)} \right| < \epsilon \right\} \quad (5)$$

where $\epsilon = 0.05$, $C_1$ is defined by (2), and $T_f$ is the simulation stopping time, i.e. the minimal time such that the cost $C_1(.)$ doesn't change.

As an example of the simulation setup, Figure 4a shows an initial configuration of a robotic swarm of $8$ robots. Both

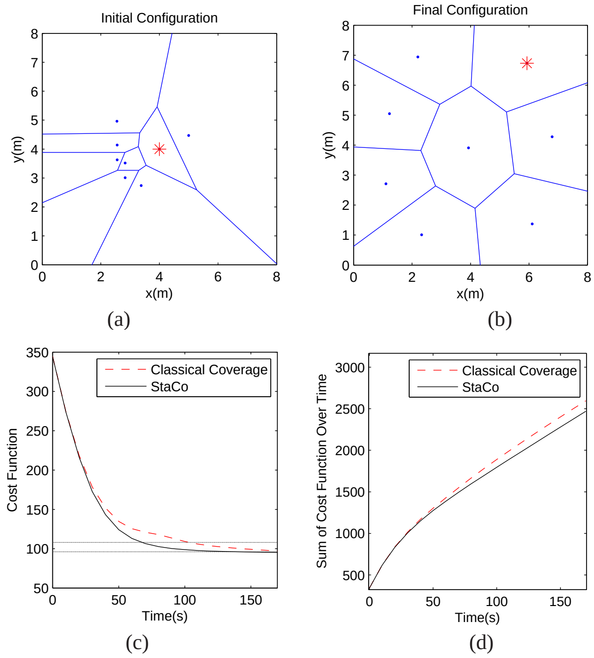


Figure 4. Comparison of performance of the proposed StaCo approach and the standard approach for a particular configuration: (a) initial configuration (b) final configuration (c) coverage cost function (d) cost function summed up over time.

the StaCo and the classical coverage approaches are applied to this configuration; with the StaCo approach the leader makes a prediction of the swarm behavior for one subsequent time step. The final configuration after $170$ s is shown in Figure 4b. Clearly, both methods reach the same final configuration; however, as shown in Figure 4c, StaCo reaches the final configuration faster than the classical approach. Finally, in Figure 4d, the cost functions for both techniques are summed up over the time. This figure shows that the StaCo approach converges to the optimal configuration faster than the classical approach. The settling time of both approaches can be easily measured via the horizontal lines in Figure 4c (The upper and lower lines refer respectively to $C_1(T_s)$ and $C_1(T_f)$, which denote the $0.05$ error bound). Therefore, in this particular case study, the settling time for the StaCo approach is $75$ s, and for the classical coverage approach it is $105$ s.

### B. Effect of Swarm Size

In order to compare both techniques in a more generic way, we have applied our simulation to groups of $2$ – $20$ robots, $20$ times for each swarm size, with random starting configurations. The convergence settling times for both techniques were accurately measured based on (5). Their statistical representation is illustrated in Figure 5. In this figure, the average value, the minimum, and the maximum of the settling time over $20$ runs are plotted with respect to the swarm size. From Figure 5 we can conclude that the StaCo approach performs better compared to the classical coverage approach. For certain initial configurations both methods achieve the final configuration with the same settling time, while for the majority of possible initial configurations the StaCo approach performs better. Such behavior is observed in simulations and is also supported by the theoretical arguments in Sections III and IV.

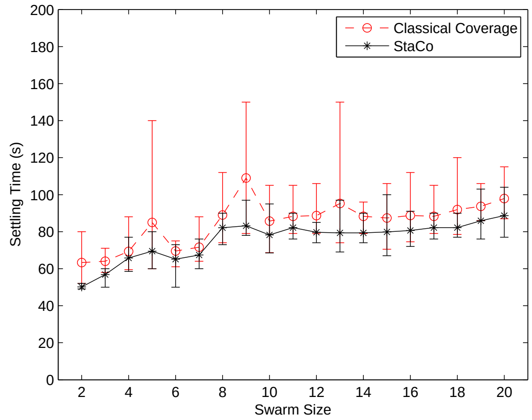


Figure 5. Comparison of the coverage settling time between the proposed StaCo approach and the classical coverage approach for robotic swarms of different sizes.

### C. Effects of Leader's Speed and Prediction Horizon

Firstly, we examine the effect of the leader's speed on the performance of StaCo. Secondly, we will investigate how the number of prediction steps influences the performance.

We employ a robotic swarm with eight robots. For each initial configuration, we increase the speed of the leader from 4 cm/s up to 16 cm/s in steps of 2 cm/s, while the followers' maximum speed remains 4 cm/s. Each simulation is repeated 20 times from random initial configurations. Afterwards, the simulations are repeated with the leader's prediction horizon being increased to up to 2 subsequent time steps, with varying leader's speed.

The results presented in Figure 6 show that increase of the leader's speed and prediction horizon can improve the convergence performance of the StaCo approach.

