# Equilibrium Strategies for Interference Free Channel Access in Cognitive Radio based WRANs

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Abstract-In this paper, we study how cognitive radio based wireless regional area networks (e.g., IEEE 802.22 networks) can adapt themselves so that they can co-exist with each other. These networks opportunistically accesses and uses underutilized bands and relinquishes them when the primary user of that band initiates transmission. Such networks, deployed and operated by competing wireless service providers have to self-coexist among themselves by accessing different parts of the available spectrum. Since there is no coordination among the networks in accessing the spectrum bands, they have to adapt such that interference from neighboring networks is minimum. When interfered, a network can adopt either one of two choices- switch to a new band hoping to find a noninterfering band, or stay with its current band hoping that the interfering network(s) will move away to a new band. Using game theory, we model the spectrum band switching process as an 'infinitely repeating' game where the aim of each network (player) is to minimize its cost of finding a clear channel. We first explore the pure strategy solution space for the game and show that a pure strategy equilibrium, though possible, is infeasible to implement in reality. Thus, we further explore the mixed strategy space and propose a mixed strategy Nash equilibrium among the networks. We analyze the game for the 2-player case, where a network is in interference with only one other network. We also provide hints on how to obtain the equilibrium for the *n*-player game.

*Keywords*-IEEE 802.22 networks, self-coexistence, game theory, mixed strategy.

#### I. INTRODUCTION

Recent experimental studies that have conclusively shown that licensed radio spectrum is highly under-utilized and that the usage is space and time dependent [17]. In order to take advantage of the spectrum availability due to the analog to digital transition [19], the FCC in the United States has defined provisions to open the sub–900 MHz TV bands for unlicensed services. However, it is mandated for the unlicensed devices to detect and avoid interference with the licensed users in a timely manner [16]. Cognitive radio (CR) based IEEE 802.22 [15] is a wireless regional area network (WRAN), that is targeted to provide a solution to this problem [18]. The aim of IEEE 802.22 is to use

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spectrum bands dynamically through incumbent sensing and avoidance. For this reason, much of the standard of the IEEE 802.22 is based on cognitive radio. The basic operating principle relies on the cognitive radio being able to sense whether a particular band is being used and, if not, utilize the spectrum without interfering with the transmission of other users (primary incumbents). The elements in these networks (i.e., cognitive radio enabled devices) continuously perform spectrum sensing, dynamically identify unused spectrum, and operate in the spectrum band when it is not used by the primary incumbent radio systems. Upon detecting incumbents, cognitive radio enabled devices are required to switch to another channel or mode. This entails the need of cognitive radio techniques not only to detect the presence/absence of incumbent signals but also to cater to the more important issue of self-coexistence among the 802.22 networks. In a typical deployment, multiple 802.22 BSs and CPEs may operate in the vicinity of each other where they compete with each other for grabbing as much spectrum as possible. Different from other IEEE 802 standards where self-coexistence is not a problem, it is so for these networks.

In this paper, we construct an "infinitely repeated" game for the networks (players) for accessing spectrum in an interference free manner where the players always believe that there is some chance the game will continue to the next period. We analyze both pure and mixed strategy solution space for the game and show that a pure strategy equilibria, though possible, is infeasible to implement in reality. We propose a mixed strategy Nash Equilibrium among the players (networks), which does not require negotiation messages to be exchanged between the players.

There are several advantages to taking a game theoretic approach: such model works in a distributed manner where a centralized allocating mechanism is not needed thus making the system scalable. Being rational entities in the game, each of the IEEE 802.22 networks tries to maximize its own payoffs (In our case, minimize the cost of finding a clear spectrum band subject to constraints on resource usage). Furthermore, negotiation messages does not need to be exchanged among the networks. Thus, our solution abides by the assumption that a network does not have any information about the other networks.

The rest of the paper has been organized as follows. In section II we provide a brief discussion of the related works on dynamic spectrum access. Section III formulates the selfcoexistence problem as a dynamic channel switching game. The game is analyzed in detail in Section IV, exploring both pure and mixed strategy spaces and their corresponding equilibria. Conclusions are drawn in the last section.

