EC4MAS: A Multi-Agent Model With Endogenous Control for Combinatorial Optimization Problem Solving

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***Abstract*—Since a couple of years, new approaches are proposed to solve combinatorial optimization problems: multi-agent systems. In this paper, we propose a new model, EC4MAS, to build self-organizing multi-agent systems with more endogenous control. We start presenting a representative set of solving methods and we highlight what are the key elements of these solving processes and how they are used to construct a new representation of the problem to solve it. Generally, this representation is based on the characteristics of the method implemented but the construction of this representation could happen in the system without so much external intervention. This has been illustrated by some work in psychology that we present. Based on this observation, we propose and illustrate in this work a self-organizing multi-agent approach that tries to construct itself this representation, in an endogenous way. It is organized into a social organization of the different local solving behaviors and a spatial organization that represents the different structural/topological characteristics of the problem. The objective of the system is thus to find a good coupling between these two organizations to get the best possible representation.**

***Keywords*-multi-agent system; self-organization; endogenous control.**

I. Introduction

In this paper, we present a model for more endogenous control in multi-agent systems for combinatorial optimization problem solving. First, we will present standard methods to solve this kind of problems. We highlight how they try to construct a new representation of the problem. Then we define how to create such representation in a multi-agent system to control it. We propose to set up an endogenous control system in order to design self-organizing systems not limited to a single kind of problem. Such control allows the system to adapt its own behavior according to its environment. This is similar to the natural process of learning and cognitive development which allow an individual to create his own knowledge to adapt more easily to problems. Our model introduces this development in multi-agent systems through the use of social representations.

We developed our model on this basis and we expose in this paper how we define all the elements to get a suitable representation for the control. Our model to create Endogenous Control for self-organized Multi-Agent System, EC4MAS, is based on a social and a spatial organization of agents and their coupling. These two organizations provide informations about the current solving strategy and on its outcome while the coupling allows the control system to dynamically adapt its own strategy.

Section II is a description of standard methods for optimization problem solving. Section III explains the notion of representation in these methods and informatics and psychology. The proposed model is then defined in Section IV. Section V presents and discusses results of experiments on a graph coloring problem. Section VI concludes the presented work and draws some perspectives.

II. Solving combinatorial optimization problems

Combinatorial optimization is a domain which main interest is the solving of complex problems with high combinatory structure. We point out in this section well-known solving methods for this kind of problem.

Constructive approaches are initialized with a partial solution, generally empty, and try to build a complete solution widening the partial solution one variable at once. In these methods the only possible mean to direct the search is often the next variable to assign and its value. Like in greedy or backtracking methods some heuristics could help to determine the next variable to consider. Branch and bound method [1] is an implicit enumeration of the solution space, that is all the possible solutions can be examined but thanks to pruning techniques it can avoid to explore large subsets of bad solutions. These methods use local information and do not consider the global optimality making these more approximative ones.

Methods using the concept of neighborhood start with a complete assignment, which is not necessarily a solution, and make some changes to reach a different configuration. Changes needed to obtain new configurations define the possible neighbors, it is called the neighborhood function. Local search or Tabu search [2] are basic ones. In these methods one variable is changed at one time and we could use mechanisms like a tabu list to forbid examination of previous variables in a fixed period of time to get out from local optimum more efficiently. In Simulated annealing [3], neighbors are generated, evaluated and selected or not. Acceptation of a neighbor is conditioned by its improvement level and the moment of the search. A

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temperature is decreased and used to get acceptation level, it is the mean to control and direct the evolution of the search. In these methods locality and local optimum are also problematic and mechanisms such as tabu list have to be used to improve the quality of solutions found. Simulated annealing tries to deal with an important problem, the balance between exploration and exploitation.

Evolutive algorithms are based on individual natural evolution principle. They are based on a population which is a set of individuals where each one represents a possible solution, an evaluation function which measures the adaptation level of one individual to its environment and an evolution process with some operators. An initial population is randomly generated, then each individual is evaluated and some of them are selected, finally new individuals are created using the evolution process. Algorithms using evolution strategies had been initially proposed by [4]. In Genetic algorithms [5] evolution operators, such as crossover and mutation are applied at random on one or several selected individuals. Genetic programming [6] uses a coding no more generic but specific to the handled problem, so is the only operator used, mutation. Here, population is the basis for the solving like in multi-agent systems but the use of individuals in these two approaches are different. Evolutive approaches make the population evolve in a centralized manner and select some individuals to survive which is not always the case in the multi-agent systems where agents are autonomous.