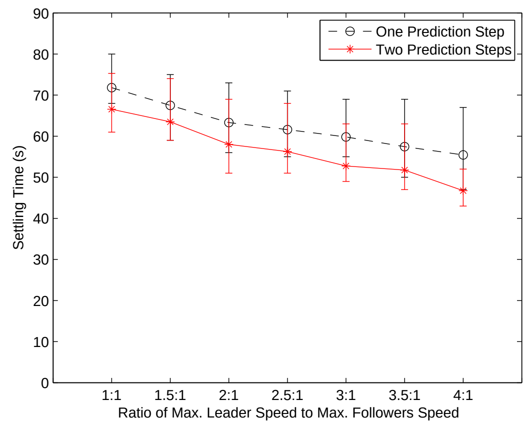


Figure 6. Coverage settling time of a robotic swarm of one leading robot and seven following robots for different leader's speeds, while the leader makes predictions for one or two future time steps.

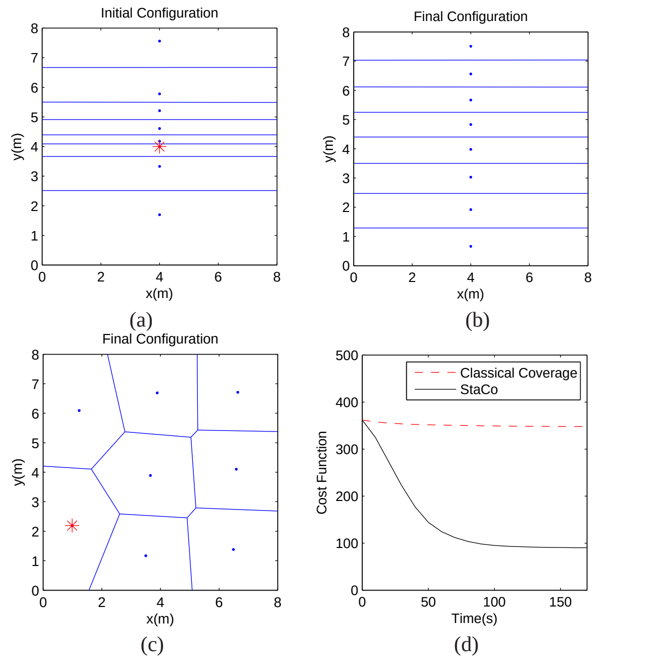


Figure 7. Comparison of coverage performance between the proposed StaCo approach and the standard coverage approach for an initial configuration close to a sub-optimal configuration: (a) initial configuration; (b) final configuration for standard coverage approach; (c) final configuration for the StaCo approach; (d) comparison of the cost functions.

### D. Escaping sub-optimal configurations

In StaCo approach the leader is able to perceive global information about the position of all swarm robots. This ability may help the swarm to escape from sub-optimal configurations. A sample initial configuration, already discussed in Section II, is shown in Figure 7a.

This initial configuration is very close to a suboptimal case, which is achieved if each robot moves a bit up or down and settles in the center of its rectangular Voronoi cell. Although the classical coverage approach terminates in this local minimum immediately (see Figure 7b), it is very easy for the StaCo approach to escape from this local minimum. The final configuration achieved by the StaCo is shown in Figure 7c. Comparison of costs over the time are illustrated in Figure 7d. Clearly, the StaCo approach performs much better.

Similarly to the results depicted in Figure 7, starting from any other initial configuration close to a sub-optimal configuration, the standard coverage approach will result in this sub-optimal position. The perception capabilities of the leader in StaCo allow for finding the globally optimal configuration.

## VI. Discussions and Conclusions

This article addressed the multi-robot coverage problem and presented a new approach called StaCo, which is based on the game-theoretic concept of Stackelberg games. StaCo takes advantage of the high perception capabilities of a small group of robots (leaders) among a large group of simple robots (followers) and allows for a very efficient coverage performance. No communication among the robots takes place. The leader(s) choose(s) a position in such a way that the other robots will, by optimizing their own objectives, improve the overall configuration of the system. Therefore, this approach is a non-intrusive way to steer the system into a desirable direction and leads to fast and effective coverage of an environment.

StaCo always performs at least as well as the classical approach, mostly StaCo performs better. This outcome was shown both theoretically and by means of case studies. Moreover, StaCo is able to escape from sub-optimal configurations when the classical approach is doomed to fail.

A possible limitation of the StaCo approach is that currently there is no explicit form of the optimal Stackelberg solution of the game due to the complexity of the cost function of the leader; however, its derivation is a subject of our ongoing research.

StaCo opens a promising new research avenue: Using heterogenous robotic swarms for coverage in complex scenarios such as those with non-convex environments (environments with obstacles or with non-convex boundaries). As described in many existing works, accomplishing a swarm robotic mission in a non-convex environment is a difficult task. However, the authors believe that the StaCo approach can be very successful in such scenarios.

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