#### II. RELATED WORK

As far as dynamic spectrum sensing and access are concerned, there is an emerging body of work that deal with different decision making aspects, issues and challenges in cognitive radio network setting. Energy detection have been largely used in [2], [4] to monitor primary spectrum usage activity. Spectral correlation based signal detection for primary spectrum sensing in IEEE 802.22 WRAN systems is presented in [5]. Signature-based spectrum sensing algorithms are presented in [1] to investigate the presence of Advanced Television Systems Committee (ATSC) DTV signals. In a similar effort, sequential pilot sensing of Advanced Television Systems Committee (ATSC) DTV signals is carried out in [9] to sense the primary usage in IEEE 802.22 cognitive radio networks. In [6], a novel frequency sensing method is proposed known as dynamic frequency hopping (DFH). In DFH, neighboring WRAN cells form cooperating communities that coordinate their DFH operations where WRAN data transmission is performed in parallel with spectrum sensing without interruptions. The aim here is to minimize interrupts due to quiet sensing. In [3], a novel metric called Grade-of-Service (GoS) is defined and the trade-off between miss-detection and false alarm is studied for optimizing spectrum sensing performance.

Though most of the above mentioned works focus on primary spectrum usage sensing, however, the issue of selfcoexistence among multiple CR networks are not considered. A broad survey on resource allocation in cellular networks using graph coloring mechanisms can be found in [8], [11] and in the references therein. However, most of these works do not consider the dynamic availability of spectrum bands due to the presence of primary users and thus can not be directly applied to IEEE 802.22 network spectrum sharing. In [13], the dynamic channel allocation problem is formulated as graph coloring problem where dynamic channel availability is observed by the secondary users. In [14], spectrum allocation and scheduling problems are studied jointly in cognitive radio wireless networks with the objectives of achieving fair spectrum sharing. However, all channel divisions are treated equally here. In [7], a distributed, real-time spectrum sharing protocol called On-Demand Spectrum Contention (ODSC) is proposed that employs interactive MAC messaging among the coexisting 802.22 cells. However, control signaling is greatly increased through extensive MAC messaging.

## III. GAME FORMULATION

In this section, we formulate the self-coexistence problem as a dynamic channel<sup>1</sup> switching game. We assume that a set of IEEE 802.22 base stations in a region *compete* for one of the M > 1 orthogonal spectrum bands not used by primary incumbents. The respective cells for each of the bases stations can be partially or completely overlapped with each other. We refer to such overlapping cells as interfering cells and assume that they can not use the same spectrum band; otherwise QoS of the users of all interfering cells may suffer.

Note that a base station needs to compete only with the neighboring base stations for which there is some overlap<sup>2</sup> for occupying a channel void of interference. If two base stations are more than one hop away from each other, then it is possible for them to occupy the same spectrum band, and thus cannot be said to be competing with each other for occupying a channel [10]. Thus, we consider a base station with its corresponding cell as a rational player and define a set of opponents for each player. The opponent set consists of the the interfering neighbors of the palyer under question. In analyzing the problem, without loss of generality, we can focus our attention on a particular base station with is cell, and analyze the game from its perspective. The same reasoning would apply for other base stations as well.

Let us consider an arbitrary base station and call it player 0 and let N be the number of its neighbors. Also let  $N' (\leq N)$  be the number of neighbors with whom player 0 is interfering (opponents of player 0), numbered as player 1, player 2 through player N'. Thus the player set corresponding to the base station under consideration is comprised of players  $0, 1, \dots, N'$ .