Besides standard solving methods, there are many other methods which want to use complex systems characteristics, like distribution of the solving process, and they need to be considered from this point of view. We can cite Ant colony optimization algorithms [7] and Particle swarm optimization [8]. Ant colony optimization algorithms are based on the ant natural behaviors where ants could solve a problem (finding the shortest path) indirectly communicating with only pheromones, it is called stigmergy. In Particle swarm optimization the first objective is to represent social interactions between agents which have a given objective in a common environment. It is important to notice that these methods are not evolutionary ones at the literal sense because they use cooperation between individuals instead of competition and finally no selection is done on the population. They are quite similar to multi-agent systems from this point of view.

III. Representation and control

In this section, we explicit the global principle used to solve a problem which is to construct a new representation of it.

A. Construction of a representation for the solving process

When we want to build a solving method, we have to construct a new representation of the problem to work with. This new representation is a mean to understand the problem and to define all the elements we want to use to get a solution.

First, the search space and its characteristics (roughness, dynamic, wideness ...) is used as a support for the solving process since it defines the elements to consider.

Then, the neighborhood function defines how the solving process gets from a solution to another, and is directly dependent on the search space characteristics.

Finally, the evaluation function, based on the nature or the type of expected objective, optimal or not for example.

B. Psychology and control

Creating a representation is a way to better understand a problem in order to solve it with limited capabilities. This principle is the one used by real individuals to solve real problems in their life. In this case the representation can be seen as the intelligence of this individual.

Cognitive development has been studied by Piaget [9]. He said that intelligence is no more than a more elaborate form of biological adaptation of an individual to its environment. It is a continuous process that rebalances structures of intelligence (schemas and operations) using two parallel processes, assimilation to interpret new facts and accommodation to change the cognitive structure.

Jean-Claude Abric [10] defines social representation "*as a functional vision of the world, which allows the individual or group to make sense of his actions, and understand the reality through its own reference system, so to adapt to it, to find its place in it*". It can be interpreted as a decision-making tool as "*It becomes the framework by which the rest of attitudes and judgments are adjusted so that everyone is on the same line as the group*" [11] so it is an attractor from individuals' point of view and it controls them. For more formal details we refer the reader to [12].

C. How to use social representation for endogenous control

Control in a self-organizing multi-agent system guides agents' choices. This guidance can not be exclusively based on purely local information but on a wider vision because it is involved in a group of agents which purpose can be seen as creating and maintaining a social representation.

Looking at the characteristics of social representations and those of an endogenous control, we see that these two concepts have much in common. Both systems have the task to control individuals' decisions in the group, dynamic and emerging characteristics. On this basis of strong ties we propose to use them to build a new model for endogenous control for multi-agent systems.

IV. Endogenous control for multi-agent system

In this section we present the construction of our model for endogenous control in self-organizing multi-agent based complex problem solvers (EC4MAS), its components, the social organization, the spatial organization and the coupling, and how they interact.

A. General organization of the model

Like seen in the previous section, to solve a problem we have to build a new representation of it, so the solving process can understand it to find a solution. The endogenous characteristics of our control means that a minimum, or in the best case no, external interventions are needed to build the control system (or representation). In order to define the general organization of our model, or the general organization of the representation of the problem, we use the problem decomposition of Fig. 1.

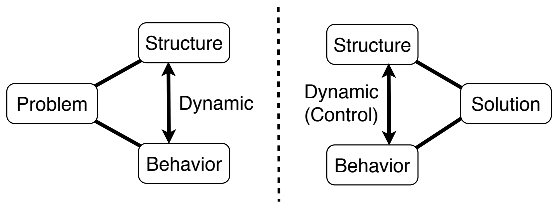

Fig. 1: Problem and solution organization

A problem is based on three key elements. First we can get informations on its structure which regroups directly available informations, like variables, domain of the variables, direct constraints and so on. The behavior mostly regroups indirect informations like influences between variables, indirect constraints or search space structure. These characteristics of the problem define how the problem will behave when we use its structure to solve it. Finally, the dynamic of the problem is the link between its structure and its behavior, it appears with the solving and it is the mechanism that links the elements of the structure to the ones of the behavior.