We investigate the dynamic channel switching game, where the aim of each base station is to capture a spectrum band free of interference from its neighbors, from both pure and mixed strategy perspectives. At the beginning of the game, each base station dynamically chooses one of the M allowable spectrum bands for its operations. If two or more overlapped base stations choose the same spectrum band, then interference will occur and their transmissions will fail. Thus each base station has to pay a price when it experiences interference from its adjoining base stations. Let this cost be  $C_I$ . Each of these base stations will then have to take decisions regarding whether to stick with the band they have acquired or to switch to a new band in the next stage of the game. When a base station switches to a new channel, it will have to reallocate the new spectrum among its users. This also entails a cost. Let this cost be  $C_S$ . We assume that  $C_S < C_I$ , otherwise it does not make sense for

<sup>&</sup>lt;sup>1</sup>Throughout this paper, we use the words "channel" and "band" interchangeably unless explicitly mentioned otherwise.

<sup>&</sup>lt;sup>2</sup>Note that, two or more neighboring base stations can operate successfully using different channels.

a base station to consider switching to a new channel when facing interference.

Next we discuss the two strategies that can be adopted by a base station, viz, the *stay* strategy and the *switch* strategy in their quest to find a clear spectrum band free of interference from its neighbors.

## A. The stage game

If interfered at any stage of the game, player i has the binary strategy set of *switching* to another band (expecting to find a free spectrum band) or *staying* on the current band (assuming the interferers will move away). Using game theoretic notation, the binary strategy set for player i can be represented as:

$$S_i = \{ \text{switch}, \text{stay} \} \tag{1}$$

To generalize, let us assume that a base station (player 0) is interfering with N' of its neighbors (players 1 through N'). Let us assume the existence of the strategy set  $S_0, S_1 \cdots S'_N$ for the players  $0, 1, 2, \dots, N'$ . In this game, at every stage, if player 0 chooses strategy  $s_0 \in S_0$ , player 1 chooses strategy  $s_1 \in S_1$  and so on, then we can describe the set of strategies chosen by all N' + 1 players as one ordered (N'+1)-tuple,  $\mathbf{s} = \{s_0, s_1, \dots, s_{N'}\}$ . This vector of individual strategies is called a strategy profile (or sometimes a strategy combination). For every different combination of individual choices of strategies, we would get a different strategy profile s. The set of all such strategy profiles is called the space of strategy profiles **S**. It is simply the cartesian product of the vectors S for each base station which can be written as:  $\mathbf{S} = S_0 \times S_1 \times \cdots \times S_{N'}$ .

Thus we can describe our stage game by the tuple G = (P, S, C), i.e, by a player set P, where  $P = \{0, 1, \dots, N'\}$ , a space of strategy profiles S, where  $S = S_0 \times S_1 \times \dots \times S_{N'}$  and a vector C of von Neumann-Morgenstern utility functions defined over S.

## B. The Repeated Game

When a base station is in interference with a subset of its neighbors, it is possible that the base station does not find a clear band in a single play of the stage game G described above. This leads us to the notion of repeated play of the stage game G. We assume that the cost incurred by each player from the repeated game is the sum of the cost incurred by the player in each play of the stage game. Since each base station would like to find a clear channel in a minimal amount of time incurring as little cost as possible, our objective is to *minimize* the cost.

Two statements are implicit when we say that in each period we are playing the stage game G:

• For each player, the *set of strategies* available to him in any period in the game G is the same regardless of which period it is and regardless of what actions have taken place in the past.

• The payoffs to the players at any stage depend only on the action profile for G which was played in that period, and this stage-game payoff to a player for a given action profile for G is independent of which period it is played.

Before we elaborate on the repeated game strategies, let us first define some notations in the context of the repeated game. We will refer to the strategy of the stage game Gwhich player *i* executes in period *t* as  $s_i^t$ . The strategy profile played in period *t* is just the N' + 1-tuple of each players stage-game strategies:

$$s^{t} = (s_{0}^{t}, s_{1}^{t}, \cdots, s_{N'}^{t})$$
<sup>(2)</sup>

In order to be able to condition the players' stage-game action choices in later periods upon actions taken earlier by other players we need to define the concept of a history as a description of all the actions taken till the previous period. Formally, the history at time t can be written as:

$$h^{t} = (s^{0}, s^{1}, \cdots, s^{t-1}) \tag{3}$$

In other words, the history at time t specifies which stagegame action profile (i.e., combination of individual stagegame actions) was played in each previous period.

Let player i play the game for T stages. Then we can write players i's strategy for the repeated game as:

$$s_i = (s_i^0, s_i^1, \cdots, s_i^T) \tag{4}$$

We refer to the strategy profile s for the repeated game as the following N' + 1 tuple profile of players' repeated game strategies:

$$s = (s_0, s_1, \cdots, s_{N'}) \tag{5}$$

1) Repeated Game Solution Approach: Whenever we consider a repeated game, we need to define for how long the game will be played. But before we do that we make note of the following two important observations about the game at hand:

- For a given number of channels M, even if a base station (say n) finds a clear channel at a stage t, it might not correspond to the final assignment for base station n. This is because some of its neighbors might still be experiencing interference, because of which they might choose the same channel that base station n is currently residing on at a later stage.
- 2) The number of available channels M may vary over time since the spectrum usage is time and space variant.

From these two observations, it is clear that a definite ending time (or criteria) for the repeated game can not be inferred. Thus, when a base station (say n) faces interference from its neighbors, the decision that base station n takes can not be conditioned by foreseeing future events. It can only be influenced by the past actions taken by its opponents

(neighboring base stations). Hence, the best that base station n can do, when facing interference, is to play a strategy which is a best response, given base station n's beliefs about the strategies of its opponents, such that its expected cost of finding a clear channel is minimized in the current stage. Since the opponents of base station n also follow this same line of reasoning, the strategies played by the base stations constitute a Nash Equilibrium at every stage of the game. We now consider the following important result:

**Theorem 1:** A sequence of stage-game Nash Equilibrium strategy profiles is also a Nash Equilibrium in the (possibly infinitely) repeated game. More formally, let  $\bar{s} = (\bar{s}_0, \bar{s}_1, \dots, \bar{s}_{N'})$  be a strategy profile for the repeated game. If each of the stage-game strategy profiles  $\bar{s}_i$  of  $\bar{s}$  is a Nash Equilibrium strategy profile for the stage game, then  $\bar{s}$  is also a Nash Equilibrium for the repeated game.

**Proof:** We will prove this by contradiction. Let us assume that  $\bar{s}$  is not a Nash Equilibrium for the repeated game. Then for some player i there is an alternative repeated game strategy  $\hat{s}_i = (\hat{s}_i^0, \dots, \hat{s}_i^T)$  which is different, in at least one period, from his part  $\bar{s}_i = (\bar{s}_i^0, \dots, \bar{s}_i^T)$  of  $\bar{s}$  such that his payoff in the repeated game is higher, given that everyone else plays their parts of  $\bar{s}$ . In order that his repeated-game payoff be higher, there must be at least one period t in which his stage-game payoff using  $\hat{s}_i^t$  is higher than the stage-game payoff he would get from playing  $\bar{s}_i^t$ . But if that is true, then  $\bar{s}_i^t$  could not have been a part of a Nash-equilibrium strategy profile of the stage game, because some other strategy for player i, viz.  $\hat{s}_i^t$ , would have been better for i. This contradicts the hypothesis that every component of  $\bar{s}$  is a Nash equilibrium of the stage game.

## IV. GAME ANALYSIS

With the strategy set and costs defined, the optimization problem is to find a mechanism of switching or staying such that cost incurred can be minimized and an equilibrium can be achieved. We typically assume all the players are rational and pick their strategy keeping only individual cost minimization policy in mind at *every stage of the game*. In this section we analyze a single repetition of the stage game 'G'. We intend to find if there is a set of strategies with the property that no base station can benefit by changing its strategy unilaterally while the other base stations keep their strategies unchanged (Nash equilibrium [12]).

## A. Exploring Pure Strategy Space

We start with the *pure strategy space* played by all the base stations. To simplify investigation of Nash equilibrium with pure strategy space, we consider the game with two players i and j coexisting on one band. The game is represented in strategic form in Table I. Each cell of the table corresponds to a possible combination of the strategies of both players and contains a pair representing the costs of players i and j, respectively. Recall that  $C_S$  is the cost incurred by a base station when it switches to a new band and  $C_I$  in the price paid by a base station experiencing interference in terms of its reduced QoS. Also  $C_S < C_I$ , as explained before.