We used this problem organization to define the global organization of our model. The representation structure of the solution is used to model topological/structural characteristics of the current solution. The representation of the behavior of the solution is used to model the solving strategy currently used by the solving system. The dynamic which can be seen as the control of the solution is used to couple the structure and the behavior of the solution, to permanently adapt the current strategy to the current solution.

In a multi-agent system, all the agents only have access to limited informations and the global solution/strategy emerges from all the local acts or interactions of agents, so we model the structure and the behavior with organizations of agents. An organization can model an agent situation/strategy, or role, in the context of a particular situation/strategy and can use these relations to mark the mutual influences between them.

B. Structure: spatial organization

The structure or spatial organization is used to model the current situation in the search space. To get spatial informations on the problem structure we have to define several sensors. These sensors are used by the agents to perceive their spatial environment, so to define their spatial role. Agents can also communicate informations of these sensors to their neighbors. The spatial role reflects the current position of the agent in the search space and allows it to apprehend the difficulty of its situation. The spatial organization which can be observed and interpreted is based on:

- a set of m spatial roles $Rsp = \{Rsp_1, ..., Rsp_m\}$
- a configuration $Csp = \{a_1, ..., a_j\}$ with $Csp \in SP$ where a_i is an agent state and SP is the search space
- a function $fRsp : SP \to Rsp$

The organization of a group of agents in the environment or physical organizations of agents Csp, can highlight basic characteristics of the problem relevant to the solving. In order to capitalize these informations, it is necessary to allow their identification and use by the system. The spatial organization is based on the $fRsp$ function which associates a spatial role to an agent from the current spatial configuration. This function uses sensors, given to the agents to capture their situation, to determine the Rsp. The sensors could be specific to the problem to solve or more generic.

Spatial roles are about the effect of the solving process in the physical environment. The role is essentially descriptive of the problem and the situation of the system during its solving. The spatial organization connects particular configurations of the problem. In some cases, the problem definition may give access to specified elements to define spatial roles such as topology of the graph for graph coloring. In other cases this information is not identifiable from the outset but may appear during the search.

C. Behavior: social organization

The multi-agent system with multiple interacting agents, must be addressed in a more extended view than of a single agent. The overall activity is dependent on all individual actions but also on the interactions between agents. The group of agents is a reflection at a given time of the search activity of the system. This activity has to be captured by the system and has to be used to direct and control the research. Observation of these perceptions is defined by:

- $RSo = \{Rso_1, ..., Rso_n\}$, a set of n social roles
- $Lso = \{Lso_{11}, ..., Lso_{nn}\}$ where $Lso_{ij} = (Rso_i, Rso_j)$,
  $\forall i, j \in [1, ..., n]$, a set of relations between social roles
- $\forall Lso_i \in Lso, Cso = \{Lso_1, ..., Lso_i\}$, a set of social contexts
- a social organization $Oso =< Rso, Lso >$

Roles Rso act as guides for the agent to determine the appropriate strategy and dictate it a predefined behavior. The adoption of a social role by an agent implies that it adopts the guidelines and directives of that role. The action of the agent, so its social role choice, could have been influenced by other agents, this result in the relation Lso_{ij}. A relation Lso_{ij} exists if an agent with the role Rso_i has encountered another agent with the role Rso_j. The social organization involves a set of roles Rso and relations Lso between them. The activity of a single agent can not be isolated from other agents and therefore the social roles are linked within the organization. The social organization Oso gives information on situation of the agents within the solving process at a given time. The situation of an agent, and more precisely the adequacy of its social role, is directly dependent on its environment. To locate an agent of a social perspective, relations between it and its neighbors define a context Cso. The context is a part of the social organization, with a limited size around one particular agent. It provides information on relations between different social roles So in a particular situation of the search.

Formally, the social organization is used to represent the solving strategy of the system. Relations of social roles are based on the activity of the system, and more particularly, on the action of the agents. This dynamic updates permanently the organization to adapt the search.

D. Dynamic: coupling

Social and spatial organizations both provide information of a different nature. The first one is particularly interested in the mechanisms of the solving looking to the fittest behavior of the agents. The second one provides information on the status of the system in search space. These two elements are strongly linked because social roles define how agents act in the environment and spatial roles represent their situation in the environment. The coupling of these two organizations is defined by:

- a coupling function $fC : Cso \times Rsp \to \mathbb{R}$ with $\forall x \in Cso$ and $\forall y \in Rsp$, $0 \leq fC(x, y) \leq 1$
- a fitness function $fT : (Cso \times Rsp) \times Time \to \mathbb{R}$ where $Time$ is the number of cycles of the solving

The coupling fC is dynamic and allows the relations between spatial and social roles to evolve according to the fitness function fT evolution. To find the best couple *(Rso, Rsp)* for an agent in a given situation, is the key to success. This coupling is determined by evaluating and storing the pairs (social role, spatial role) created by agents during the search in the previous cycles (a cycle is an amount of time where each agent acts one time). A look back at previous choices with fT allows to update the coupling fC to adapt the control system.

E. EC4MAS principle

The general principle of our model is, to first represent the current situation (spatial organization), then to represent the current strategy (social organization) and finally, to couple these two organizations to permanently and dynamically adapt the solving process.

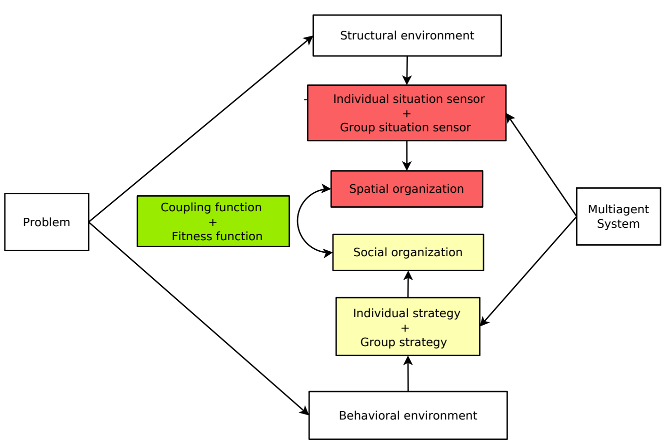

Fig. 2: EC4MAS Meta-Model

V. Results and discussion

In this section, we present an example of the use of two different implementations of EC4MAS to solve a graph coloring problem.

A. Simplified version of EC4MAS

This version of the model is qualified as simplified because some shortcuts had been made as the roles and the organizations are predefined and explicitly implemented in the system.

1) Experimental Setup: Agents of the system represent the nodes of the graph to color. A solution is found when all the agents have no conflicts with their neighbors, so each two connected nodes are assigned different colors.

The main solving strategy of the agents is based on the Min Conflict heuristic [13]. Two social roles are used, the first is the exploitation strategy and the second is the exploration strategy. Exploitation tends to decrease the number of conflicts between an agent and its neighbors. Exploration can randomly take a color or apply the exploitation strategy (Min Conflict with exploration). The social organization is modeled with a static tree, where one role is represented at a level. The children are based on the representation amount percentage of the role in a situation divided in two (less or more 50%).

Spatial roles are based on the degree of the nodes (static). Each role regroups nodes with similar degree. The spatial organization follows the graph topology.

Coupling is done using matrices to link social and spatial organization. Matrices could be found as leafs in the social organization tree. These matrices have one column per social role and one row per spatial role. The values in the matrix are float numbers between 0 and 1 and are normalized on the row. An agent finds from its spatial role and its social context the associated matrix to get a social role. Higher the value in a social column, higher the probability for the social role to be selected is.

2) Results and analysis: To test our model we generated 100 different graphs with different seed. We used equi-partite 4-colorable graphs with 300 nodes, that means that the 4 color sets have the same size or can only be different from one node. An edge connectivity (ec) of 0,02333 (7/300) is used to get hard problems like seen in [14]. Each solving is done 1000 times with a maximum of 1000 cycles. The performance is the number of cycles to get a solution, if no solution is found 1001 is used.