$i \setminus j$	Switch	Stay	
Switch	$(C_S, C_S)$	$(\mathbf{C_S}, 0)$	
Stay	$(0, \mathbf{C_S})$	$(C_I, C_I)$	
Table I			
PAYOFF MA	TRIX FOR PLA	YERS <i>i</i> AND	

As is evident from the table, this game has two pure strategy Nash equilibriums – one corresponding to the strategy profile (*switch*, *stay*) with player *i* choosing to *switch* and player *j* opting to *stay* and the other corresponding to (*stay*, *switch*) with player *i* choosing to *stay* and player *j* opting to *switch*. These two cases have been shown in boldface in the table. In both of these cases neither player can reduce his cost by playing a different strategy if the other player plays his part.

However, in reality, neither player can ascertain the strategy to be played by the other player. This inhibits each player from playing a pure strategy. For instance, consider the equilibrium (*switch*, *stay*). Player *i* would *always* choose to *switch* if and only if he knew for sure player *j* would *always* choose *stay* and vice versa. Similar argument also applies for the other equilibrium, (*stay*, *switch*). In practise, all a player can do is to develop a *belief* about the strategy of the other player by forming a probability distribution over the strategies of his opponent based on history of the past stages of the game and act accordingly. This leads us to investigate the mixed strategy solution space of the game.

#### B. Exploring Mixed Strategy Space

In the mixed strategy space for the base stations we assign probabilities to each of the strategies in the binary strategy space. We define the mixed strategy space of player i as:

$$S_i^{mixed} = \{(\text{switch} = p_i), (\text{stay} = (1 - p_i))\}$$
 (6)

where, player *i* chooses the strategy "switch" with probability  $p_i$  and chooses the strategy "stay" with probability  $(1 - p_i)$ .

Let us first show the 2-player case (two base stations residing on the same band) before we generalize to a N-player case.

1) Mixed Strategy with 2 players: The 2-player game is represented in strategic form in Table II, with the corresponding mixed strategy probabilities shown.

$i \backslash j$	Switch $(p_j)$	Stay $(1-p_j)$		
Switch $(p_i)$	$(C_S, C_S)$	$(\mathbf{C_S}, 0)$		
Stay $(1-p_i)$	$(0, \mathbf{C_S})$	$(C_I, C_I)$		
Table II				

PAYOFF MATRIX FOR PLAYERS i and j

In order to find the mixed strategy equilibria, we need to first find each player's best response correspondence. Player *i*'s best-response correspondence specifies, for each mixed strategy  $p_j$  played by player *j*, the set of mixed

strategies  $p_i$  which are best responses for player i, i.e, it is a correspondence  $p_i^*$  which associates with every  $p_j \in [0, 1]$ a set  $p_i^*(p_j) \subset [0, 1]$  such that every element of  $p_i^*(p_j)$  is a best response by player i to j's choice  $p_j$ . The graph of  $p_i^*$  is the set of points:  $\{(p_i, p_j) : p_j \in [0, 1], p_i \in p_i^*(p_j)\}$ Similarly the graph of  $p_j^*$  is the set of points:  $\{(p_i, p_j) : p_i \in [0, 1], p_j \in p_i^*(p_i)\}$ .

To find *i*'s best-response correspondence we first compute his expected payoff for the stage game for an arbitrary mixed-strategy profile  $(p_i, p_j)$  by weighting each of *i*'s purestrategy profile payoffs by the probability of that profile's occurrence as determined by the mixed-strategy profile  $(p_i, p_j)$ (see Table II):

$$E_i[C] = p_i p_j \frac{M-2}{M-1} C_S + p_i p_j \frac{1}{M-1} (C_S + C_I) + p_i (1-p_j) C_S + (1-p_i) p_j 0 + (1-p_i) (1-p_j) C_I$$
(7)

Note that the first two terms in equation (7) both correspond to the strategy profile (*switch*, *switch*). This is because when both players *i* and *j* chooses to *switch*, there can be two possible associated costs – (i) *i* and *j* choose different channels (from M - 1 possible channels) after switching. The probability of *i* and *j* choosing different channels after switching is (M - 2)/(M - 1) and the cost associated for each player is  $C_S$ . (ii) *i* and *j* choose the same channel (from M - 1 possible channels) after switching. The probability of *i* and *j* choosing the same channel after switching is 1/(M-1). The cost associated for each player is  $(C_S + C_I)$ since even after switching they still interfere each other.

Player *i*'s minimization problem is:

$$min_{p_i \in [0,1]} E_i[C] \tag{8}$$

Since  $p_i$  is *i*'s choice variable, it will be convenient to rewrite equation (7) as an affine function of  $p_i$ . Simplifying equation (7) we get:

$$E_{i}[C] = p_{i} \left[ \frac{M}{M-1} p_{j}C_{I} + (C_{S} - C_{I}) \right] + [C_{I} - p_{j}C_{I}]$$
  
=  $p_{i}\delta(p_{j}) + [C_{I} - p_{j}C_{I}]$  (9)

where,  $\delta(p_j) = \frac{M}{M-1} p_j C_I + (C_S - C_I).$ 

For a given  $p_j$ , the expected cost function  $E_i[C]$  will be minimized with respect to  $p_i$  at either:

- 1) The unit interval's right endpoint (viz.  $p_i = 1$ ) if  $\delta(p_j)$  is negative, or,
- 2) The unit interval's left endpoint (viz.  $p_i = 0$ ) if  $\delta(p_j)$  is positive, or,
- 3) For every  $p_i \in [0, 1]$  if  $\delta(p_j)$  is zero, because  $E_i[C]$  is then constant with respect to  $p_i$

Let  $\delta(p_j)$  be zero at  $p_j^*$ . To find  $p_j^*$ , let us equate  $\delta(p_j^*)$  to zero:  $\frac{M}{M-1}p_j^*C_I + C_S - C_I = 0$ . Solving for  $p_j^*$  we get:

$$p_j^* = \left(1 - \frac{C_S}{C_I}\right) \left(1 - \frac{1}{M}\right) \tag{10}$$



Figure 1. a) Player i's best response for the game; b) Player j's best response for the game.



Figure 2. The players best response correspondences and the Nash Equilibrium set.

Now, since we considered that  $C_S < C_I$  and M > 1, thus  $p_j^*$  lies in the interval (0, 1). Since  $\delta(p_j)$  is an increasing function in  $p_j$ , player *i* will choose the pure strategy  $p_i = 1$  against  $p_j$ 's on the interval  $[0, p_j^*)$  and the pure strategy  $p_i = 0$  against  $p_j$ 's on the interval  $(p_j^*, 1]$ . Against  $p_j = p_j^*$ , player *i* is free to choose any mixing probability. Player *i*'s best response correspondence is shown in Figure 1(a).

Similarly, player *j*'s expected payoff function is:

$$E_{j}[C] = p_{j} \left[ \frac{M}{M-1} p_{i}C_{I} + (C_{S} - C_{I}) \right] + [C_{I} - p_{i}C_{I}]$$
  
=  $p_{j}\delta(p_{i}) + [C_{I} - p_{i}C_{I}]$  (11)

Let  $\delta(p_i)$  be 0 at  $p_i^*$ . Then by equating  $\delta(p_i^*)$  to 0 we get:

$$p_i^* = \left(1 - \frac{C_S}{C_I}\right) \left(1 - \frac{1}{M}\right) \tag{12}$$

which again lies in the interval (0,1). Since  $\delta(p_i)$  is an increasing function in  $p_i$ , player j will choose the pure strategy  $p_j = 1$  against  $p_i$ 's on the interval  $[0, p_i^*)$  and the pure strategy  $p_j = 0$  against  $p_i$ 's on the interval  $(p_i^*, 1]$ . Against  $p_i = p_i^*$ , player j is free to choose any mixing probability. Player j's best response correspondence is shown in Figure 1(b).