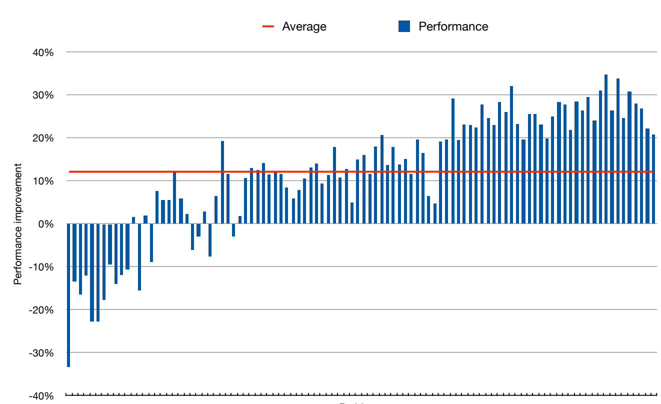

Fig. 3: Performance improvement of EC4MAS on 100 problems with 300 nodes, ec = 0,02333, 4 colors.

a) Performance: First, we randomly picked a problem (*ref-problem*) among generated graphs, a genetic algorithm is used to get coupling values and to find the best exploring rate for Min Conflict with exploration (17,7% for *ref-problem*). Then we used this tuning to solve all the generated problems. Fig. 3 shows the performance improvement on all the 100 problems which is the difference between Min Conflict with exploration results and EC4MAS results. The problems are ordered by the number of cycles of the solving process. We can see that EC4MAS can generally improve the solving on a set of problems with similar characteristics since the average improvement of EC4MAS is about 12,4%. There is a big difference of performance between easier problems (at the beginning) and more difficult ones (at the end). The number of cycles to solve *ref-problem* with EC4MAS is 234, most of the problems with a negative improvement are under this value. This is due to the coupling used for *ref-problem* which is a representation of the problem and gives specific informations on it. With problems much easier than that, the solving process is too specialized and instead of guiding the search process it introduces too much perturbations.

TABLE I: Performance and efficiency

Method	Perf.	Tuning time	Efficiency
Min Conflict (17,7%)	100%	4	-
Optimal Min Conflict	124,71%	333	1,50%
EC4MAS (17,7%)	112,14%	22	20,39%

b) Tuning: In addition to the performance gain, we also focus on the tuning time of the system. Table I presents the performance gain of three different tunings and the efficiency of each method. The Min Conflict with 17,7% of exploration is taken as a reference for the measures. The tuning time is the sum of the time to find the optimal exploring rate for each problem for Optimal Min Conflict, and is the time to find the optimal exploring rate and the coupling values for *ref-problem* for EC4MAS. In the first case the tuning time is dependent on the number of problems and their hardness, in the second case only on the hardness of *ref-problem*. We can see here that the performance is much higher with optimal exploring rate, about 2 times more than EC4MAS, but the tuning time (in minutes) is about 15 times higher. In the end, the global efficiency (performance gain divided by tuning time) of EC4MAS is almost 13 times higher than Optimal Min Conflict. This shows that EC4MAS can learn the characteristics of a particular problem and is able to use this knowledge to really well solve similar problems with a limited tuning cost.

c) Genericity: ECM4AS uses the degree of nodes to create spatial roles. EC4MAS has been developed to be as generic as possible. To illustrate the genericity we introduce here a new type of sensor for spatial role, the local clustering coefficient. It is a good indicator for graph coloring problem hardness as seen in [15]. This coefficient is based on triangles between neighbors of the node and the node itself.

Table II shows the performance on *ref-problem* with the Min Conflict with exploration, EC4MAS with degree and clustering coefficient for spatial role. We can see that the most specific sensor which is the clustering coefficient is more efficient than the others, about 17% than Min Conflict while the degrees are only 11,5% better than Min Conflict. EC4MAS is generic

TABLE II: Improvement of spatial roles over Min Conflict

Spatial role	Perf. (cycle)	Improvement
Degree	234	11,5%
Clustering coefficient	223	17,04%

so it could support several types of sensors for spatial roles, from more general one like degree to more specific one like clustering coefficient.

B. EC4MAS

We present a second version of EC4MAS with no explicit spatial/social roles and organizations.

1) Experimental setup: Organizations are seen here like graphs, oriented graph for spatial organization, weighted graph for social organization and weighted and oriented graph for coupling.