The Nash Equilibria are the intersection points in the graphs of player i's and j's best-response correspondences. This comes directly from the fact that a mixed strategy profile  $\{\bar{p}_i, \bar{p}_j\}$  is a Nash Equilibrium if and only if  $\bar{p}_i$  is a best response by player i to player j's choice of  $\bar{p}_j$  and also  $\bar{p}_j$  is a best response by player j to i's choice of  $\bar{p}_i$ . Figure 2

shows the intersection of the graphs of player *i*'s and *j*'s best response correspondence. We see that the intersection of the graphs of the two best response correspondences contains exactly three points, each corresponding to mixed strategy profile  $(p_i, p_j)$ : (stay,switch),  $(p_i^*, p_j^*)$  and (switch,stay). The first and last of these correspond to the two pure strategy Nash equilibria we identified earlier in Section IV-A. The additional one corresponds to strategy profile  $(p_i^*, p_j^*) = ((1 - \frac{C_S}{C_I})(1 - \frac{1}{M}), (1 - \frac{C_S}{C_I})(1 - \frac{1}{M})).$ 

Note that, at any stage game, player i does not know what player j will choose  $p_j$  to be. Thus what player i does is, based on history, estimates the value of  $p_j$ . For example, if iobservers j switches 7 out of 10 times, then  $p_j$  is estimated to be 0.7. Based on this, it takes an appropriate decision depending on where in the interval  $((0, p_j^*], p_j^* \text{ or } (p_j^*, 1])$ the estimated value of  $p_j$  lies. Likewise, player j on his part also estimates the value of  $p_i$  and takes his decision regarding whether to switch or not. Thus both players i and j play their best response against each other, based on their beliefs of what the other player's strategy is.

2) Mixed Strategy with N players: So far, we have shown the equilibrium for the 2-player case. Now, let us provide some insights on finding the equilibrium for the N-player game. Let us consider an arbitrary base station and call it player 0. Let player 0 be interfering with N' of its neighboring base stations, numbered 1 through N'.

Let p be the 'switching' probability of each one of player 0's N' opponents (i.e., base stations which are interfering with 0). Similar to the arguments for the 2-player case, for player 0 to be indifferent between choosing its two pure strategies, the expected cost when it chooses to 'switch' must be same with that of when it chooses to 'stay'. When player 0 chooses to 'switch', the strategies taken by the opponents of player 0 can result in any  $k \in [0, N']$  opponents of player 0 choosing to 'switch'. Let this cost be  $E[c_0^{switch}]$ . Similarly, when player 0 chooses to 'stay', the strategies taken by the opponents of player 0 can again result in any  $k \in [0, N']$  opponents of player 0 choosing to 'switch'. Let this cost be  $E[c_0^{stay}]$ . As discussed above,  $E[c_0^{switch}]$  and  $E[c_0^{stay}]$  must be same for player 0 to randomize between his two pure strategies. Thus, player 0's mixed strategy NE probability of switching will correspond to the roots of the equation obtained by equating  $E[c_0^{switch}]$  to  $E[c_0^{stay}]$ . This equation can be easily solved by using numerical methods (e.g., bisection method).

#### V. CONCLUSIONS

In this paper, we use game theory to devise the strategies of cognitive radio based IEEE 802.22 networks such that multiple networks can co-exist even without any coordination among them. A base station's choice of either switching to a new channel or staying with its current channel been modeled as an 'infinitely repeating' game, where each player always believes that there is some chance the game will continue to the next period. The aim of each base station is to minimize its cost of finding a clear channel. Analysis of the game reveals that there exists a pure strategy Nash Equilibria. However, since we cannot assume coordination among the players before a play, implementing such a strategy in reality is infeasible. Thus, we explore the mixed strategy space of the game and propose a solution based on the same. In the mixed strategy space, each player takes an appropriate decision, based on the switching probabilities of its interfering players. The proposed mechanism is distributed in nature without the need for any negotiation messages, thus making the solution scalable.

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