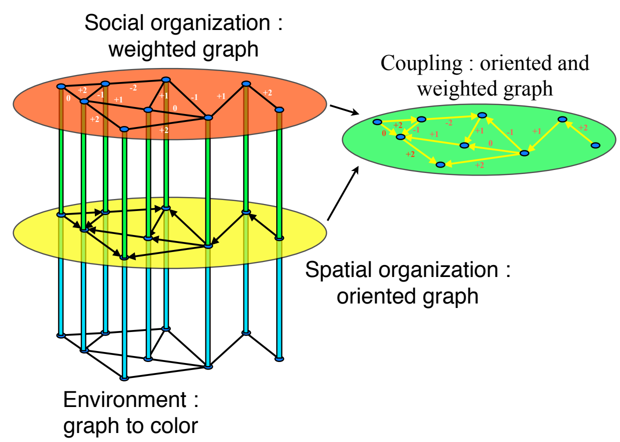

Fig. 4: Organizations of EC4MAS

In this version, spatial sensors are used to construct an oriented graph on the basis of the graph to color. They provide information on degree of the node and its color, so an agent can get a degree of freedom which can be interpreted as spatial role. When a color is selected by an agent, it may create some new conflicts. Social roles are interpreted as the global action of the agent when it chooses its color. If the color is not in conflict with the neighbors we can say that the conflicts are removed by the agent, if the number of conflicts of the agent is increased/decreased we can say that the agent add/remove some conflicts from the system. The impact of the choice of a color on the system is marked through the weights in the social organization (weight of the links between neighbors), increase of the weight if perturbations are removed and decrease of the weight if perturbations are created or transmitted. The coupling associates weights of the social organization to the oriented edges of the spatial organization, it represents the flow of the perturbations in the system. It may direct the perturbations through the best path (nodes and edges) to be sure that they will be removed as quickly as possible.

2) Results and analysis: We present here the first results of this version, further experimentations will be made in the

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future. We compared this version to a model exposed in [16]. In this model agents compute in a centralized way, a value for their environment in a fixed range to get informations on the possibles conflicts introduced by a specific color, as in EC4MAS agents use the coupling graph.

TABLE III: Improvement of EC4MAS over Min Conflict

Model	Perf. (cycle)	Improvement
MinConflict	211	
multi-agent for K-coloring	180	15%
EC4MAS	201	5%

Table III presents results with Min Conflict, the model described in [16] and E4CMAS. We can see that the specific model for k-coloring improves the solving by 15% while our model only over 5%. We can explain this result by the high dynamic of the system during the solving. The evolution of the weights in the social organization has to evolve quickly to be always adapted to the current strategy otherwise, it slows down the solving process providing it no more appropriate data. A good setting of the different changes of the weights have to be found to get optimal informations for the system. The flow graph, or coupling, gives informations to decrease the number of perturbations (or conflicts) in the system but it has to be updated as quick as the system to be controled evolves, which is not totally the case in this version.

VI. Conclusion and future work

In this paper, we present our model, EC4MAS, for more endogenous control in multi-agent based solvers for combinatorial optimization problems. We started from well-know methods of solving for this kind of problems and used them to define a general approach to solve combinatorial optimization problems. The main idea is to define a new representation of the problem more easily understandable and more adapted to the limited knowledge we have at the moment we conceive the solving process. This representation is based on the problem structure, its behavior and its dynamic. If we want to build a new representation of the problem we have to take into account these elements and to base the conception process on them. Some works in psychology have shown that this process is the one used by an individual to develop his intellect. This emergence of a representation could also be seen in groups of individuals to understand and work with some concepts.

Our model, EC4MAS, is based on theses observations and uses it to construct a control system in an endogenous way. This model is based on a social and a spatial organization and their coupling. These two organizations provide information on the current solving strategy of the system and on its result, and the coupling allows the control system to dynamically adapt the strategy of the system more efficiently.

EC4MAS is a generic model and could be used to solve different kind of problems. The spatial organization could be adapted to use specific informations to let EC4MAS be able to solve different kind of problems. On more harder problems EC4MAS gives good improvements since the characteristics of the problem are used to better tackle it. EC4MAS makes the tuning of the system robust in front of changes in a problem and the coupling of social and spatial organization provides pertinent solutions to already encountered situations with a specific strategy. The tuning has not been changed when new problems are submitted. This is a great point because individual optimization is very expensive and could not be always used, more particularly when problems are dynamic. The endogenous self-organized characteristic of EC4MAS could efficiently limit the amount of time and resources to solve a problem, this is shown by the simplified version.

The second presented version shows us the importance of the speed of the adaptation of the coupling (or control) system. To get the system oriented the coupling must be as quick as possible to construct the representation, because it has to represent the current situation/strategy and also the consequences of the previous ones to guide the system.

In Future work, we will focus on this speed characteristic and we will implement the model with new problems like the jobshop problem.

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